The Chiral Magnetic Effect in Hot QCD
and
The Isobar run in 2018

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What are we doing in the 2018 Run?

STAR is proposing to run ^{96}Zr + ^{96}Zr collisions and ^{96}Ru + ^{96}Ru collisions at RHIC in 2018. 3 weeks each. Zr and Ru are isobars.

Isobars are nuclei with the same mass, nearly the same shape, but with different number of protons

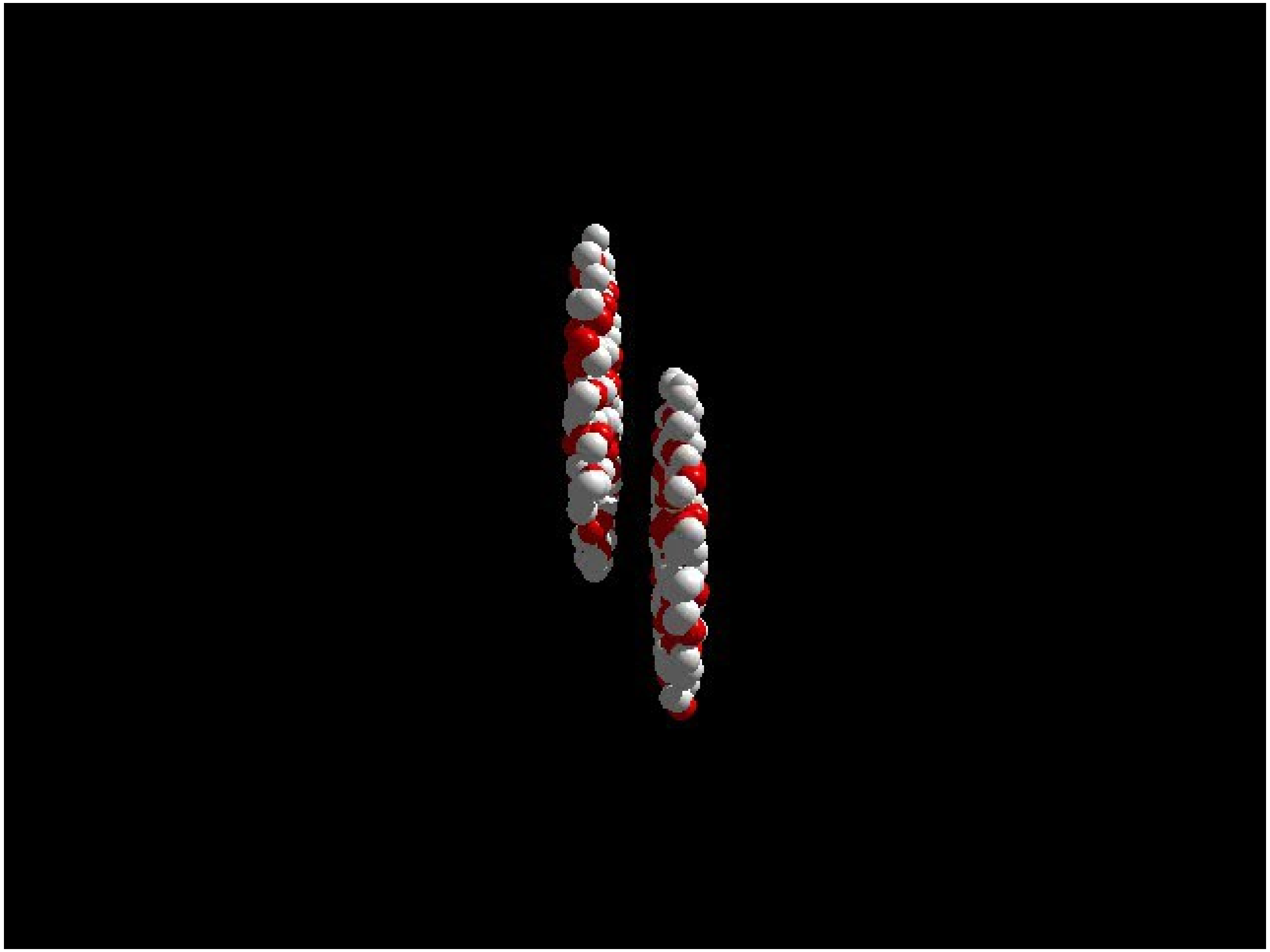
There exist 4 stable isobaric systems where the change in Z is large

Element	Z	Mass	Abundance
Zr	40	96	2.80%
Mo	42	96	16.68%
Ru	44	96	5.54%
Sn	50	124	5.79%
Te	52	124	4.74%
Xe	54	124	0.09%
Te	52	130	34.50%
Xe	54	130	4.07%
Ba	56	130	0.11%
Xe	54	136	8.86%
Ba	56	136	7.85%
Ce	58	136	0.19%

$\Delta Z = 4$

All are stable isotopes and they make good beams in the accelerator

Charges in motion



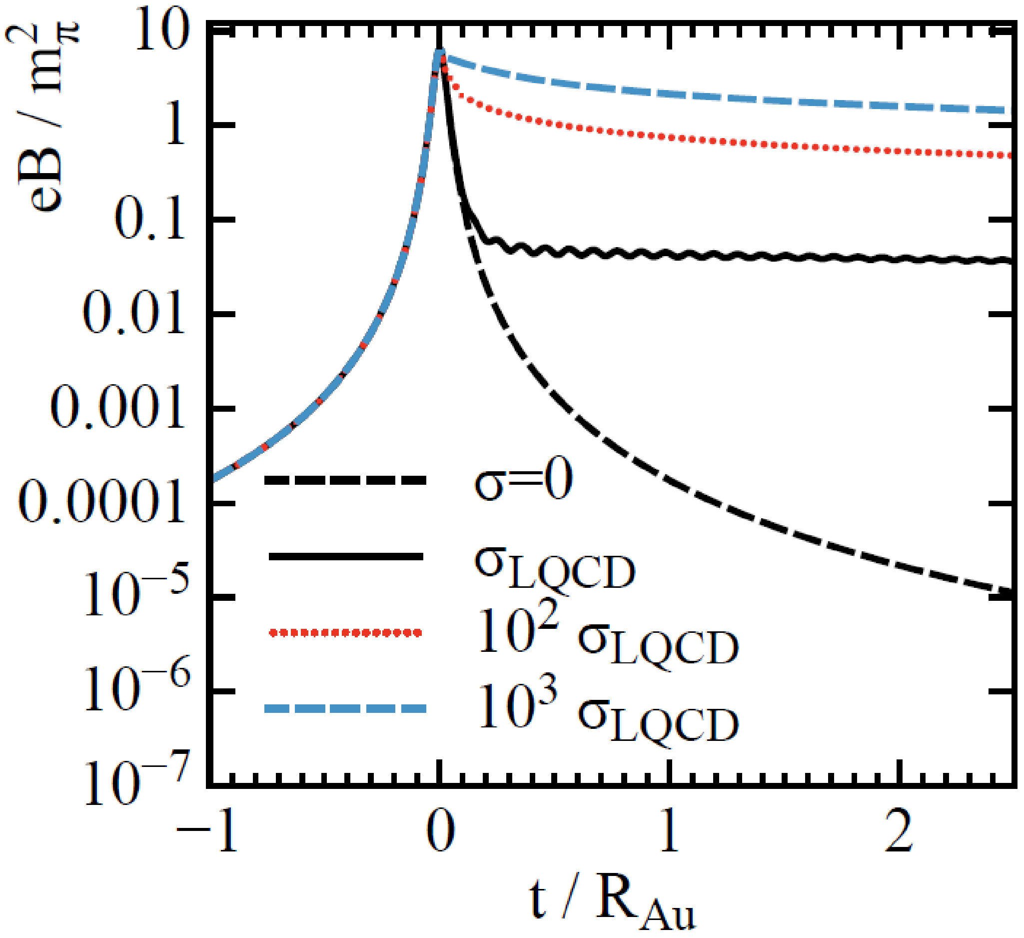
Charges in motion create strong magnetic fields

The B_{EM} field – 10^{18} gauss at the peak

- **The B field is strong and short duration due to the velocity of the passing ions**
  - **MRI uses 10^4 gauss**
  - **1000x MagnetoStar**

- **Magneto hydrodynamic effects in the QGP extend the lifetime of the B field**
  - **aka Lenz's Law**
  - **Finite conductivity**

- **Recent calculations suggest the lifetime is extended in a plasma but the magnitude is reduced x50 from the peak at the relevant time scale**

