ExB Notes and a Discussion about the Magnetic Field Shape Distortions

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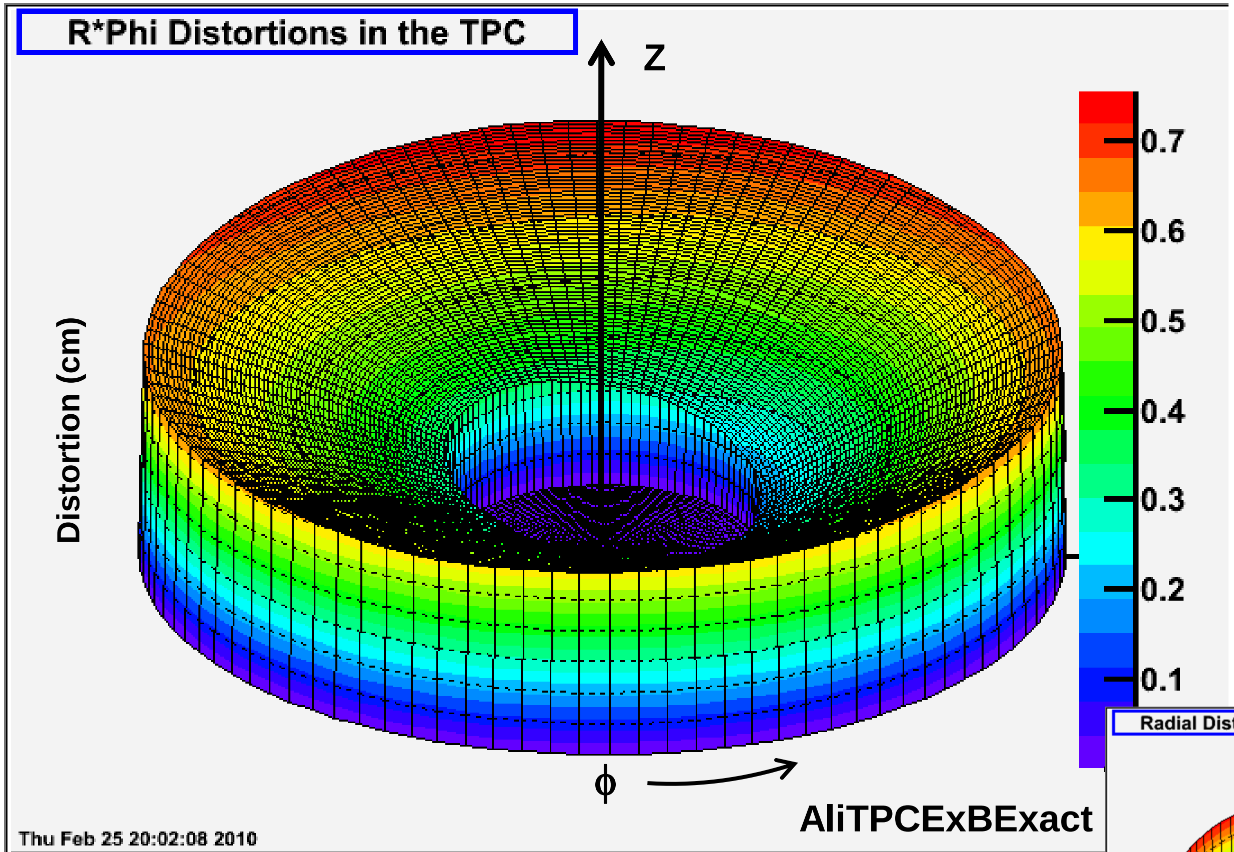
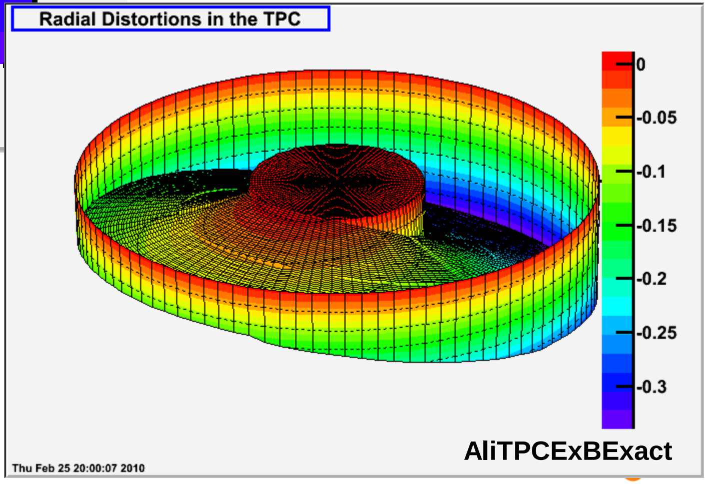
also see: http://rnc.lbl.gov/~jhthomas/public/ALICE/ExBEquations.pdf

