# Some Achievements and Challenges in Particle Correlations

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**Twenty Years of Collective Expansion**
**TBS – Berkeley – May/23/05**

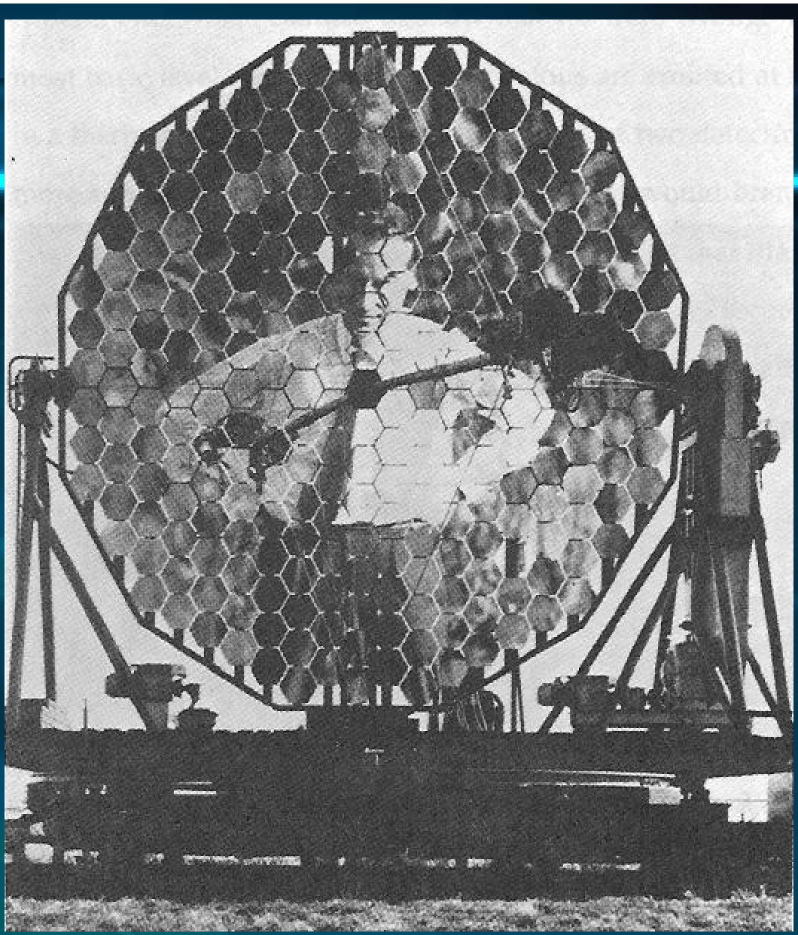
# In the beginning…

- **Hanbury Brown – Twiss**
  - **Mid-50's → proposed new interferometric technique for measuring stellar radii:**
    - **For solving limitations in Michelson type of Interferometry in determining stellar sizes (resolution ↑ for increasing distance between receivers, which decreased phase stability) ⮩**
    - **Apparatus measured amplitude and phase of oscillations (both necessary to determine uniquely the source distrib.).**
  - **Designed for reducing need of phase stability → output of the 2 receivers COMBINED in CORRELATOR (took into account the time delay between the two receivers) →**
    - » **relative phases of the 2 signals were lost**
    - » **Measured correlation in intensity fluctuations only**
  - **R. Q. Twiss developed mathematical tool for 2$^{nd}$ order interference (intensity interferometry)**

# 50 years of HBT

**R. Hanbury Brown & R. Q. Twiss, Phil. Mag. 45 (1954) 663; Nature 177 (56) 27; 178 (56) 1447**

- First: experiments with radio sources
- Later experiments in optical astronomy
  - pilot model → Jodrell Bank (UK)
  - HB & T built apparatus and made the experiment (Sirius) in Narrabri, Australia

