ExB Corrections for the STAR and ALICE TPCs

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The Langevin Equation – (see Blum, Riegler and Rolandi)

Solve:

$$m\frac{d\overline{u}}{dt} = e\overline{E} + e\left[\overline{u}\times\overline{B}\right] - K\overline{u}$$

Microscopic Lorentz force with "Friction"

substituting:

$$\tau = \frac{m}{K}\ , \quad \omega = \frac{e}{m}\,|\,\overline{B}\,|\ , \quad \mu = \frac{e}{m}\tau\ , \quad \text{and}\ \ \hat{E} = \frac{\overline{E}}{|\,\overline{E}\,|}$$

subject to the steady state condition

$$\frac{d\overline{u}}{dt} = 0$$

yields

$$\overline{u} = \frac{\mu|\,\overline{E}\,|}{\left(1+\omega^2\tau^2\right)}\left(\ \hat{E} + \omega\tau\left[\hat{E}\times\hat{B}\right] + \omega^2\tau^2\left(\hat{E}\bullet\hat{B}\right)\hat{B}\ \right)$$

where B is a unit vector pointing in the direction of $\overline{\text{B}}$.

An Example: The Case of E ≈ Parallel to B

$$\delta_r = \int \frac{u_r}{u_z}\, dz, \qquad \delta_\phi = \int \frac{u_\phi}{u_z}\, dz$$

Electric field strength cancels out!

Small terms ⇒ 2nd order corrections

$$u_\phi = \frac{\mu\left|\overline{E}\right|}{\left(1+\omega^2\tau^2\right)} \left(\hat{E}_\phi - \omega\tau\left(\hat{E}_r\hat{B}_z - \hat{E}_z\hat{B}_r\right) + \omega^2\tau^2\hat{B}_\phi\right)$$

etc…so

Assuming cylindrical coordinate and E = E_z and B_z >> B_r or B_ϕ

$$\delta_r = \frac{\omega^2\tau^2}{\left(1+\omega^2\tau^2\right)} \int \frac{\hat{B}_r}{\hat{B}_z}\, dz$$

$$\delta_\phi = \frac{\omega\tau}{\left(1+\omega^2\tau^2\right)} \int \frac{\hat{B}_r}{\hat{B}_z}\, dz$$

Simple first order equations

Distortion Equations – related to integrals of the fields

Distortion in x or y

Integral over distorted field

$$\begin{pmatrix}\delta_x\\ \delta_y\end{pmatrix} = \begin{pmatrix} c_0 & c_1 \\ -c_1 & c_0\end{pmatrix}\begin{pmatrix}\int \frac{E_x}{E_z}\, dz \\ \int \frac{E_y}{E_z}\, dz\end{pmatrix}$$

$$\begin{pmatrix}\delta_x\\ \delta_y\end{pmatrix} = \begin{pmatrix} c_2 & -c_1 \\ c_1 & c_2\end{pmatrix}\begin{pmatrix}\int \frac{B_x}{B_z}\, dz \\ \int \frac{B_y}{B_z}\, dz\end{pmatrix}$$

Master Equations
for E or B field distortions
simple linear equations

with $c_0 = \frac{1}{(1+T_2^2\omega^2\tau^2)}$, $c_1 = \frac{T_1\omega\tau}{(1+T_1^2\omega^2\tau^2)}$, and $c_2 = \frac{T_2^2\omega^2\tau^2}{(1+T_2^2\omega^2\tau^2)}$

$$\omega\tau = BField[kGauss] * \frac{-10. * |Drift\, Velocity[cm/\mu sec]|}{|Electric\, Drift\, Field\, Strength\, [V/cm]|}$$

- **Distortions in the TPC are directly related to integrals of the E and B fields**
- **These solutions of the Langevin Equation are exact to 2nd order**
  - **The matrices are rotations (with a pre-factor) related to the Lorentz angle**
  - **Since rotations commute in 2 dimensions, these matrices commute with other rotations. These same equations are valid in Cylindrical Coordinates, $\delta x \Rightarrow \delta r$, $\delta y \Rightarrow r * \delta\phi$**

If you have a well defined model, and good data, then the distortion can be removed with great precision