and: /afs/cern.ch/user/j/jhthomas/public/AliDistort/AliceDistortions.cxx

R-Phi and Radial ExB Shape Distortions

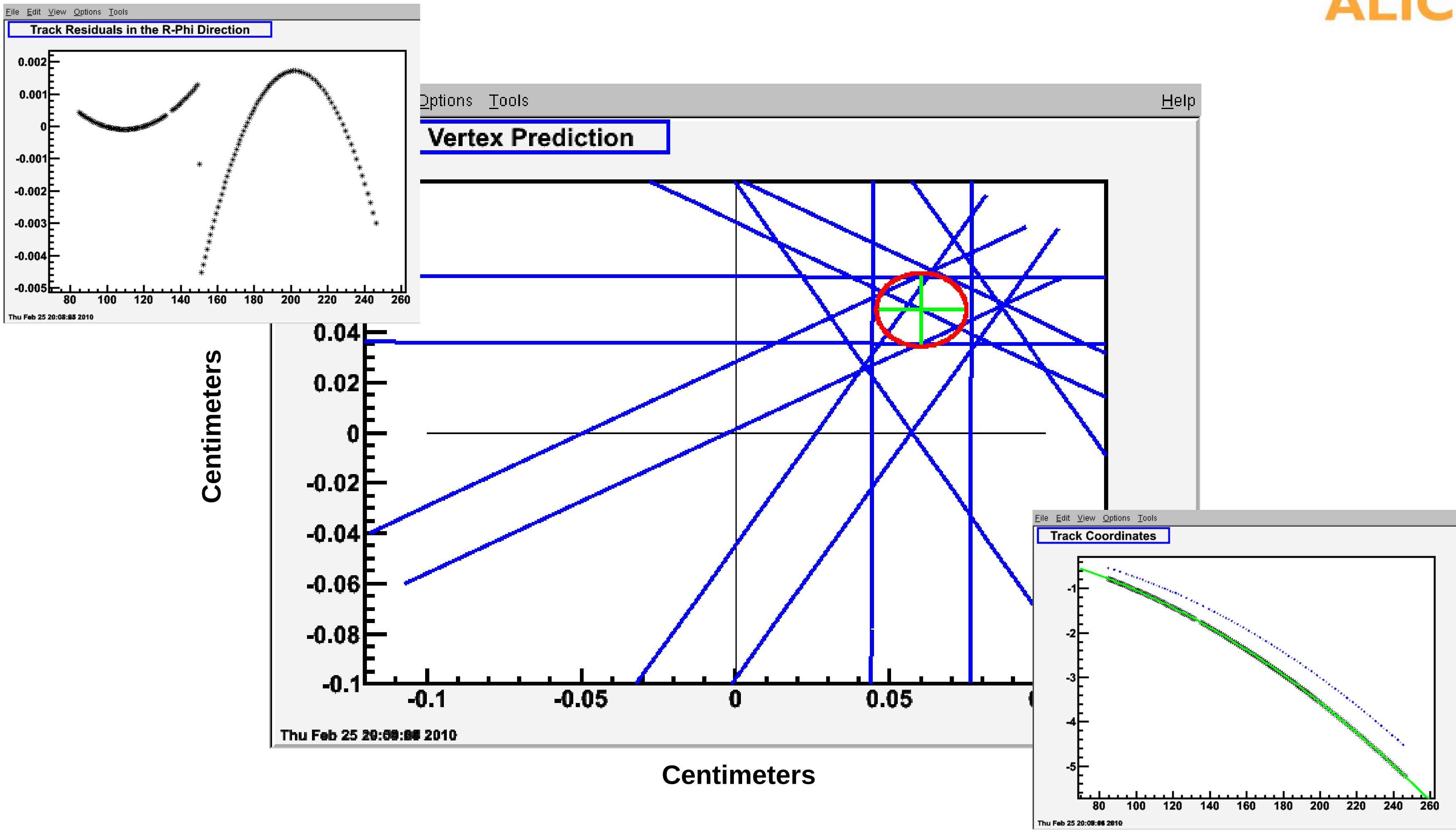


- $\pm$100 gauss B_r fluctuations
- $\pm$50 gauss B_ϕ fluctuations
- The tracks will suffer distortions in the R-Phi direction as well as in the Radial direction



- Note that $\omega\tau$ is a tensor and so the components of $\omega\tau$ in the R-Phi and Radial directions can be different
- We can measure these components for $Ne/CO_2/N_2$ with the lasers &/or GG scan

Simulated ExB Shape distortions in the TPC



The Magnetic Field is non-uniform, shifts the apparent vertex and minimizing residuals doesn't give much guidance

Distortion Equations – (see Blum & Rolandi)

Solve:

$$m\frac{d\overline{u}}{dt} = e\overline{E} + e\left[\overline{u}\times\overline{B}\right] - K\overline{u}$$

Langevin Equation with "Friction"

substituting:

$$\tau = \frac{m}{K}\ , \quad \omega = \frac{e}{m}\,|\,\overline{B}\,|\ , \quad \mu = \frac{e}{m}\tau\ , \quad \text{and}\ \ \hat{E} = \frac{\overline{E}}{|\,\overline{E}\,|}$$

subject to the steady state condition

$$\frac{d\overline{u}}{dt} = 0$$

yields