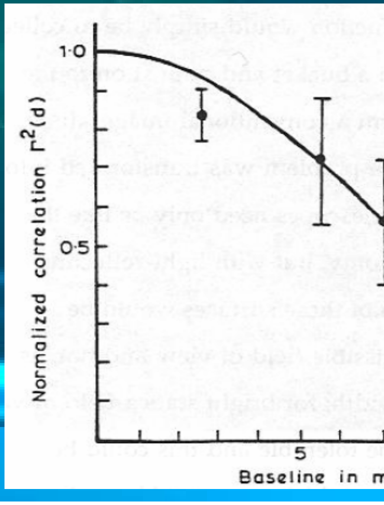


## 1st achievement

**Normalized Correlation coeff.**

$$\boxed{\Gamma^2(d) = [2J_1(x)/x]^2 \;\; ; \;\; x = \pi\theta d/\lambda}$$

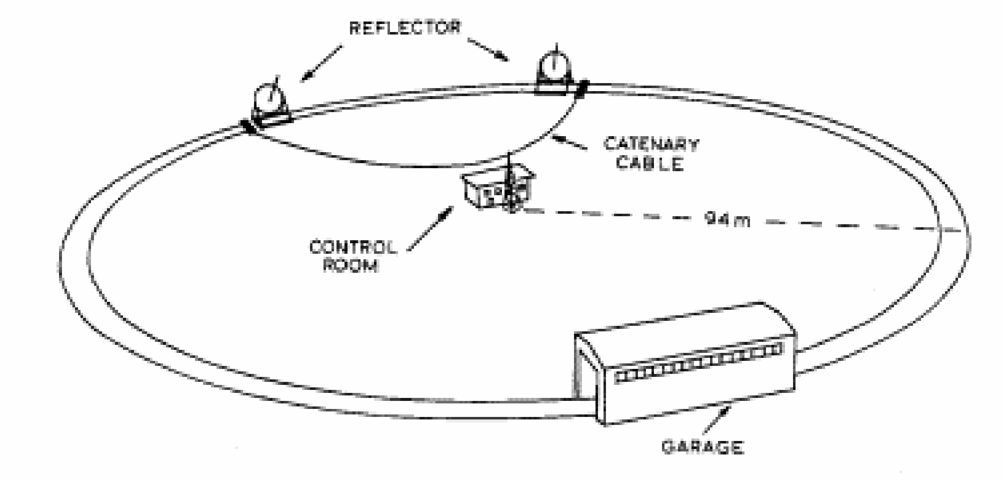
$\lambda \rightarrow$ <wave-length> observed light
$d \rightarrow$ distance between 2 mirrors
$\theta \rightarrow$ angle subtended by star
(stars ⇔ luminous disks)
Plot → $\theta = 6.3\ 10^{-3}$ sec of arc




**Example of experimental Result (Sirius)**

# Original HBT apparatus

…"Collecting light as rain in a bucket…" (RHB)




➡ no need for conventional image: telescopes ≈ paraboloids used for radio-astronomy (but with light-reflecting surfaces) ↦ necessary precision of surfaces governed by maximum permissible field of view.

▸ two mirrors ⇔ each focusing the light of a star onto a photo-multiplie ⊕ Correlator (electronic device: receives signals from 2 mirrors and multiplies).

➡ Difficulties to convince the community ⇔ photons tended to arrive in pairs at the two correlators [helped by Purcell, Nature 178 (1956) 1449].

**1st challenge**

## Hanbury Brown & Twiss:

**Nature 178 (**56) 1447

- **Argue that no incompatibility existed among the HBT experiment and the others perfomed by 2 groups that could no reproduce their results**

# THE QUESTION OF CORRELATION BETWEEN PHOTONS IN COHERENT LIGHT RAYS

By R. HANBURY BROWN

Jodrell Bank Experimental Station, Cheshire

AND

R. Q. TWISS

Radiophysics Laboratory, Sydney

A RECENT article by Brannen and Ferguson[1] casts doubt on our experimental demonstration[2] that the times of arrival of photons, defined as the times of ejection of photoelectrons, are correlated in two coherent beams of light. These authors quote the null result obtained by Adám, Jánossy and Varga[3] and describe in detail an experiment which they have performed themselves and which also gave a null result. They state that the existence of such a correlation would call for "a major revision of some fundamental concepts in quantum mechanics".

This last statement is in direct contradiction to our own conclusions; however, further argument should perhaps be left until our analysis, based on quantum mechanics, has been published. In the present communication we are concerned only to show that the results of Brannen and Ferguson, together with those of Adám *et al.*, are to be expected, and that they are not inconsistent with our own experiment.

The optical system described by Brannen and Ferguson was practically identical with our own, but they used a different method of detecting the correlation. A beam of light from a pinhole was divided by a half-silvered mirror into two coherent beams which illuminated independent photomultipliers. The rate of coincidence in time between the photoelectrons emitted from the two photocathodes was measured by conventional coincidence counters and compared with the random rate. It was found that the measured rate did not differ significantly from the random value, and that less than 0·01 per cent of the photons in the two beams could be in true time coincidence. On the basis of a similar experiment, Adám *et al.*[3] concluded that there was no evidence for correlation of photons and that less than 0·6 per cent could be in true coincidence.

The theoretical coincidence-rate for these two experiments does not appear to have been evaluated, and so we have derived it as follows. The average number $N_0$ of photoelectrons emitted by a photocathode in unit time is given by:

$$N_0 = \int_0^\infty \alpha(\nu) n_0(\nu) d\nu = B\alpha(\nu_0) n_0(\nu_0) \qquad (1)$$

where $\alpha(\nu)$ is the quantum efficiency of the cathode surface at a frequency $\nu$; $n_0(\nu)$ is the number of quanta of frequency $\nu$ incident on the cathode in unit time and unit band-width; $\nu_0$ is the frequency of the light at which $\alpha(\nu)n_0(\nu)$ has its maximum value; $B$ is the equivalent band-width of the light in cycles per second. If $\tau$ is the resolving time of the equipment and $2\tau N_0 \ll 1$, a condition which must apply to this technique, then the average number of random coincidences $\bar{C}_R$ to be expected in a time $T_0$ is given by:

