- Distortion Coefficients -
Do they need an update?

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Mustafa's Work

STAR iTPC

Distortions due to non-uniform magnetic field

Macroscopically, the motion of an electron in the gas can be described by

$$m_e \frac{d\vec{v}}{dt} = e\vec{E} + e(\vec{v} \times \vec{B}) - K\vec{v}$$

where frictional force is assumed to be proportional to the speed with a proportionality constant K.

The solution for steady state is

$$\vec{v} = \frac{\mu}{1 + (\omega\tau)^2} \left[\vec{E} + (\omega\tau) \frac{\vec{E} \times \vec{B}}{|\vec{B}|} + (\omega\tau)^2 \frac{\vec{B}(\vec{E}.\vec{B})}{|\vec{B}|^2} \right]$$

where

$$\tau = \frac{m_e}{K}, \quad \mu = \frac{e\tau}{m_e}, \quad \omega = \frac{e}{m_e}|\vec{B}|$$

are, mean free time, electrons mobility and cyclotron frequency, respectively. 2

STAR electric and magnetic fields distortions

For the STAR TPC the following are orders of magnitude of the fields distortions [See appendix in "TPC Distortion Equations in the Sub-100 \mum Era", G.V.Buren, J.H.Thomas].

$$B_\phi/B_z \sim 10^{-6},\ B_r/B_z \sim 10^{-2},\ E_\phi/E_z \sim 10^{-3},\ E_r/E_z \sim 10^{-1}$$

Assuming a straight line trajectory along z, the distortions in the φ and r directions are

$$x_\phi = \int_z dz \frac{v_\phi}{v_z}, \quad x_r = \int_z dz \frac{v_r}{v_z}$$

In STAR, the distortions due to electric and magnetic fields are separated, the cross terms are practically irrelevant [see Gene Jim paper]

$$x_\phi^E = \int_z dz \left[C_0 \frac{E_\phi}{E_z} - C_1 \frac{E_r}{E_z}\right],\ x_r^E = \int_z dz \left[C_0 \frac{E_r}{E_z} + C_1 \frac{E_\phi}{E_z}\right]$$

$$x_\phi^B = \int_z dz \left[C_1 \frac{B_r}{B_z} + C_2 \frac{B_\phi}{B_z}\right],\ x_r^B = \int_z dz \left[-C_1 \frac{B_\phi}{B_z} + C_2 \frac{B_r}{B_z}\right]$$

where

$$C_n = \frac{(\omega\tau)^n}{1 + (\omega\tau)^2}$$

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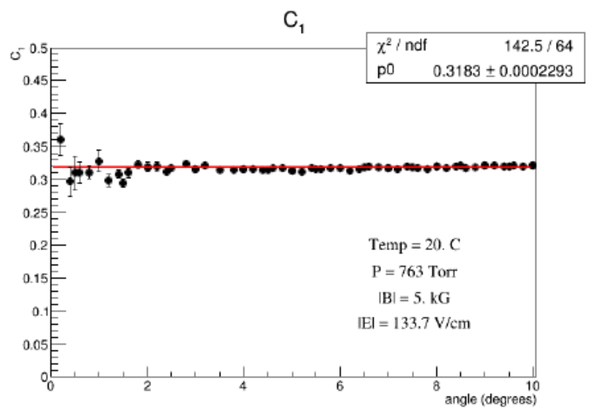
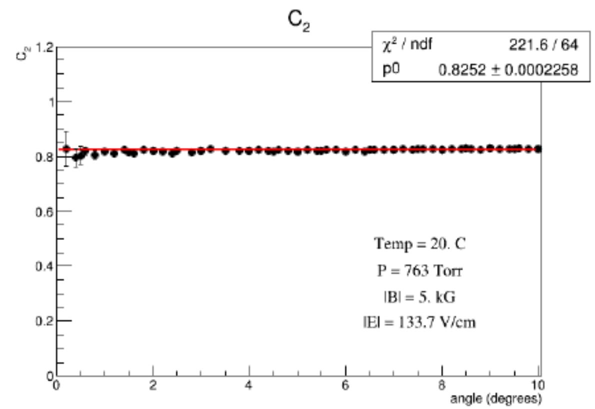
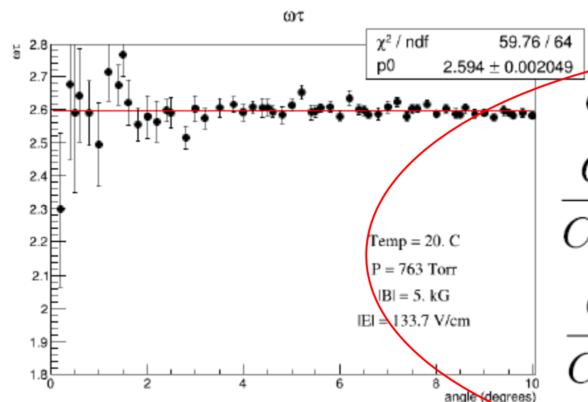
Magboltz results

These results are using Magboltz 10.0.2 (edition of 28 Aug 2013).