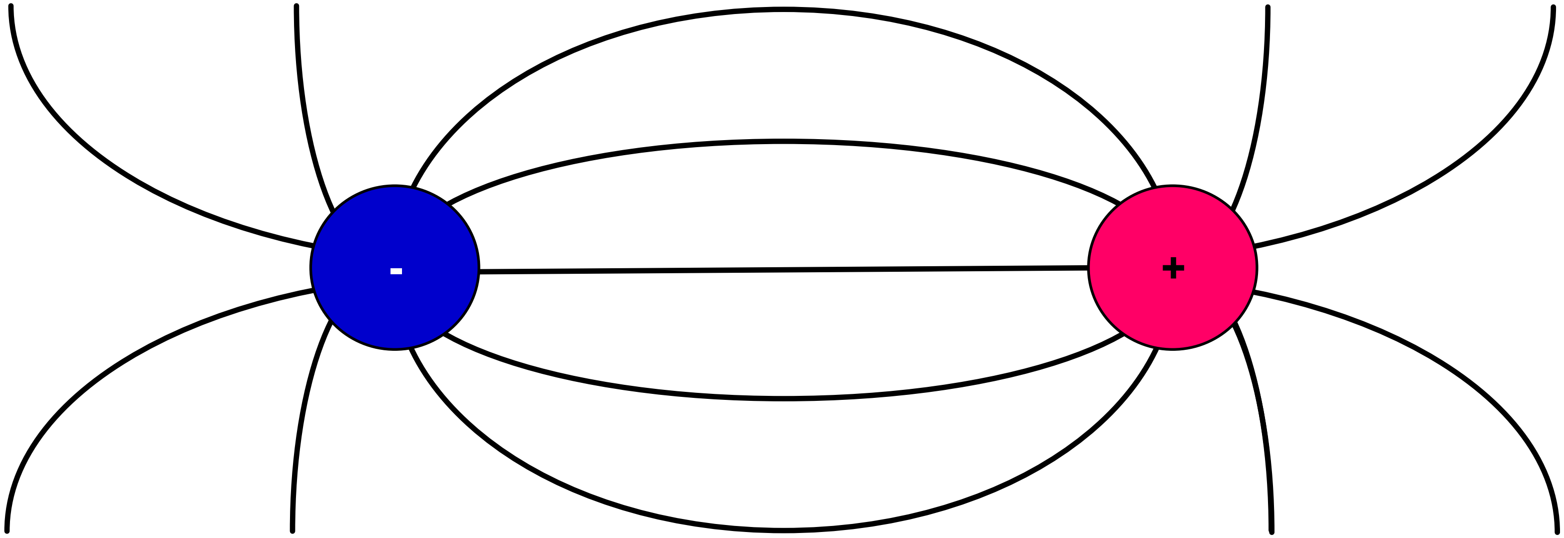


L. McLerran, V. Skokov, Nucl.Phys. A929 (2014) 814-190

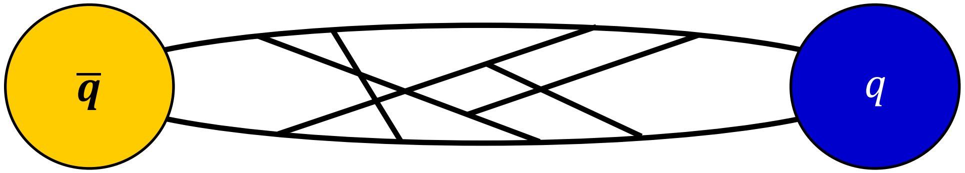
Wow! … but who cares?

- The Nucleus is a laboratory to study fundamental physics
  - Strong B_{EM} fields do useful work

Mnemonic for remembering the strong interaction



The E&M interaction is mediated by
a non-self interacting particle



The Strong interaction is mediated
by a self interacting particle

Gluons form flux tubes and other more complex topological configurations

Quark and Gluon interactions

- **The “Force Law” for quarks and gluons depends on whether the quarks are found in isolation or in a dense environment**
  - **In isolation: stretch a flux tube and it will get longer and eventually break. The longer the tube, the more energetic (massive) the system.**
  - **In a dense environment, short distance interactions become important. These short range interactions dissolve the flux tubes into a soup of q and g interactions: a quark gluon plasma**
  - **Quarks become “free” particles in a QGP and they also become (nearly) massless in a QGP due to Chiral symmetry restoration**
  - **This “freedom” is what makes Ultra Relativistic Heavy Ion Collisions interesting to the experimentalist and calculations possible for the theorist**

Chiral Symmetry

- **We expect Chiral symmetry restoration in a QGP**
  - **There is no direct experimental evidence for this, but it is not a controversial expectation**

- **What is Chiral Symmetry: Vector gauge theories with massless Dirac fermion fields Ψ exhibit chiral symmetry**
  - **Rotating the left-handed and the right-handed components of the wave function, independently, makes no difference to the theory**

$$\Psi_L \rightarrow e^{i\theta_L}\,\Psi_L \quad and \quad \Psi_R \rightarrow \Psi_R$$

or

$$\Psi_L \rightarrow \Psi_L \quad and \quad \Psi_R \rightarrow e^{i\theta_R}\,\Psi_R$$

  - **Massive fermions do not exhibit chiral symmetry, since the mass term in the Lagrangian, $m\,\overline{\Psi}\,\Psi$, breaks chiral symmetry explicitly**

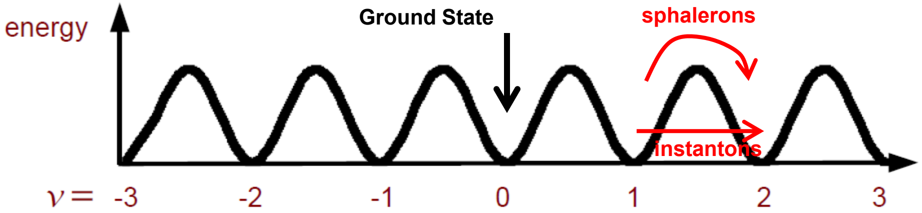
- **Helicity is identical to Chirality for massless particles**
  - **A particle has right handed helicity when the $\overline{p}$ vector is parallel to the spin**
  - **A particle has left handed helicity when the $\overline{p}$ vector is anti-parallel to spin**
  - **For a massive particle, there is always a Lorentz frame moving closer to the speed of light so that the $\overline{p}$ vector appears reversed in the new frame**

Gluon Fields in a QGP

- **Complex gluon field configurations are common in a QGP**
  - **Examples include: Links, knots, and *anti*-screening due to gluon loops**
  - **QCD supports many topologically distinct vacuum states**
    - **And some of these topologically distinct states violate P and CP**
      - **What? Doesn’t the strong interaction conserve Parity?**
      - **Parity is conserved in the strong interaction … but only in cold QCD.**

Hot QCD allows for metastable states … lots of them

QCD has an infinite number of vacua which can distinguished by a winding number $\nu = 0, \pm 1, \pm 2, ...$



$$Q_w = \frac{g^2}{8\pi^2} \int d^4x\, \vec{E}_a \cdot \vec{B}_a = 0, \pm 1, \pm 2, ...$$

In chiral limit (m=0): $[N_L - N_R]_{t=\infty} - [N_L - N_R]_{t=-\infty} = 2 N_f Q_w$

- **Moving from one vacuum state to another is the result of changing the topological charge of the system**
- **Topological charge flips helicity and thus counts the difference between the number of right and left handed quarks**
- **Topological charge changing transitions also violate local P and CP conservation**

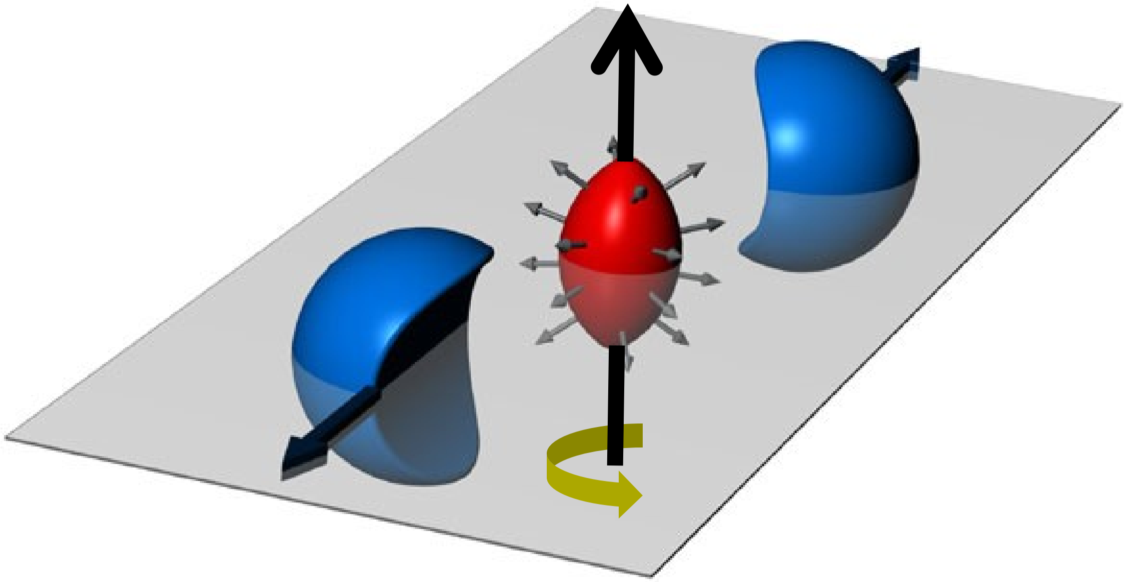
- **What every experimentalist likes to see in a theory publication …**
  - **"The consequences and magnitude of these effects are subject to experimental study and verification"**
    - **Kharzeev, McLerran, and Warringa arXiv:0711.0950 and Nucl. Phys. A803 (2008) 227.**

From a humble experimentalist's point of view … these theories appear to be fully vetted; CP and P violating domains almost certainly occur in ultra-relativistic HI collisions. The question is whether the effects are large enough to be observed …

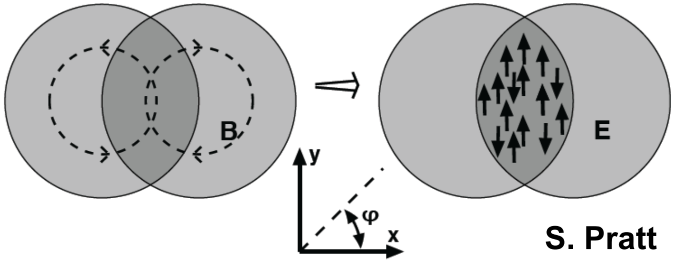
The Chiral Magnetic Effect (CME)

- Now that we have our tools in place, lets start discussing the Chiral Magnetic Effect
  - Theorists call it the Chiral Magnetic Effect. I am an experimentalist so call it Chiral Magnetic Engineering
- Three things must come together, simultaneously, to make the Chiral Magnetic Effect work
  - Strong magnetic fields (10^{15} - 10^{18} gauss!)
  - Chiral Symmetry restoration
  - Topological Charge changing transitions in hot QCD

