Design for Kinematic Mounts

In the design below, using 1/16" alumina or zirconia rods as spring contacts...

Spring Parameters:

Design Equations

Beam profile (curved bottom defined by equation, horizontal flat on top):

$$y(x) = -2\sqrt{\frac{3}{2}\frac{F}{\eta\sigma_y b}(L-x)}$$

Beam length:

$$L = \frac{3}{2}\frac{1}{\eta\sigma_y}\left(\frac{4}{5}E(1.15\delta)\right)^{2/3}\left(\frac{F}{b}\right)^{1/3}$$

- The adjustment factor 1.15 applied to δ adds a little extra length to help account for the discrepancy between the small deflection beam theory calculation and the actual large deflection behavior, as well as the practical necessity of non-zero thickness near the tip of the beam.
- Derivation of these design formulas is shown below.

Top Spring (which forces ball into groove)

Spring contact radius	R_S = 1.58mm
Max spring deflection	δ = 2.0mm
Max spring force	F = 15N
Yield stress	σ_y = 880MPa
Max allowed stress ratio	η = 0.55 (i.e. 484MPa)
Modulus	E = 113GPa
Spring beam width	b = 4.5mm
Required beam length	L = 16.25mm (calculated)

Bottom Spring (when present, counteracts weight of hanging mass)

Spring contact radius	R_S = 1.58mm
Max spring deflection	δ = 1.75mm
Max spring force	F = 2.0kg * 9.81m/s = 19.6N
Yield stress	σ_y = 880MPa
Max allowed stress ratio	η = 0.55 (i.e. 484MPa)
Modulus	E = 113GPa
Spring beam width	b = 4.5mm
Required beam length	L = 16.25mm (calculated)

Constant Stress "Tapered" Cantilever Spring

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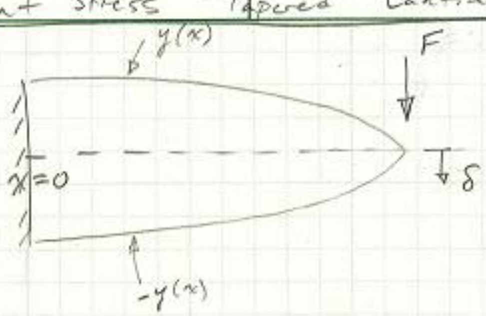

DESCRIBES THE CORRECT CONSTANT STRESS PROFILE

CONSTANT STRESS CONDITION:

from $\sigma = \frac{My}{I} \implies \eta\sigma_y = \frac{M(x)\,y(x)}{I(x)}$

(σ_y ← yield stress; η ← Factor of safety)

$$\Rightarrow \eta\sigma_y = \frac{F(L-x)\cdot y(x)}{\frac{1}{12} b\,(2y(x))^3}$$

$$= \frac{3}{2}\frac{F}{b}\frac{L-x}{y(x)^2} \implies \boxed{y(x) = \sqrt{\frac{3}{2}\frac{F}{\eta\sigma_y b}(L-x)}}$$

SPECIFICATION of $F = k\delta$ (SPRING CHARACTERISTICS):

from $\delta = \frac{FL^3}{3EI}$, approximate $I \approx \bar{I} = \frac{1}{L}\int_0^L I(x)\,dx$

$$\Rightarrow \frac{FL^4}{3E\delta} = \int_0^L \frac{1}{12} b\,(2y(x))^3\,dx = \frac{2}{3}b\left(\frac{3}{2}\frac{F}{\eta\sigma_y b}\right)^{3/2}\int_0^L (L-x)^3\,dx$$

$$= \frac{2}{3}b\left(\frac{3}{2}\frac{F}{\eta\sigma_y b}\right)^{3/2}\left(-\frac{2}{5}(L-x)^{5/2}\right)\Bigg|_0^L = \frac{4}{15}b\left(\frac{3}{2}\frac{F}{\eta\sigma_y b}\right)^{3/2} L^{5/2}$$

$$\Rightarrow L^{3/2} = \frac{12}{15}\frac{E\delta}{F}b\left(\frac{3}{2}\frac{F}{\eta\sigma_y b}\right)^{3/2} = \frac{4}{5}E\delta F^{1/2} b^{-1/2}\left(\frac{3}{2}\frac{1}{\eta\sigma_y}\right)^{3/2}$$

$$\Rightarrow L = \left(\frac{4}{5}E\delta\left(\frac{F}{b}\right)^{1/2}\right)^{2/3}\frac{3}{2}\frac{1}{\eta\sigma_y}$$

$$\Rightarrow \boxed{L = \frac{3}{2}\frac{1}{\eta\sigma_y}\left(\frac{4}{5}E\delta\right)^{2/3}\left(\frac{F}{b}\right)^{1/3}} \;\star$$

SETS THE REQUIRED BEAM LENGTH FOR A SPECIFIED COMBINATION OF

- $\eta\sigma_y$ = max fraction of yield stress
- E = modulus
- δ = max deflection
- F = max force
- b = beam width

★ Note that this design analysis tends to under predict δ by about 15%. Thus it may be useful to replace δ with $\delta^* = C\,\delta_{desired}$, where $C \approx 1.15$

Contact Stress Calculations

- For the top kinematic mounts (control X,Y,Z):
  - The spherical ball (5/32" diameter, silicon nitride) rides in an internal radius cylindrical groove (Ti 6Al-4V) of radius 2.5mm.
  - The spring contacts are ceramic (alumina or zirconia) cylinders (1/8" radius) riding on a generally planar guide surface (Ti 6Al-4V).

