The Equation of State for the Hagedorn Thermostats

L. G. Moretto[1], K. A. Bugaev[1,2], J. B. Elliott[1] and L. Phair[1]

[1] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*
[2] *Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*

The concept of the Hagedorn thermostat, which was introduced in [1], allowed us to explicitly demonstrate very special thermodynamic properties of the systems with the Hagedorn mass spectrum [2, 3] (H-systems hereafter):

$$\rho_H(m) = M^{-1} \exp\left(m/T_H\right), \tag{1}$$

where m is the mass of the hadron in question and T_H is the Hagedorn temperature, M is the normalization constant with the dimension of the mass.

Here we study the equation of state (EOS) of N Hagedorn thermostats and its peculiar properties. This EOS can be found directly from the microcanonical partition of N hadrons with the mass spectrum (1):

$$\Omega = \frac{V^N}{N!}\left[\prod_{k=1}^{N}\int d\,m_k\ \rho_H(m_k)\int\frac{d^3p_k}{(2\pi)^3}\right]\delta\left(E-\sum_{i=1}^{N}\epsilon_i\right), \tag{2}$$

where the quantity $\epsilon_k(m_k,p_k) \approx m_k + \frac{p_k^2}{2m_k}$ denotes the non-relativistic energy of the Hagedorn resonance with the 3-momentum $\vec{p}_k$, whereas V and E denote the system's volume and energy, respectively.

The evaluation of the microcanonical partition (2) can be done by the procedure used in [1]. Then the concentration of the Hagedorn resonaces of mass m reads as

$$\frac{N(m)}{V} = \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}}. \tag{3}$$

This expression gives the equilibrium value of particle density for H of mass m. For a known probability density $1/M$ of H states in mass, the energy/mass conservation

$$\begin{aligned} E &= \int\limits_{m_{min}}^{m_{max}}\frac{d\,m}{M}\,N(m)\left(m + \frac{3}{2}T_H\right) \\ &= V\int\limits_{m_{min}}^{m_{max}}\frac{d\,m}{M}\left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}}\left(m + \frac{3}{2}T_H\right), \end{aligned} \tag{4}$$

determines an upper value m_{max} for the Hagedorn mass.

Given E, the total mass/energy of the initial H particle, Eq. (4) is an implicit equation for mass vs. E. The physical implication of these results is interesting:

1) Since the Hagedorn is assumed to transform only into other Hagedorns, a gas of Hagedorns must be at saturation with itself, i.e. the concentrations of the various masses below m_{max} cannot change with volume, and, of course, the temperature remains fixed at T_H;

2) However, as the volume increases/decreases, the upper cut-off m_{max} decreases/increases according to the conservation law expressed by (4). It can be shown that for large volumes the total number of Hagedorns in the system saturates (see Fig. 1. for details).

In any case, the distribution is dominated by the largest Hagedorn mass m_{max}. We can also, trivially, define the Hagedorn gas equation of state, which for $m_{min} = 0$ acquires a simple form (P is the pressure)

$$N_{tot} = \int\limits_{0}^{m_{max}}\frac{d\,m}{M}N(m) = \frac{2\,m_{max}}{5\,M}V\left(\frac{m_{max}T_H}{2\pi}\right)^{\frac{3}{2}} \tag{5}$$

$$P = \frac{N_{tot}\,T_H}{V}, \quad \frac{E}{V} = \frac{3}{2}P + \gamma P^{\frac{7}{5}}. \tag{6}$$

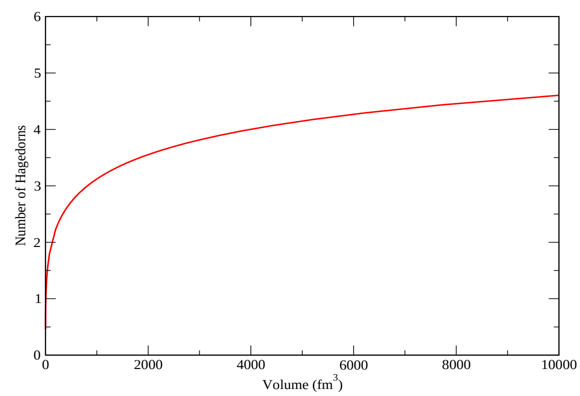

FIG. 1: A typical behavior of the total number of Hagedorns N_{tot} for $E = 4$ GeV as function of the volume V. For large volumes one can see the saturation due to energy conservation. The value of M was chosen to reproduce the number of all hadronic resonances lighter then 1.8 GeV.

Here the constant γ is defined as $\gamma = \frac{5}{7T_H^2}\left[50\pi^3M^2\right]^{1/5}$. From the equation of state (6) one can determine the speed of sound c_s as

$$c_s^2 = V\frac{dP}{dE} = 1/\left[\frac{3}{2} + \gamma\,P^{2/5}\right].$$

The latter vanishes in the high pressure limit or at small volumes $V \to 0$ and fixed E.

[1] L. G. Moretto *et. al.*, arXiv:nucl-th/0504010 (2005); K. A. Bugaev *et. al.*, arXiv:hep-ph/0504011 (2005).
[2] Hagedorn R., Suppl. Nuovo Cimento **3**, 147 (1965).
[3] Hagedorn R. and Ranft J., Suppl. Nuovo Cimento **6**, 169 (1968).