A collision has Angular Momentum and a B field

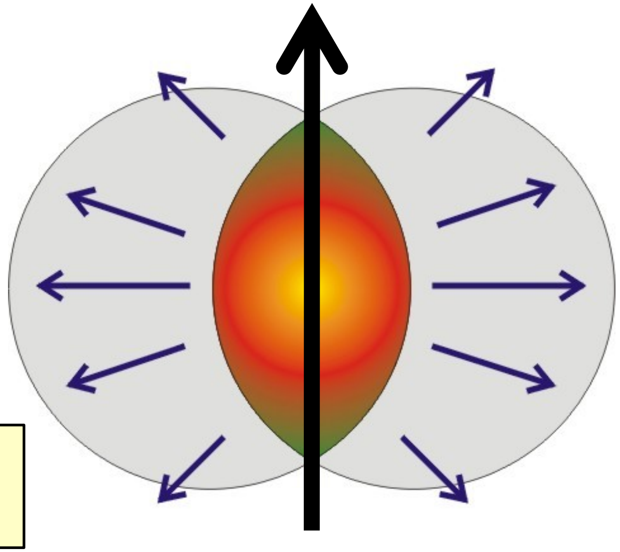


- Electromagnetic charges in motion create an E&M magnetic field (not a color magnetic field)



S. Pratt

- The magnetic fields can reach 10^{18} gauss. Stronger than on the surface of a neutron star.



- Quarks interact with the magnetic field via their spin and magnetic moment
- Quarks will align themselves parallel or anti-parallel to the B field direction
- The B field is a long range phenomena; it affects <u>every</u> quark in the de-confined system … every quark is spin-aligned

Strong fields, but also large amounts of angular momentum parallel to the B field axis (1000 $\hbar$)

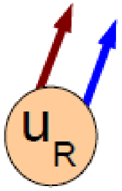
Quarks interact with the B field via their spin

- **Assume: chiral symmetry is restored in a QGP**
- **Assume: quark masses drop to ~0 after chiral symmetry restoration**

In chiral limit:

Particles/Antiparticles
with right-handed helicity

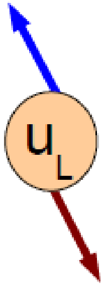
have spin and momentum parallel



In chiral limit:

Particles/Antiparticles
with left-handed helicity

have spin and momentum anti-parallel

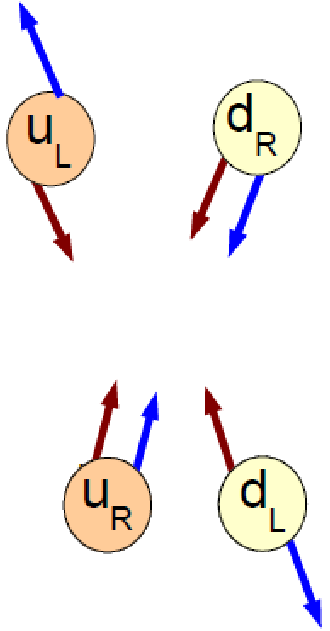


- **Chirality and helicity are the same for massless particles … so in the limit of zero mass, it is easy to define chirality (not so easy for non-zero mass)**
- **The QCD Lagrangian is chirally symmetric for massless particles**
  - **The pion is the Goldstone boson for Chiral symmetry**
  - **Chiral symmetry is spontaneously broken & gives mass to hadrons**

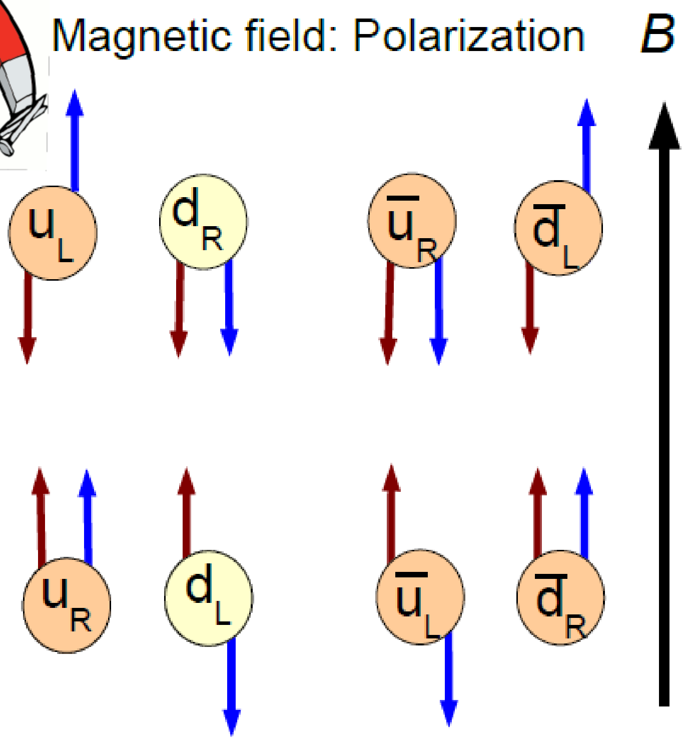
How does the B field affect the Quarks?

A magnetic field will align the spins, depending on their electric charge

No Magnetic Field: No polarization



Magnetic field: Polarization



The momenta of the quarks align along the magnetic field

A quark with right-handed helicity will have momentum opposite to a left-handed one

In this way the magnetic field can distinguish between right and left

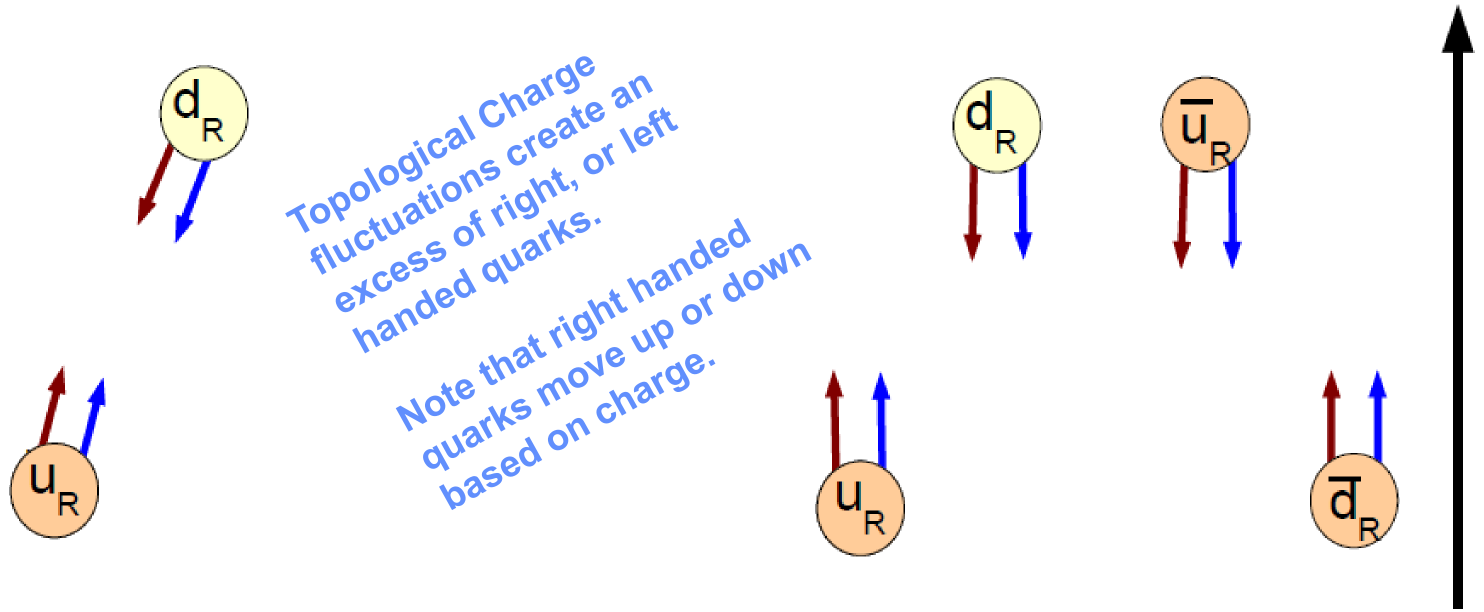
H. Warringa

Topological Charge flips chirality: L to R

A magnetic field will align the spins, depending on their electric charge

No Magnetic Field: No polarization

Magnetic field: Polarization



Topological Charge fluctuations create an excess of right, or left handed quarks.

Note that right handed quarks move up or down based on charge.

