The Hagedorn Thermostat: The Source of Trouble with the Limiting Temperature

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Recently we showed [1] that a system $\mathcal{H}$ possessing a Hagedorn-like spectrum,

$$\rho_{\mathcal{H}}(m) \approx \exp\left(m/T_{\mathcal{H}}\right), \qquad (1)$$

characterized by an entropy of the form

$$S = \epsilon V/T_B = m/T_B \qquad (2)$$

not only has a unique microcanonical temperature $T_{\mathcal{H}}$

$$T_{\mathcal{H}} = \left(dS/dE\right)^{-1} = T_B, \qquad (3)$$

but also imparts this same temperature to any other system to which $\mathcal{H}$ is coupled. In the language of thermodynamics, $\mathcal{H}$ is a perfect thermostat with constant temperature $T_{\mathcal{H}}$.

In Eq. (1), m is the mass of the hadron in question and $T_{\mathcal{H}}$ is the parameter (temperature) controlling the exponential rise of the mass spectrum [2, 3], and in Eq. (2), V denotes the bag volume.

Such form of the entropy, defined by Eq. (2), leads to a bag mass spectrum $\exp(S)$ identical to Eq. (1) [4, 5]. This property implies that any surface energy associated with the bag is negligible.

As we discussed in [1], a perfect thermostat is indifferent to the transfer of any portion of its energy to any parcel within itself, no matter how small. In other words, it is at the limit of phase stability and the internal fluctuations of its energy density are maximal. Therefore it does not matter whether this thermostat is one large bag or it is fragmented in an arbitrary number of smaller bags or, equivalently, it is a system of hadrons with a spectrum given by Eq. (1). This fact has several important consequences on the thermodynamic properties of $\mathcal{H}$.

It was found long ago [6] that the exponential mass spectrum (1) leads to nonequivalence between the microcanonical and (grand)canonical ensembles, but the striking consequences of this fact along with the source of the trouble were never thoroughly analyzed until recently. This led to multiple confusions and erroneous conclusions based on results obtained by canonical and grand canonical treatments of the Hagedorn mass spectrum (1). Here we briefly discuss the source of trouble on a well-known example *which has nothing to do with hadronic resonances* and can be easily tested in every kitchen.

The insertion of an exponential mass spectrum (1) in the canonical partition function

$$\mathcal{Z}(T) = \int\limits_0^\infty \rho_{\mathcal{H}}(E)\, e^{-\frac{E}{T}}\, dE \qquad (4)$$

led to the incorrect conclusion that the entire range of temperatures $0 \le T < T_{\mathcal{H}}$ is accessible with $T_{\mathcal{H}}$ as the limiting temperature of the system.

In order to see the origin of this erroneous conclusion, let us consider the following illuminating example. Consider a system A composed of ice and water at standard pressure. For such a system the coexistence temperature (in Kelvin) is $T_A = 273$ K. Because of coexistence, we can input or extract heat to/from the system without changing T_A. The system A is a thermostat.

If a quantity $Q = E - E_0$ of heat is added to the system with the initial energy E_0, the change in entropy is

$$\Delta S = Q/T_A. \qquad (5)$$

The level density of A is then

$$\rho(Q) = e^{S_0}\, e^{Q/T_A} = K\, e^{E/T_A}. \qquad (6)$$

The level density, or spectrum, is exponential in E and depends only on the intrinsic "parameter" T_A. The partition function of A is:

$$\begin{aligned} Z(T) &= \int\limits_0^\infty K\, e^{E/T_A} e^{-E/T} dE \\ &= \int\limits_0^\infty K\, e^{-\left(\frac{1}{T}-\frac{1}{T_A}\right)E} dE = K \frac{T_A T}{T_A - T}. \end{aligned} \qquad (7)$$

This seems to indicate that A can assume *any* temperature below T_A, which at first sight looks like the limiting value for the temperature. However, by original assumption, the only temperature possible for A is T_A.

Using this simple example we demonstrated that the microcanonical system with the exponential density of states (6) with *a single value of temperature* T_A generates the canonical partition function (7) which has no physical sense for any value of parameter $T \neq T_A$, and diverges at the true temperature T_A of the microcanonical system.

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