

# Thermodynamics of the MIT Bag with Surface Tension

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The standard bag model [1] expresses the “energy” of the bag as

$$\varepsilon V = H \equiv (\sigma T^4 + B)V, \quad \sigma = \frac{\pi^2}{30} \left[ g_g + \frac{7}{8} g_{q\bar{q}} \right] \quad (1)$$

where  $\sigma T^4$  is the energy density of translational motion of free massless quarks and gluons in the bag,  $\sigma = \frac{\pi^2}{30} \frac{95}{2}$  is the Stefan-Boltzmann constant counting the degrees of freedom of gluons  $g_g$  (spin, color) and (anti-)quarks  $g_{q\bar{q}}$  (spin, color and  $u, d, s$ -flavor), respectively, and  $B$  is the bag constant. The corresponding pressure is

$$p = \frac{\sigma}{3} T^4 - B. \quad (2)$$

The equilibrium condition,  $p = 0$ , leads to a bag temperature

$$T_B = \left[ \frac{3B}{\sigma} \right]^{\frac{1}{4}}. \quad (3)$$

Pictorially, the bag constant represents the pressure exerted by the hadronic vacuum on the partonic vacuum. In so far as  $B$  is a constant,  $T_B$  is a constant, irrespective of the bag size. Thus, the entropy can be written as

$$S = \int \frac{dH}{T_B} = \frac{H}{T_B} \quad \text{or} \quad \frac{m}{T_B}, \quad (4)$$

where  $m$  is the bag mass. The degeneracy of the bag,  $\rho$ , is then

$$\rho(m) \sim e^S = e^{\frac{m}{T_B}}. \quad (5)$$

This extraordinary result (Hagedorn spectrum [2]), leads to the conclusion that any system coupled to a bag must be at temperature  $T_B$ . No other temperature is admissible. The consequences of this entropy form on the equilibrium of bags with other particles or among themselves has been described elsewhere [3, 4].

The bag expressions reported above contain only volume terms. Given the finite size of the bags that are interpreted as resonances, it may be of interest to consider finite size effects and their role in the description of the bags properties. The simplest generalization, assuming that the bags are leptodermous (which is supported by the short range of hadron-hadron interaction and by the saturating properties implicit in Eq. (1)) is that of introducing a surface energy.

Thus, the pressure of a spherical bag can be written as

$$p = \frac{\sigma}{3} T^4 - B - a_s(T) V^{-\frac{1}{3}} = \frac{\sigma}{3} T^4 - B - \frac{a_s(T)}{\alpha R}, \quad (6)$$

where  $a_s(T)$  is the temperature dependent surface energy coefficient,  $R$  is the bag radius and  $\alpha \equiv \left[ \frac{4\pi}{3} \right]^{\frac{1}{3}}$ . Using the thermodynamic identities for the free energy  $F$  and entropy  $S$

$$p = - \left( \frac{\partial F}{\partial V} \right)_T, \quad \text{and} \quad S = - \left( \frac{\partial F}{\partial T} \right)_V, \quad (7)$$

one can find all thermodynamic functions as follows

$$F = - \left[ \frac{\sigma}{3} T^4 - B \right] V + \frac{3}{2} a_s(T) V^{\frac{2}{3}}, \quad (8)$$

$$S = \frac{4\sigma}{3} T^3 V - \frac{3}{2} \frac{da_s(T)}{dT} V^{\frac{2}{3}}, \quad (9)$$

$$\varepsilon V = [\sigma T^4 + B] V + \frac{3}{2} \left[ a_s(T) - \frac{da_s(T)}{dT} \right] V^{\frac{2}{3}}. \quad (10)$$

In evaluating the expression (8) we fixed the integration constant (an arbitrary function of  $T$ ) to zero because for the bag of zero volume the free energy should vanish.

While theoretical input on the magnitude of  $a_s(T) > 0$  is needed the consequences of this surface term are surprising. In Eq. (6) the surface term appears as an additional pressure to the bag pressure. Therefore, setting the total pressure to zero  $p = 0$ , we obtain for the bag temperature

$$T = T(R) = \left[ \frac{3}{\sigma} \left( B + \frac{a_s(T)}{\alpha R} \right) \right]^{\frac{1}{4}}. \quad (11)$$

When  $R$  is large we recover the previous bag temperature and the associated physics. When  $R$  becomes small, however, the bag temperature increases! The implications of his dependence are strange indeed. The first is the peculiar behavior of the bag’s heat capacity. The second is the stability of the gas of bags (or lack thereof). The third is the signature of a bag’s decay.

[1] A. Chodos *et. al.*, Phys. Rev. **D 9**, 3471 (1974).

[2] Hagedorn R., Suppl. Nuovo Cimento **3**, 147 (1965).

[3] L. G. Moretto *et. al.*, arXiv:nucl-th/0504010 (2005).

[4] K. A. Bugaev *et. al.*, arXiv:hep-ph/0504011 (2005).