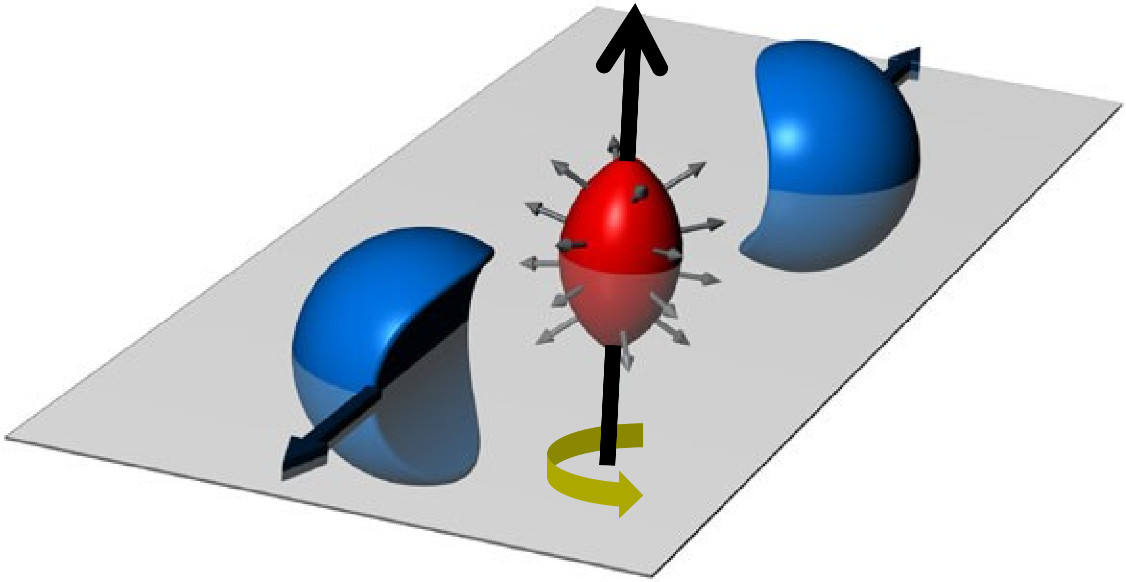
Positively charged particles move parallel the magnetic field

Negatively charged particles move to antiparallel to magnetic field

An electromagnetic current is created along the magnetic field

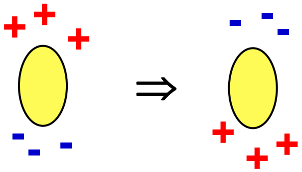
H. Warringa

Separation of Charge wrt the reaction plane

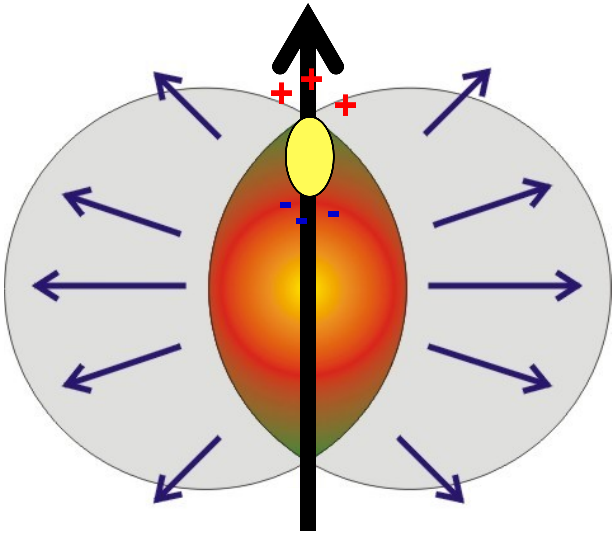


If a chirally restored bubble is created in a heavy ion collision, the positively charged quarks will go up … then hadronize … and yield an excess of positive pions above the plane

Unfortunately, it could be just the opposite in the next event depending on the topological charge in the bubble



- The signal is manifestly parity odd
  x ⇒ -x , p ⇒ -p
  but the observable will be even
- The charge-flow asymmetry is too small to be seen in a single event but may be observable with <u>correlation techniques</u>



Hypothesis Testing

- **Theoretical Expectations**
  - The Chiral Magnetic Effect will cause a separation of charge, above and below the reaction plane
    - If the CME effect occurs in HI collisions, then it would be evidence for local P and CP violation in the strong interaction
    - If observed, it is the smoking gun for chiral symmetry restoration

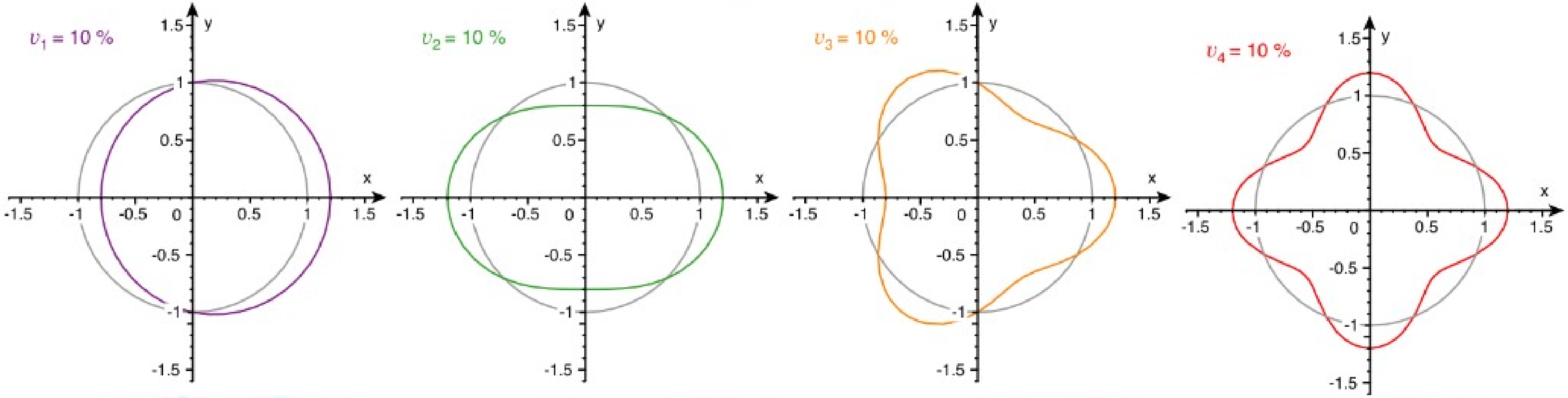
- **The Null Hypothesis**
  - Heavy Ion reactions do not cause a separation of charge, above and below the reaction plane

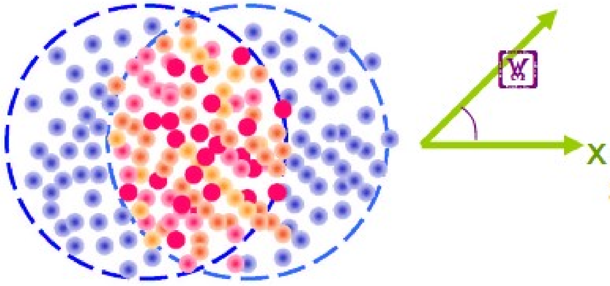
Our Tools: Anisotropic Flow

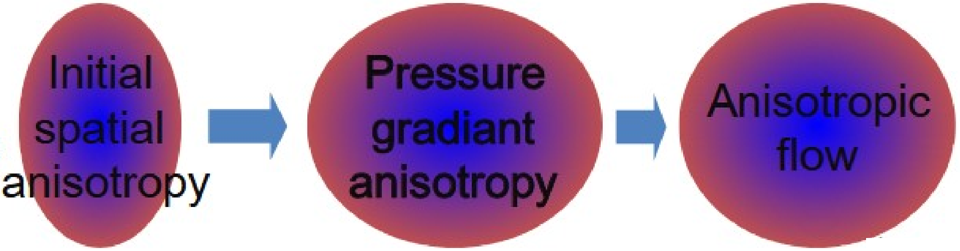
Anisotropic flow v_n

$$E\frac{d^3N}{d^3\vec{p}} = \frac{dN}{p_T dp_T d\varphi dy} = \frac{1}{2\pi}\frac{dN}{p_T dp_T dy}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T, y)\cos(n\varphi)\right]$$

Sine terms vanish because of the symmetry Φ → -Φ in A+A collisions







Initial spatial anisotropy

Pressure gradiant anisotropy

Anisotropic flow

Che-Ming Ko, Texas A&M University

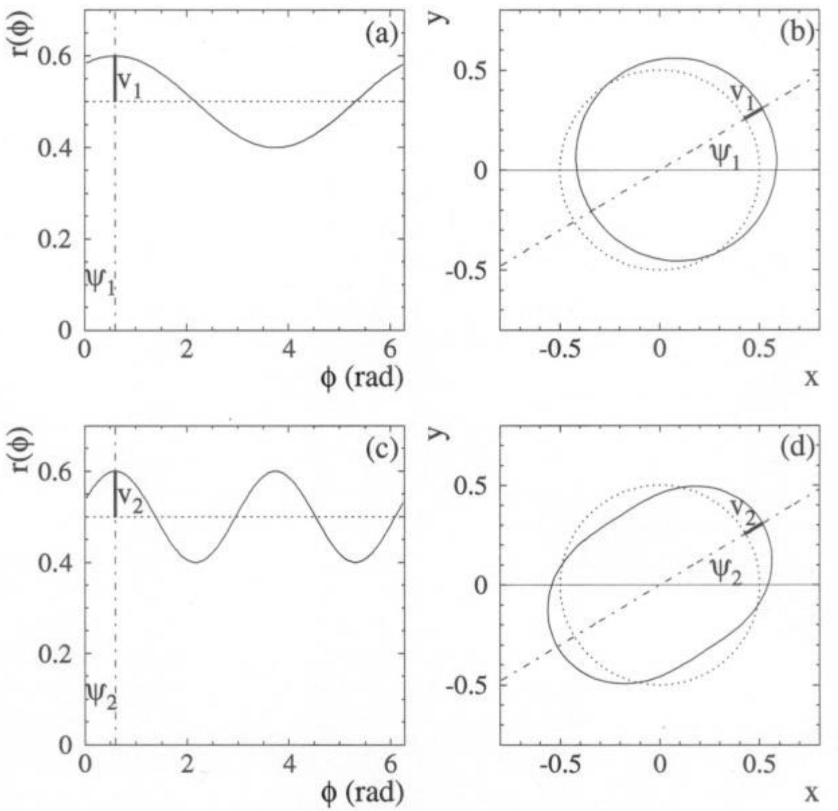
Interpreting Flow – order by order

n=1: Directed Flow has a period of 2π (only one maximum)