$$\overline{u} = \frac{\mu|\,\overline{E}\,|}{\left(1+\omega^2\tau^2\right)}\left(\ \hat{E} + \omega\tau\left[\hat{E}\times\hat{B}\right] + \omega^2\tau^2\left(\hat{E}\bullet\hat{B}\right)\hat{B}\ \right)$$

where B-hat is a unit vector pointing in the direction of $\overline{\text{B}}$.

The Case of E Parallel to B

$$u_x = \frac{\mu\,|E|}{(1+\omega^2\tau^2)}\left(\hat{E}_x + \omega\tau\left(\hat{E}_y\hat{B}_z - \hat{E}_z\hat{B}_y\right) + \omega^2\tau^2\,\hat{B}_x\right)$$

Electric field strength cancels out

$$\delta_x = \int \frac{u_x}{u_z}\,dz \qquad \delta_y = \int \frac{u_y}{u_z}\,dz$$

Taking the ratio and expanding the denominator yields exact equations to 2nd order

$$\delta_x = \frac{1}{(1+\omega^2\tau^2)}\int\frac{E_x}{E_z}\,dz + \frac{\omega\tau}{(1+\omega^2\tau^2)}\int\frac{E_y}{E_z}\,dz - \frac{\omega\tau}{(1+\omega^2\tau^2)}\int\frac{B_y}{B_z}\,dz + \frac{\omega^2\tau^2}{(1+\omega^2\tau^2)}\int\frac{B_x}{B_z}\,dz$$

Simple linear equations

$$\delta_y = \frac{1}{(1+\omega^2\tau^2)}\int\frac{E_y}{E_z}\,dz - \frac{\omega\tau}{(1+\omega^2\tau^2)}\int\frac{E_x}{E_z}\,dz + \frac{\omega\tau}{(1+\omega^2\tau^2)}\int\frac{B_x}{B_z}\,dz + \frac{\omega^2\tau^2}{(1+\omega^2\tau^2)}\int\frac{B_y}{B_z}\,dz$$