In Star StMagUtilities we use

$$C_0^{St} = 0.167,\ C_1^{St} = 0.317,\ C_2^{St} = 0.838$$

$$\frac{C_0^{St}}{C_0^{Mag}} \sim 0.95 \qquad \frac{C_1^{St}}{C_1^{Mag}} \sim 1.0$$

$$\frac{C_2^{St}}{C_2^{Mag}} \sim 1.02$$

Are these % level deviations important?

An Alice technical note … (by JT and friends)

STAR iTPC

Cylindrical Coordinates:

$$\begin{pmatrix} \delta_{rE} \\ r\delta_{\phi E} \end{pmatrix} = \begin{pmatrix} c_0 & c_1 \\ -c_1 & c_0 \end{pmatrix} \begin{pmatrix} \int \frac{E_r}{E_z}\, dz \\ \int \frac{E_\phi}{E_z}\, dz \end{pmatrix}$$

$$\begin{pmatrix} \delta_{rB} \\ r\delta_{\phi B} \end{pmatrix} = \begin{pmatrix} c_2 & -c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \int \frac{B_r}{B_z}\, dz \\ \int \frac{B_\phi}{B_z}\, dz \end{pmatrix}$$

- Only C_0 and C_2 are uncertain
- Typically only radial integrals are large (shorted ring, B field) because ϕ distortions are small
- $\delta(r\bullet\phi)$ are most important in STAR
- Fortunately the uncertainties in C_0 and C_2 only affect radial distortions … where we are not very sensitive

Conclusions:

Space point distortions in a TPC are accurately described by the Langevin equation. The distortions in the transverse plane, to 2nd order, are given by the following equations:

Cartesian Coordinates:

$$\begin{pmatrix} \delta_{xE} \\ \delta_{yE} \end{pmatrix} = \begin{pmatrix} c_0 & c_1 \\ -c_1 & c_0 \end{pmatrix} \begin{pmatrix} \int \frac{E_x}{E_z}\, dz \\ \int \frac{E_y}{E_z}\, dz \end{pmatrix}$$

$$\begin{pmatrix} \delta_{xB} \\ \delta_{yB} \end{pmatrix} = \begin{pmatrix} c_2 & -c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \int \frac{B_x}{B_z}\, dz \\ \int \frac{B_y}{B_z}\, dz \end{pmatrix}$$

Cylindrical Coordinates:

$$\begin{pmatrix} \delta_{rE} \\ r\delta_{\phi E} \end{pmatrix} = \begin{pmatrix} c_0 & c_1 \\ -c_1 & c_0 \end{pmatrix} \begin{pmatrix} \int \frac{E_r}{E_z}\, dz \\ \int \frac{E_\phi}{E_z}\, dz \end{pmatrix}$$

$$\begin{pmatrix} \delta_{rB} \\ r\delta_{\phi B} \end{pmatrix} = \begin{pmatrix} c_2 & -c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \int \frac{B_r}{B_z}\, dz \\ \int \frac{B_\phi}{B_z}\, dz \end{pmatrix}$$

and the z distortion, to first order, is:

$$\delta_z = \int \frac{v'(E)}{v_0} (E - E_0)\, dz$$

where v_0 is the nominal drift velocity and the first order variation of the drift velocity, $v'(E)$, can be obtained from direct measurements or through MagBoltz simulations. (e.g. see Fig. 1)

The tensor terms, needed above, can be measured by fitting the constants of motion, c_i, to real data or through MagBoltz simulations. Thereafter, any other distortion can be calculated by using the constants of motion as shown below:

$$c_0 = \frac{1}{(1+T_2^2\omega^2\tau^2)}, \qquad c_1 = \frac{T_1\omega\tau}{(1+T_1^2\omega^2\tau^2)}, \quad \text{and} \quad c_2 = \frac{T_2^2\omega^2\tau^2}{(1+T_2^2\omega^2\tau^2)}$$

with

$$\omega\tau = \frac{-10 * B * v_0}{\|E\|} \quad \frac{[kGauss][cm/\mu\text{sec}]}{[V/cm]}$$

Note that $\omega\tau$ is a signed quantity and the sign of the B field is important in order to obtain the correct sign for the constants of the motion when the B field polarity is reversed. The drift velocity, v_0, and the magnitude of the electric field, $\|E\|$, are always positive in this equation. The minus sign comes about because we have assumed that the drifting particle is an electron and so $q = -1$.

Conclusions

- Uncertainty in C_0 affect electric distortions in the radial direction
  - We are not very sensitive to radial distortions due to direction of track and due to the discretized location of pads in the radial direction (i.e. we only interpolate along the pad rows)
  - $C_1 \sim 2^*C_0$ and so the radial distortions are further suppressed by the magnitude of C_0
- Uncertainty in C_2 affect magnetic distortions in the radial direction
  - We are not very sensitive to radial distortions due to direction of track and due to the discretized location of pads in the radial direction (i.e. we only interpolate along the pad rows)
  - The magnitude of the distortions in radial direction is magnified relative to $r\bullet\phi$ distortions … but driven by $B\phi$ distortions which are always small
- Conclusion
  - 5% error on C_0 affects radial distortions, only, and is suppressed relative to $r\bullet\phi$ distortions … this does not significantly affect our tracking
  - 2% error in C_2 does not significantly affect our overall goal of ~2% error