- v_1 measures whether the flow goes to the left or right – whether the momentum goes with or against a billiard ball like bounce. For collisions of identical nuclei, symmetry forces v_1 to be an odd function of η

n=2: Elliptic flow has a period of π (two maximums)

- v_2 represents the elliptical shape of the momentum distribution. It is an even function of η for identical nuclei



If parity is conserved, sin() terms drop out

$$E\frac{dN^3}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(\underset{\uparrow}{1} + \underset{\uparrow}{2a_1}\sin(\Delta\phi) + \underset{\uparrow}{2\mathrm{v}_1}\cos(\Delta\phi) + \underset{\uparrow}{2\mathrm{v}_2}\cos(2\Delta\phi) + \underset{\uparrow}{2\mathrm{v}_4}\cos(4\Delta\phi) + \mathrm{K}\right)$$

isotropic parity non-conserving directed elliptic higher order terms

The Experimental Observable

- **The coefficients for the Fourier expansion of the invariant yield are**

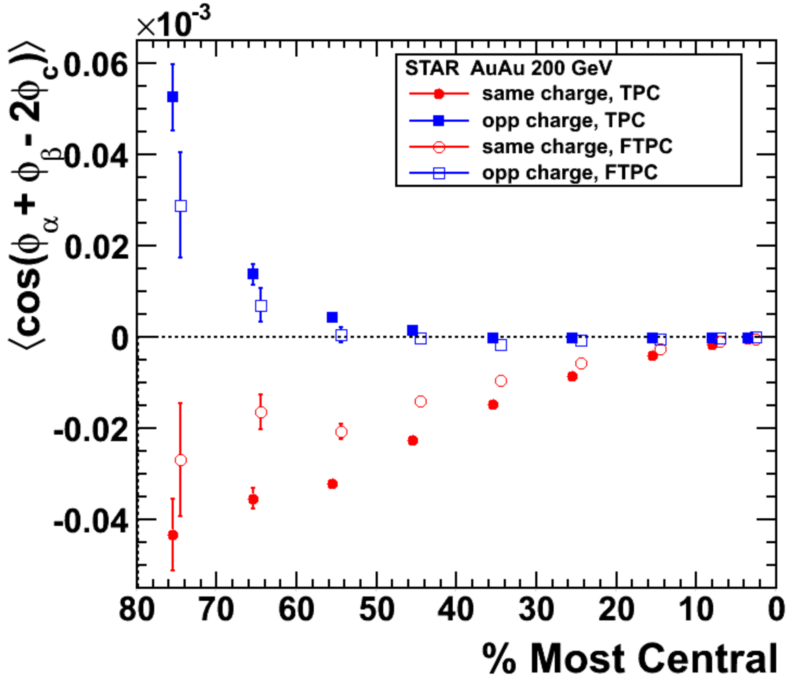
  $$v_n \equiv \langle \cos(n(\phi - \Psi_R)) \rangle \quad \text{or} \quad v_n^2 = \langle \cos(n(\phi_i - \phi_j)) \rangle$$

  - where the average is taken over all particles in the event and ψ_R is the known reaction plane angle (e.g. from a forward detector if we are using TPC data)
  - The second method is a true two particle correlation (many details left out)

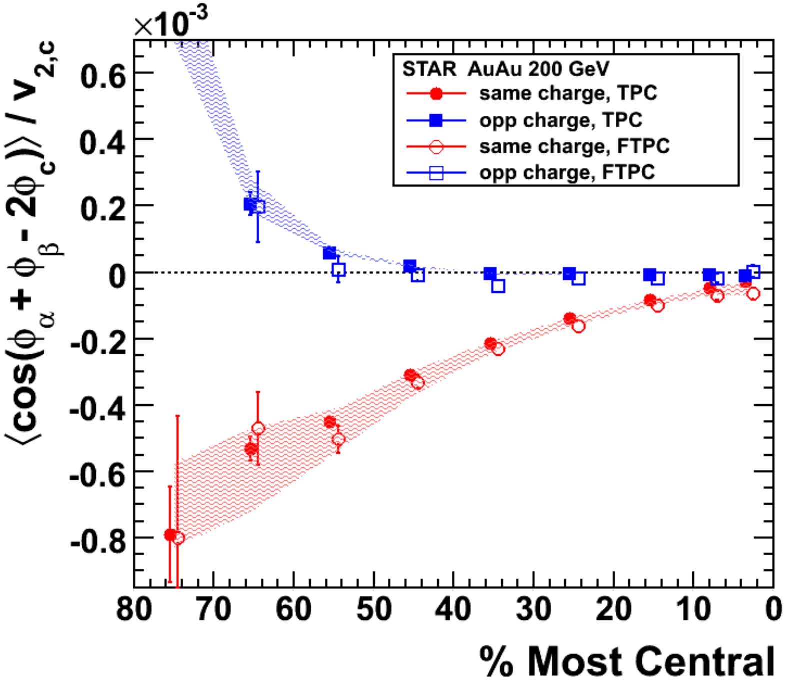
- **A clever observable: $\langle \cos(\phi_i + \phi_j - 2\phi_k) \rangle$** … a triple correlation
  - Mixed Harmonics: $\langle \cos(\phi_i - \phi_k)\cos(\phi_j - \phi_k) - \sin(\phi_i - \phi_k)\sin(\phi_j - \phi_k) \rangle = (v_1^2 - a_1^2)\, v_2$
  - Measure $(v_1^2 - a_1^2) * v_2$ because v_2 is large and it amplifies the parity non-conserving signal, a_1, while preserving reasonable statistical errors.
  - The signal is parity odd, but the observable $(v_1^2 - a_1^2) * v_2$ is even. Best way to measure charge sensitive flow because $v_1 \Rightarrow 0$ and $\underline{(v_1^2 - a_1^2) * v_2 \;\Rightarrow\; -a_1^2 * v_2}$

- **Under certain assumptions v_1 is directed flow**
  - Note that a 'normal' v_1 measurement for pions in a Au-Au reaction has an intrinsic symmetry that suggests weighting by sign(η)
  - **Don't do this for CME work:** We are looking for charge flow that goes up/down so choose to do the sum without sign(η) weighting and thus the 'normal' v_1 will cancel out. (See next bullet). This assumes symmetric η acceptance.

Like Sign and Opposite Sign Correlations



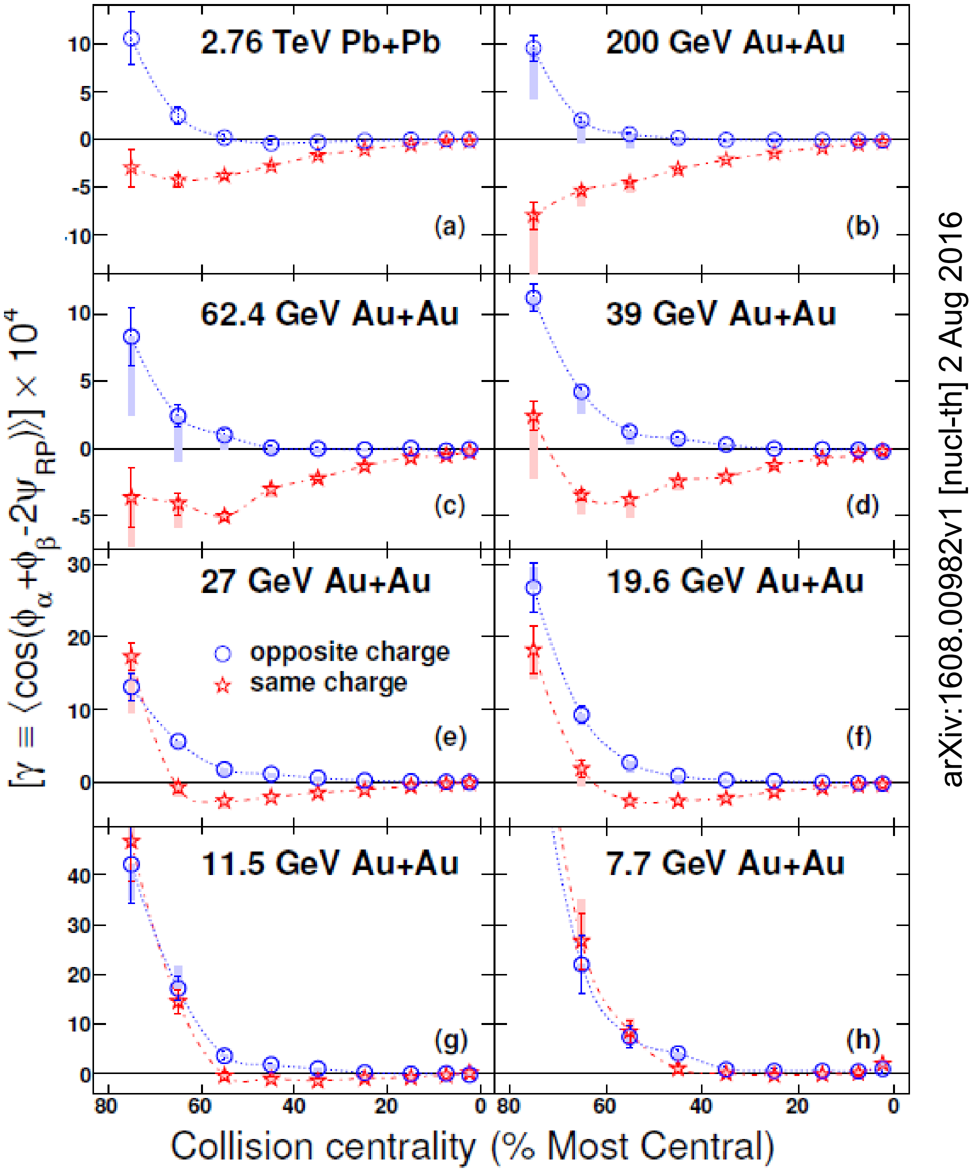
Abelev et al. (STAR Collaboration), PRL 103, 251601 (2009)