No approximations required if we write $\int(B_x/B_z)\,dz$ instead of $\int B_x\,dz \,/ \int B_z\,dz$

Separable equations, additive in all terms

$$\begin{pmatrix}\delta_{xE}\\ \delta_{yE}\end{pmatrix} = \begin{pmatrix}c_0 & c_1\\ -c_1 & c_0\end{pmatrix}\begin{pmatrix}\int \frac{E_x}{E_z}\,dz\\ \int \frac{E_y}{E_z}\,dz\end{pmatrix}$$

$$\begin{pmatrix}\delta_{xB}\\ \delta_{yB}\end{pmatrix} = \begin{pmatrix}c_2 & -c_1\\ c_1 & c_2\end{pmatrix}\begin{pmatrix}\int \frac{B_x}{B_z}\,dz\\ \int \frac{B_y}{B_z}\,dz\end{pmatrix}$$

Form is very similar to a rotation matrix, with constants of the motion

$$c_0 = \frac{1}{(1+T_2^2\omega^2\tau^2)}\,,\quad c_1 = \frac{T_1\omega\tau}{(1+T_1^2\omega^2\tau^2)},\quad \text{and}\quad c_2 = \frac{T_2^2\omega^2\tau^2}{(1+T_2^2\omega^2\tau^2)}$$

where I have taken the liberty of adding the two tensor terms T_1 and T_2 from the microscopic theory in Blum and Rolandi's book

$$\omega\tau = -10.0 * BField[kGauss] * \frac{Drift\ Velocity[cm/\mu sec]}{Electric\ Drift\ Field\ Strength\ [V/cm]}$$

For practical applications, the integrals can be pre-computed, which is very fast, yet constants of motion are decoupled and can be updated run by run (or event by event)

Comments

- E and B field distortions can be calculated separately and summed
  - No cross terms and no coupling of E and B fields
- Integrals such as $\int (B_x/B_z)\, dz$ are easy to compute, so why not?
  - Makes the difference between a 1st and 2nd order calculation
- 2nd order calculations are very good
  - B field shape distortions are accurate to 200 μm at 1st order, 4 μm at 2nd
  - Electric field distortions, and their errors, will be much smaller unless something really bad happens to the field cage
- The tensor terms, T_1 and T_2, can be measured
  - In methane gases, they have been measured to be $T_1 = 1.34$, $T_2 = 1.11$
  - Which if the same holds true in Ne, could lead to 20% errors … or ~2 mm
  - Tensor terms are unknown (as far as I know) for Ne based gas mixtures
- We have taken the liberty of asking Ruben S. to update AliMagF
  - Precompute the B field integrals $\int (B_x/B_z)\, dz$ and put them in AliMagF
  - Clean up some other glitches

backup slides

Ruben Integrals will yield very compact code

```
void AliceDistortions::UndoExBShapeDistortion( const Float_t x[], Float_t Xprime )
{

  Double_t BIntegralStart[3], BIntegralEnd[3] , xStart[3], xEnd[3] ;
  Double_t BxOverBzIntegral, ByOverBzIntegral, Denominator ;

  Int_t xStart[0] = x[0] ;  xStart[1] = x[1] ;  xStart[2] = x[2] ;
  xEnd[0] = x[0] ;  xEnd[1] = x[1] ;  xEnd[2] = TPC_Z0 ;

  AliceMagField -> GetTPCnewInt(xStart, BIntegralStart) ;
  AliceMagField -> GetTPCnewInt(xEnd  , BIntegralEnd  ) ;

  BxOverBzIntegral = (BIntegralStart[0] - BIntegralEnd[0]);
  ByOverBzIntegral = (BIntegralStart[1] - BIntegralEnd[1]);

  Xprime[0] +=  ( Const_2 * BxOverBzIntegral  -  Const_1 * ByOverBzIntegral )  ;
  Xprime[1] +=  ( Const_2 * ByOverBzIntegral  +  Const_1 * BxOverBzIntegral )  ;
  Xprime[2] +=  0.0 ;

}

// Const_1 = OmegaTau/(1+OmegaTau**2) and Const_2 = OmegaTau**2/(1+OmegaTau**2)
```