Finite volume corrections to $\pi\pi$ scattering

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Except in the case of infinitely heavy baryons, where an adiabatic potential can be defined, the interaction between two hadrons is studied with lattice QCD by Monte Carlo calculation of energy levels of the entire system. In the infinite volume limit and away from kinematical thresholds, the two-hadron Euclidean correlator gives no information about the physical observable in Minkowski space. Thus the simulation in the finite box is necessary, as is the case with numerical calculations anyway. Energy level of two-hadron system is given not only by their masses, but also their interaction energies. The later has a inversely proportional relation, which we call the power law relation, to the lattice volume by the discretization. Furthermore, there is a relation between energy level shifts and the S-wave scattering phase shifts [1]. This relation, valid for energies below the inelastic threshold, is a consequence of unitarity and therefore is model independent.

Finite volume effects arise because the propagation of intermediate states is altered by the periodic boundary. Such effects are dominated by pions, the lightest particle of the theory. On-shell intermediate particle can live long and propagator a distance of order $1/m_\pi$, therefore is most sensitive to the size of the volume. Among all one-loop diagrams of two-flavor chiral perturbation theory, only the s-channel diagrams provide such on-shell contribution of pion propagation.

In addition to this power law shift in the energy levels, there are exponentially suppressed corrections which are not model independent. Such exponential volume effects arise because the off-shell propagation of intermediate state is altered by the periodic box. Since off-shell intermediate states are short-lived, the signals are suppressed by a factor $e^{-m_\pi L}$ with m_π the pion mass and L the linear dimension of the cube. Such propagations exist in all channels in one-loop calculation. Analysical calculation of exponentially suppressed corrections on $I = 2$ pion-pion system is reported in Ref. [2], using two-flavor chiral perturbation theory. For simulations done with small enough quark masses such that the pions are within the chiral regime, these soft pion effects can be computed using the chiral perturbation theory.

We have computed finite volume correction to the function $k\cot\delta$, where k is the momentum and δ is the S-wave scattering phase shift, as given in Fig. 1. This amount has to be subtracted when one determines phase shifts from finite volume study of two-pion scattering by the universal power-law relation. We find that the finite volume corrections are relatively small, a few times smaller than the statistical and systematic errors quated in the past. However, the finite volume corrections grow fast with the approach to the chiral limit and they become non-neglibible as smaller pion masses are used and numerical errors are reduced in simulations.

[1] M. Lüscher, Nucl. Phys. B **354**, 531 (1991).

[2] P. F. Bedaque, I. Sato and A. Walker-Loud, Phys. Rev. D **73**, 074501 (2006) [arXiv:hep-lat/0601033].

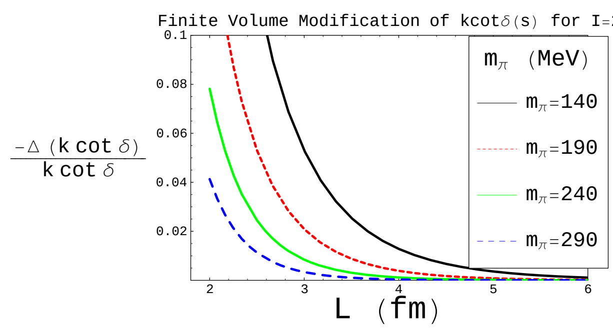

FIG. 1: Ratio of the magnitude of the exponential correction term $\Delta(k\cot\delta)$ to $k\cot\delta$ for different values of the pion mass.