Abelev et al. (STAR Collaboration), PRC, 81, 054908 (2010)

- **A charge separation signal appears in the data; independent of how we determine the reaction plane with different estimates of Ψ_R (i.e. ϕ_c)**
  - **Signal is present if Ψ_R is found with the TPC, FTPC, or even ZDC.**
  - **Systematic errors in panel II, above, cover the range introduced by using $v_2\{2\}$ or $v_2\{4\}$ in the calculation**
- $\left\langle \cos(\phi_i + \phi_j - 2\phi_k) \right\rangle / \mathrm{v}_{2,c} \approx -1 * \left\langle a_{1,\alpha}\, a_{1,\beta} \right\rangle$ **and so is a candidate CME signal**
  - **Same sign $a_{i,\gamma}$ flow is negative … Opposite sign $a_{i,\gamma}$ flow is positive**

ALICE, STAR and the Beam Energy Scan



- **Significant charge separation observed at all but the lowest energies**
- **$\gamma_{sc} < 0$ and $\gamma_{oc} > 0$ as expected**
- **The data demonstrates importance of background contributions**
- **CME - suppression of signal at low energies due to a short lifetime for the QGP or perhaps even a phase transition**

The Background Hypothesis

- Charge separation in HI collisions has definitely been observed. This falsifies the null hypothesis and is consistent with the expectations for the CME over a wide range of conditions

- **Background Hypothesis:** The observed charge separation is due to non-CME background effects
  - If a comprehensive background model can be found that explains all the data then this would rule out the CME
  - This needs to be investigated but is much harder to falsify

- Many investigations have been started and the literature is full of proposed models that explain some of the data
  - But to the best of my knowledge, no model has been proposed that can accurately explain all of the charges separated data

Background – a few words

- **Structure of correlator allows control of a wide class of backgrounds**

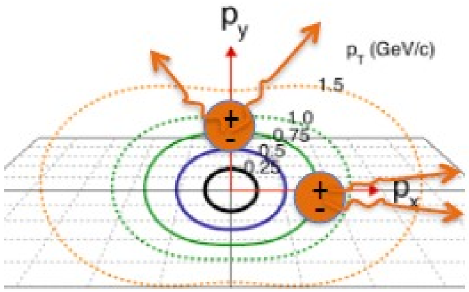
$$\left\langle \cos(\phi_i - \phi_k)\ \cos(\phi_j - \phi_k) \right\rangle - \left\langle \sin(\phi_i - \phi_k)\ \sin(\phi_j - \phi_k) \right\rangle = \left(\left(\mathrm{v}_1^2 + B_{in} \right) - \left(a_1^2 + B_{out} \right) \right) \mathrm{v}_2$$

- **As previously noted, the magnitude of v_1 is small and the directed flow terms sum to zero due to our choice to *not* weight the sum by sign(η)**

- **The correlator represents the difference between correlations projected onto an axis in the reaction plan and onto an axis perpendicular to the reaction plane**
  - **This removes correlations among particles that are not related to the reaction plane orientation**

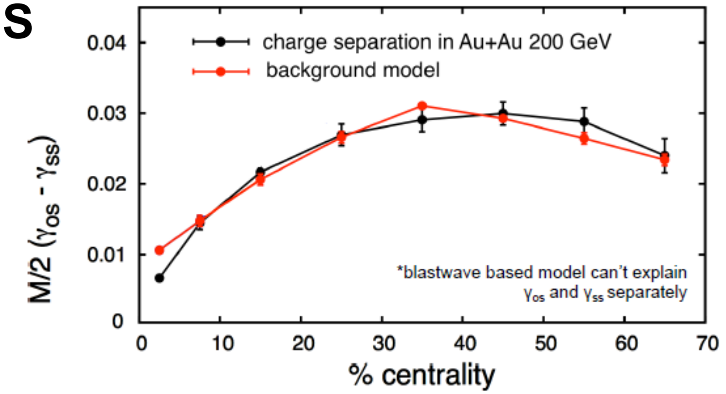
- **A source of background that may persist in the data are particles from a cluster (resonance decay or jet) where the cluster is flowing with respect to the reaction plane**

Flow boost collimates pairs more strongly in-plane than out-of-plane; potentially leading to false correlations.



Serious Challenges to the CME interpretation

- **Two particle decays, such as resonance decays, are background**
  - Back to back decays look like charge separated events, but not correlated to the reaction plane. The γ observable should take care of most of the resonance decays … but perhaps not all of them
  - Charge and momentum conservation, which are required in each event, can be thought of as 2 particle decays that separate charges in each event. Unfortunately, suitable tuning of hydro models can describe the charge separated data for (γ_{oc} - γ_{sc}) but not for γ_{oc} and/or γ_{sc}, separately.
- **The signal (γ_{oc} - γ_{sc}) should be independent of v_2 .   It isn't.**
  - The experimentally observed signal decreases linearly as $v_2 \Rightarrow 0$
  - Understanding this observation is difficult because v_2 is a positive definite quantity and it does not average to zero for central collisions w/ fluctuations
- **The most recent challenge comes from CMS**
  - In an about to published paper, they report Pb-Pb and p-Pb data that shows similar results for both datasets
  - Expect to extinguish the QGP in p-Pb
  - Expect to disorient the direction of the B field in p-Pb



S. Schlichting, S. Pratt, 1009.4283

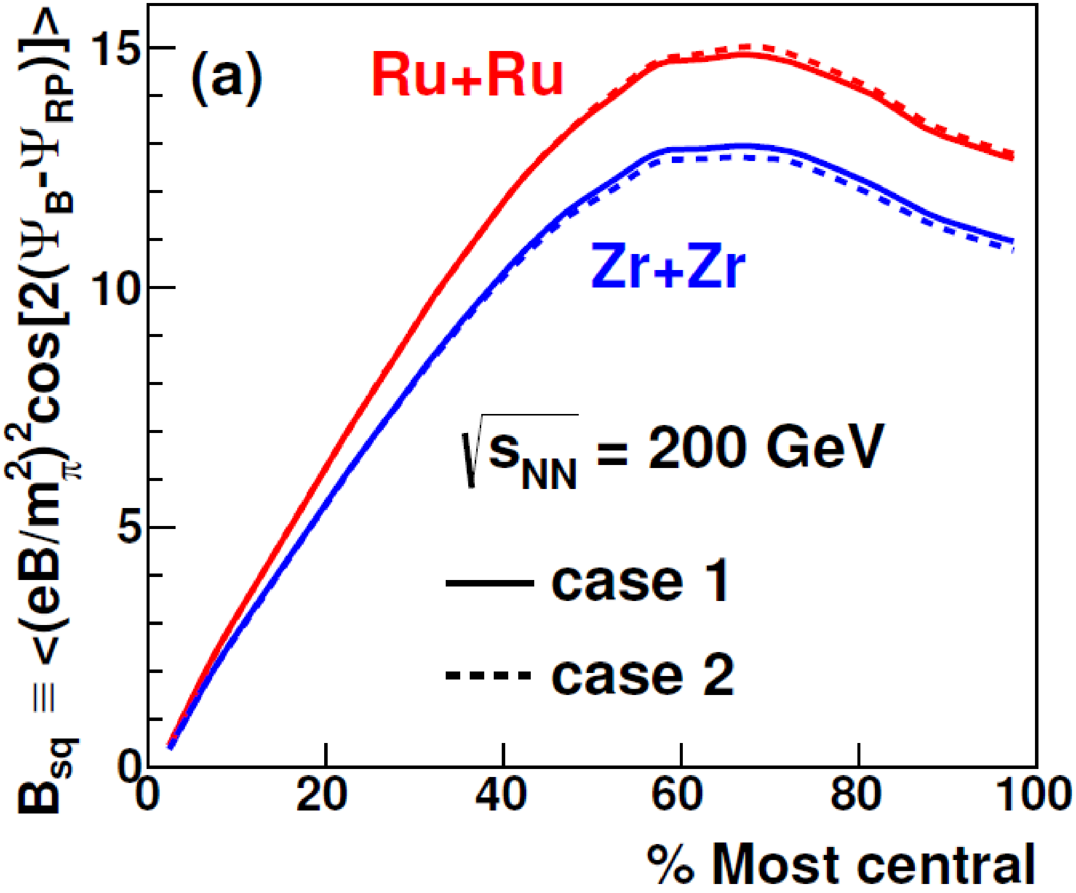
An Isobar Run at RHIC

- **Adjusting the B field in these collisions, in a controlled way, is a way to distinguish signal from background**

- **If we can control the B field, but hold all other parameters constant, then we could see if the signal changes with as the B field changes**

- **Fortunately, this is possible with nuclear isobars**
  - **Compare two identical systems but with different atomic number (Z)**

RHIC will run Isobaric Beams in 2018



arXiv:1608.00982v1 [nucl-th] 2 Aug 2016

We have […] investigated the case for colliding nuclear isobars […] and find the case compelling. We recommend that a program of nuclear isobar collisions to isolate the chiral magnetic effect from background sources be placed as a high priority item in the strategy for completing the RHIC mission.