- For the bottom kinematic mounts (control X only):
  - The ball (1/2" diameter, silicon nitride) runs against a planar flat guide.

Sphere-on-Cylinder Contact

BALL			GROOVE																	
E1	**nu1**	**D1**	**E2**	**nu2**	**R2**	**D2**	**Syc2**	**V1**	**V2**	**Q**	**A/B**	**1/A**	**- 1/e dE/dE**	**a**	**theta**	**F (applied)**	**F**	**Pmax**	**~Taumax**	**~FOS2 (tresca)**
GPa	-	*mm*	*GPa*	-	*mm*	*mm*	*MPa*	*1/Pa*	*1/Pa*	*1/Pa*	-	*m*	-	*mm*	*deg*	*N*	*N*	*MPa*	*MPa*	-
345.0	0.24	3.97	113.8	0.34	-2.50	-5.00	880	8.7E-13	2.5E-12	######	0.206	0.019	1.5450	0.138	45	25.0	17.7	443	133	3.31
345.0	0.24	12.70	113.8	0.34	1E+06	2E+06	880	8.7E-13	2.5E-12	######	1.000	0.013	0.7854	0.063	n/a	5.0	5.0	602	181	2.44

The value -1/e dE/de is from a lookup table (Puttock and Thwaite 1969).

Cylinder-on-Plane Contact

SPRING CONTACT			GUIDE SURFACE					WIDTH						
E1	**nu1**	**D1**	**E2**	**nu2**	**R2**	**D2**	**Syc2**	**L**	**b**	**F**	**Pmax**	**~Taumax**	**~FOS2 (tresca)**	
GPa	-	*mm*	*GPa*	-	*mm*	*mm*	*MPa*	*mm*	*mm*	*N*	*MPa*	*MPa*	-	
113.8	0.34	3.18	300.0	0.21	1E+06	2E+06	880	4.50	0.011	25.0	319	96	4.60	Alumina
113.8	0.34	3.18	200.0	0.32	1E+06	2E+06	880	4.50	0.012	25.0	302	90	4.86	Zirconia

Force Calculations During Travel

PARAMETERS			
static friction coefficient	mu	-	0.2
exit angle of upper spring contact	ang	deg	45
exit angle of upper spring contact	ang	rad	**0.785**
final distance past start of exit angle	a	mm	1.00
radius of upper spring contact	rUmax	mm	1.59
spring constant at top mount's upper surface	kU	N/mm	7.5
max deflection of top mount's upper spring	dUmax	mm	2.0
final deflection of top mount's upper spring	dUfinal	mm	**1.658**
spring constant at top mount's lower surface	kL	N/mm	11.2
max deflection of top mount's lower surface	dLmax	mm	1.75
spring constant at bottom mount	kB	N/mm	10.0
max deflection of bottom mount's spring	dBmax	mm	0.5
supported mass	m	kg	2.5
supported weight	mg	N	**24.5**
TOP EAST KIN MOUNT			
ball contact force at end of travel	Bfinal	N	**19.8**
max ball contact force during travel (conservative)	Bmax	N	**22.9**
max upper spring contact force during travel	Umax	N	**15**
max lower spring contact force during travel	Lmax	N	**19.6**
max insertion force during travel	Fappmax	N	**18.4**
insertion force at end of travel (negative --> self-slip)	Fappfinal	N	**-2.1**
retraction force at end of travel	Fretfinal	N	**22.8**
TOP WEST KIN MOUNT			
ball contact force at end of travel	Bfinal	N	**14.9**
max ball contact force during travel (conservative)	Bmax	N	**18.0**
max upper spring contact force during travel	Umax	N	**15**
max lower spring contact force during travel	Lmax	N	**0.0**
max insertion force during travel	Fappmax	N	**9.6**
insertion force at end of travel (negative --> self-slip)	Fappfinal	N	**-7.0**
retraction force at end of travel	Fretfinal	N	**21.8**
BOTTOM KIN MOUNT			
max ball contact force during travel (conservative)	Bmax	N	**5.0**
max insertion force during travel	Fappmax	N	**1.0**
TOTAL			
max insertion force during travel	Fappmax	N	**29.0**
insertion force at end of travel (negative --> self-slip)	Fappfinal	N	**-8.0**
retraction force at end of travel	Fretfinal	N	**45.6**

Result for Single Mount in Independent Test Stand