Summary

- **STAR & ALICE see clear evidence for charge separation in Au-Au, Pb-Pb, Cu-Cu and U-U collisions at 27, 39, 62.4, 200 GeV/A and 2.76 TeV/A**
  - **The signal is not so small … |a| ~ 10^{-2} , and is accessible with standard tools**
  - **The signal goes away at energies below 19.6 GeV/A**

- **Charge separation may be an indicator of the Chiral Magnetic Effect . It could also be due to background effects.**

- **The Chiral Magnetic Effect, if it has been observed,**
  - **Provides for the study of topologically complex gluon configurations**
  - **Strong magnetic fields**
  - **and Chiral Symmetry restoration**

- **However, there are poorly understood backgrounds that could mimic or mask the expected signal**
  - **The result is of such fundamental importance that more controlled experiments are justified and required**

- **RHIC will be running ^{96}Zr and ^{96}Ru beams in 2018 to try to sort this out**
  - **Approximately 500 M events, each, and/or about 3 weeks each**

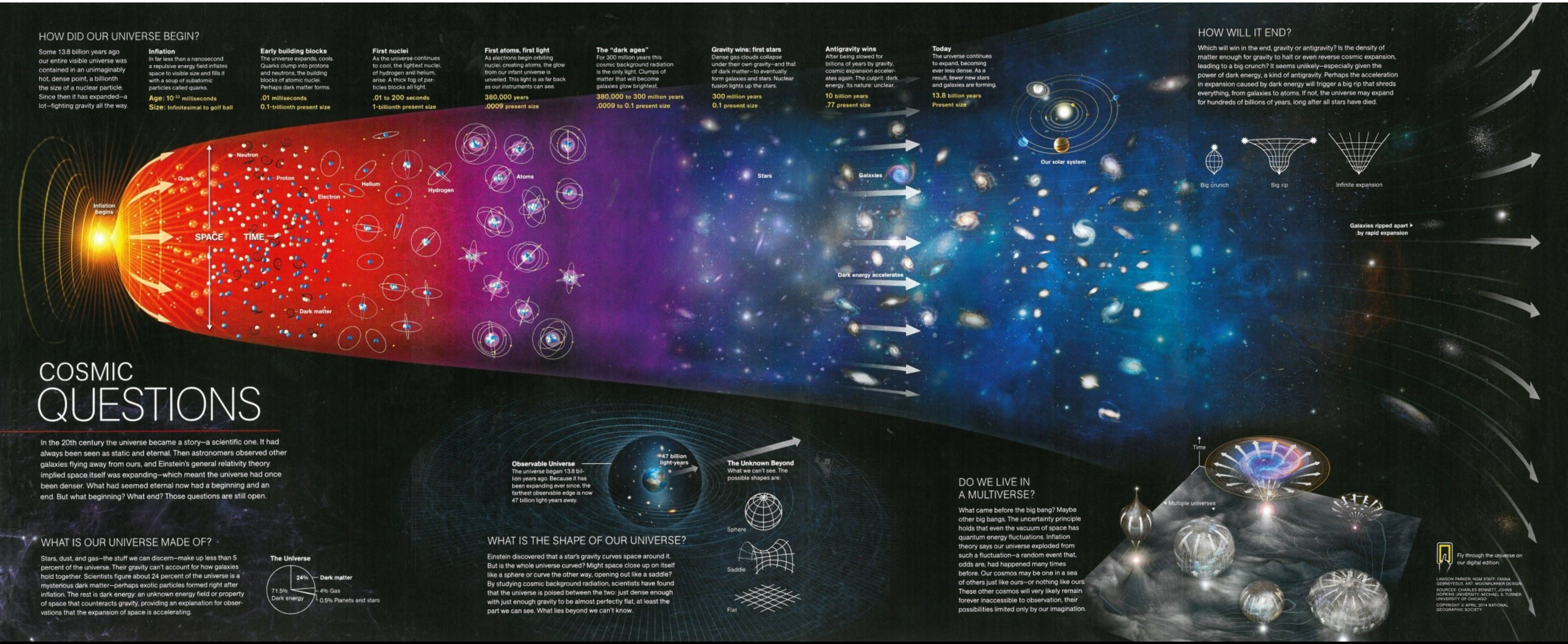
For more information see: Volker Koch *et al.* 2017 *Chinese Phys. C* **41** 072001 and/or arXiv:1608.00982v1 [nucl-th] 2 Aug 2016

Backup Slides

^{96}Zr + ^{96}Zr and ^{96}Ru + ^{96}Ru collisions

- Isobaric collisions allow us to compare two colliding systems with
  - the same mass
  - (nearly) the same shape and geometry
  - but different number of protons (and thus different E and B fields)
- Study phenomena as a function of the neutron/proton ratio
  - Fundamental physics with excellent control over background effects
- Flow, specifically directed flow v_1, has been shown to have a strong n/p ratio dependence at 55 MeV / nucleon
  - Due to larger cross section for n-p collisions compare to n-n or p-p
  - Pak, Benenson, et al., PRL 78 (1997) 1022 (hadronic dof, non-QGP)
- What happens at 200 GeV/nucleon? (QGP dof)
  - Sensitivity to the equation of state (EOS)
  - Advances in relativistic hydrodynamics
  - Study of phenomena that are sensitive to the strong electric and magnetic fields that are created when two nuclei collide – such as CME, Lamdba Polarization, perhaps even ultra-peripheral collisions

The early Universe



HOW DID OUR UNIVERSE BEGIN?

Some 13.8 billion years ago our entire visible universe was contained in an unimaginably hot, dense point, a billionth the size of a nuclear particle. Since then it has expanded—a lot—fighting gravity all the way.

Inflation
In far less than a nanosecond a repulsive energy field inflates space to visible size and fills it with a soup of subatomic particles called quarks.
Age: 10^{-32} milliseconds
Size: Infinitesimal to golf ball

Early building blocks
The universe expands, cools. Quarks clump into protons and neutrons, the building blocks of atomic nuclei. Perhaps dark matter forms.
.01 milliseconds
0.1-trillionth present size

First nuclei
As the universe continues to cool, the lightest nuclei, of hydrogen and helium, arise. A thick fog of particles blocks all light.
.01 to 200 seconds
1-billionth present size

First atoms, first light
As electrons begin orbiting nuclei, creating atoms, the glow from our infant universe is unveiled. This light is as far back as our instruments can see.
380,000 years
.0009 present size

The "dark ages"
For 300 million years this cosmic background radiation is the only light. Clumps of matter that will become galaxies glow brightest.
380,000 to 300 million years
.0009 to 0.1 present size

Gravity wins: first stars
Dense gas clouds collapse under their own gravity—and that of dark matter—to eventually form galaxies and stars. Nuclear fusion lights up the stars.
300 million years
0.1 present size

Antigravity wins
After being slowed for billions of years by gravity, cosmic expansion accelerates again. The culprit: dark energy. Its nature: unclear.
10 billion years
.77 present size

Today
The universe continues to expand, becoming ever less dense. As a result, fewer new stars and galaxies are forming.
13.8 billion years
Present size

HOW WILL IT END?

Which will win in the end, gravity or antigravity? Is the density of matter enough for gravity to halt or even reverse cosmic expansion, leading to a big crunch? It seems unlikely—especially given the power of dark energy, a kind of antigravity. Perhaps the acceleration in expansion caused by dark energy will trigger a big rip that shreds everything, from galaxies to atoms. If not, the universe may expand for hundreds of billions of years, long after all stars have died.

COSMIC QUESTIONS

In the 20th century the universe became a story—a scientific one. It had always been seen as static and eternal. Then astronomers observed other galaxies flying away from ours, and Einstein's general relativity theory implied space itself was expanding—which meant the universe had once been denser. What had seemed eternal now had a beginning and an end. But what beginning? What end? Those questions are still open.

WHAT IS OUR UNIVERSE MADE OF?

Stars, dust, and gas—the stuff we can discern—make up less than 5 percent of the universe. Their gravity can't account for how galaxies hold together. Scientists figure about 24 percent of the universe is a mysterious dark matter—perhaps exotic particles formed right after inflation. The rest is dark energy: an unknown energy field or property of space that counteracts gravity, providing an explanation for observations that the expansion of space is accelerating.

Observable Universe
The universe began 13.8 billion years ago. Because it has been expanding ever since, the farthest observable edge is now 47 billion light-years away.

The Unknown Beyond
What we can't see. The possible shapes are:

WHAT IS THE SHAPE OF OUR UNIVERSE?

Einstein discovered that a star's gravity curves space around it. But is the whole universe curved? Might space close up on itself like a sphere or curve the other way, opening out like a saddle? By studying cosmic background radiation, scientists have found that the universe is poised between the two: just dense enough with just enough gravity to be almost perfectly flat, at least the part we can see. What lies beyond we can't know.

DO WE LIVE IN A MULTIVERSE?

What came before the big bang? Maybe other big bangs. The uncertainty principle holds that even the vacuum of space has quantum energy fluctuations. Inflation theory says our universe exploded from such a fluctuation—a random event that, odds are, had happened many times before. Our cosmos may be one in a sea of others just like ours—or nothing like ours. These other cosmos will very likely remain forever inaccessible to observation, their possibilities limited only by our imagination.

Fly through the universe on our digital edition.

LAWSON PARKER, NGM STAFF; FANNA GEBREYESUS. ART: MOONRUNNER DESIGN
SOURCES: CHARLES BENNETT, JOHNS HOPKINS UNIVERSITY; MICHAEL S. TURNER, UNIVERSITY OF CHICAGO

- The Universe was filled with hot and dense matter at the earliest times
- Nanoseconds – the Inflation epoc w/ topologically complex gluon fields
- Microseconds – a Quark Gluon Plasma; flux tubes decay and form quarks
- Seconds – first Nuclei are formed. Atoms come 380,000 years later.

A Mystery

- Where does the energy come from to drive inflation?
- We don’t know.
- We presume that the early Universe is filled with topologically complex gauge fields (perhaps in 3 dimensions) that look a lot like gluon fields
  - This promises to lead to the discovery of lots of interesting phenomenology

Summary II

- QCD is a non-abelian gauge theory that is filled with fascinating phenomenology
- Heavy Ion Colliders make some of this phenomenology accessible to the experimentalist
- It is important to understand non-abelian gauge theories (in all dimensions) because you can only have knots in 3 spatial dimensions and perhaps that is why we live in a 3D world

"We suggest a structure for the vacuum comprised of a network of tightly knotted/linked flux tubes formed in a QCD-like cosmological phase transition and show that such a network can drive cosmological inflation. As the network can be topologically stable only in three space dimensions, this scenario provides a dynamical explanation for the existence of exactly three large spatial dimensions in our Universe."

Berera et al. arXiv:1508.01458 [hep-ph]

Full Fourier Transform of the Invariant Yield

$$f(\phi) = \frac{b_0'}{2} + \sum_{n=1}^{\infty} \left(a_n' \sin(n\phi) + b_n' \cos(n\phi) \right)$$

where

$$a_n' = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\phi) \sin(n\phi)\ d\phi \quad \text{for} \quad n = 1, 2, \ldots$$

$$b_n' = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\phi) \cos(n\phi)\ d\phi \quad \text{for} \quad n = 0, 1, 2, \ldots$$

If we want to test if parity is conserved then we should keep the extra terms

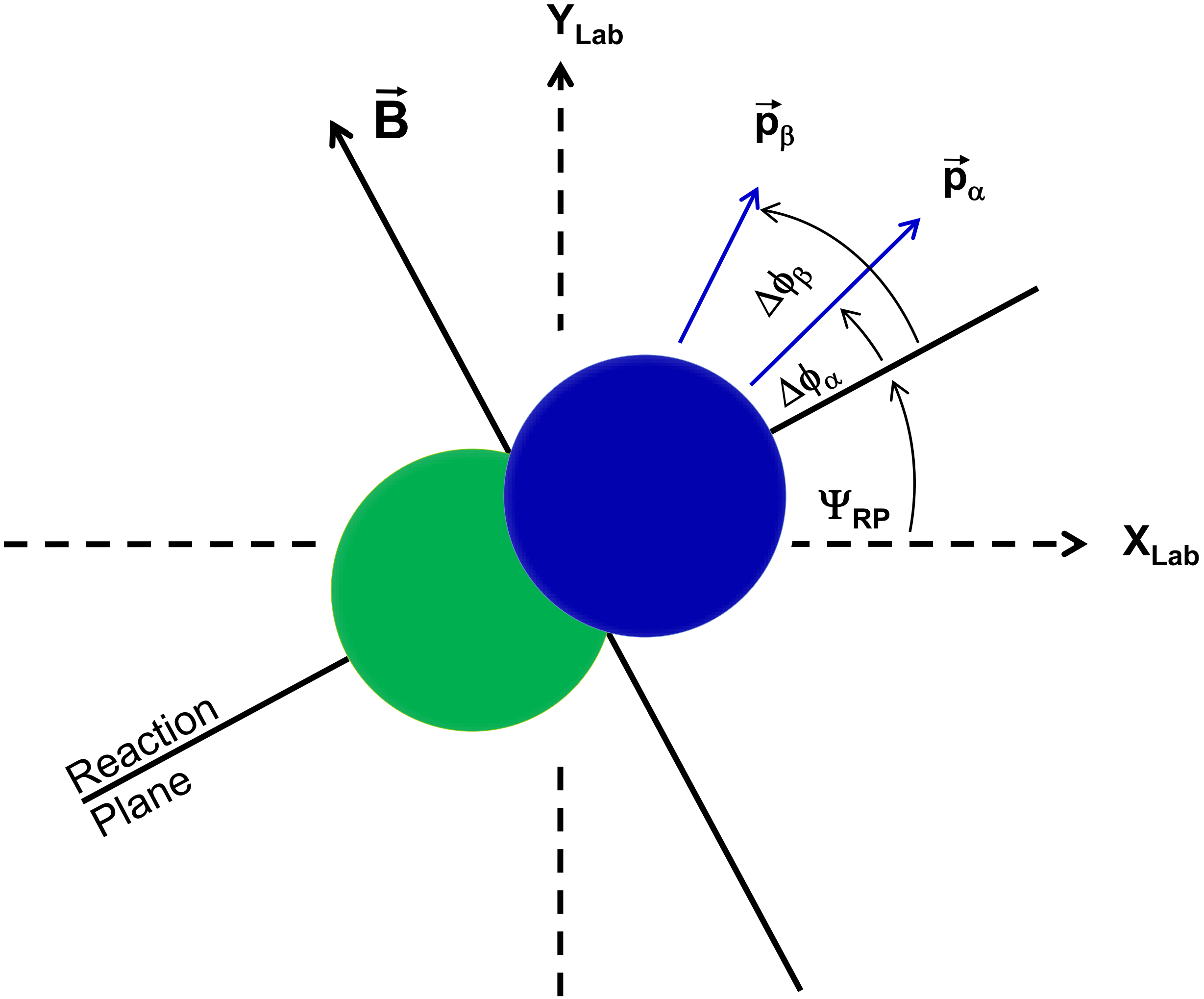
$$E\frac{dN^3}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left(1 + \underline{2a_1 \sin(\Delta\phi)} + 2b_1 \cos(\Delta\phi) + \underline{2a_2 \sin(2\Delta\phi)} + 2b_2 \cos(2\Delta\phi) + \cdots \right)$$

where

$$a_n = \pi\, a_n' = \sum_i \sin(\, n(\phi_i - \Psi_R)\,)\ , \qquad b_n = \pi\, b_n' = \sum_i \cos(\, n(\phi_i - \Psi_R)\,)$$

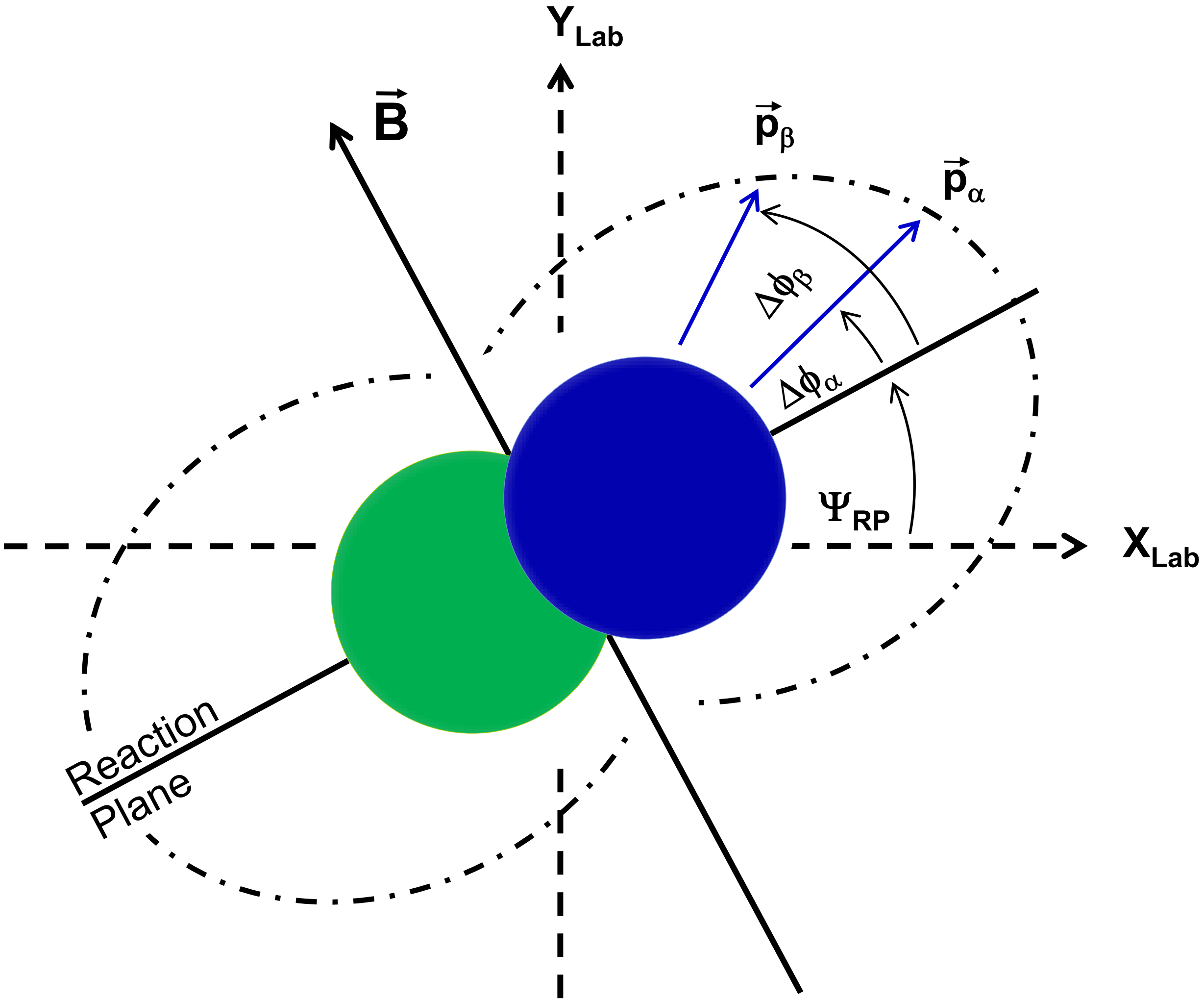
The standard HI flow analysis assumes a = 0 and assigns $b_n \equiv v_n$

Analysis Uses Standard Flow Tools



- **The line between the centers of the nuclei and the beam axis define the reaction plane – perpendicular to angular momentum vector and B field**

Analysis Uses Standard Flow Tools



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