$$\bar{C}_R = 2\tau N_0^2 T_0 \qquad (2)$$

When the light beams are correlated we have shown (unpublished work) that, under practical conditions, there will be an additional average number of coincidences $C_S$, where

$$C_S = \frac{T_0}{2}\int_0^\infty \alpha^2(\nu) n_0^2(\nu) d\nu \,.\, \Phi(\theta D/\lambda_0) \qquad (3)$$

and where $\theta$ is the apparent angular diameter of the source viewed from the photocathodes; the photocathodes are squares of size $D \times D$; $\lambda_0$ is the mean wave-length of the light; it is assumed that the light is unpolarized and that $B \gg 1/\tau$. The quantity $\Phi(\theta D/\lambda_0)$ is a function which represents the degree of coherence of the light over the area of the photocathodes and is equal to unity when $(\theta D/\lambda_0) \ll 1$.

It is convenient to introduce a parameter $\sigma$, the spectral density of the light, defined by:

$$\alpha(\nu_0) n_0(\nu_0)\sigma = \int_0^\infty \alpha^2(\nu) n_0^2(\nu) d\nu \Big/ \int_0^\infty \alpha(\nu) n_0(\nu) d\nu \qquad (4)$$

- demonstrate by estimates that the method to measure the correlation used by the others were inefficient (despite similar optical devices)
  → Would require data taking interval extremely large

Table 1. A COMPARISON OF THE THEORETICAL PERFORMANCE OF THREE EQUIPMENTS USED TO TEST THE CORRELATION BETWEEN PHOTONS IN TWO COHERENT BEAMS OF LIGHT

| | | Brannen and Ferguson (ref. 1) | Adám, Jánossy and Varga (ref. 3) | Hanbury Brown and Twiss (ref. 2) |
|---|---|---|---|---|
| Total number of photoelectrons incident per sec. on each cathode | $N_0$ | $5 \times 10^4$ ($\alpha = 0 \cdot 1$) | 100 ($\alpha = 0 \cdot 003$) | $3 \times 10^9$ ($\alpha = 0 \cdot 15$) |
| Resolving time of the equipment (sec.) | $\tau$ | $5 \times 10^{-9}$ | $2 \times 10^{-8}$ | $10^{-8}$* |
| A factor which expresses the degree of coherence of the light over each cathode | $\Phi\ \frac{\theta D}{\lambda_0}$ | $0 \cdot 1$ | $0 \cdot 1$ ? | $0 \cdot 5$ |
| Effective band-width of the light (c./s.) | $B$ | $\geqslant 1 \cdot 5 \times 10^{11}$ ($\sigma = 1$) | $\geqslant 1 \cdot 5 \times 10^{11}$ ($\sigma = 1$) | $10^{11}$ ($\sigma = 0 \cdot 5$) |
| Fractional number of photons to be expected in true time-coincidence | $\rho_c$ | $\leqslant 1 \cdot 7 \times 10^{-3}$ | $\leqslant 10^{-3}$ | $2 \cdot 5 \times 10^{-4}$* |
| Time of observation to obtain a significant correlation (3 times r.m.s. fluctuation) | $T$ | $\geqslant$ 1,000 years | $\geqslant 10^{11}$ years | 5 min. |

* A coincidence counter was not used in this experiment. The correlation was measured in a linear multiplier, using a band-width of about 30 Mc./s.

## Purcell (same letter)
## [Nature **178** (56) 1447]

- refutes criticism of the two experiments; calls HBT "notable achievement"
- demonstrates that the HBT experiment does NOT contradict Quantum Mechanics. On the contrary, the observed photon behavior reflects the Bose-Einstein statistics
- antecipates that, in the case of fermions, "negative cross-correlation" would be observed, since they obey Fermi-Dirac stat.

No. 4548 December 29, 1956 NAT

Brannen and Ferguson[1] have reported experimental results which they believe to be incompatible with the observation by Hanbury Brown and Twiss[2] of correlation in the fluctuations of two photoelectric currents evoked by coherent beams of light. Brannen and Ferguson suggest that the existence of such a correlation would call for a revision of quantum theory. It is the purpose of this communication to show that the results of the two investigations are not in conflict, the upper limit set by Brannen and Ferguson being in fact vastly greater than the effect to be expected under the conditions of their experiment. Moreover, the Brown–Twiss effect, far from requiring a revision of quantum mechanics, is an instructive illustration of its elementary principles. There is nothing in the argument below that is not implicit in the discussion of Brown and Twiss, but perhaps I may clarify matters by taking a different approach.

Consider first an experiment which is simpler in concept than either of those that have been performed, but which contains the essence of the problem. Let *one* beam of light fall on *one* photomultiplier, and examine the statistical fluctuations in the counting-rate. Let the source be nearly monochromatic and arrange the optics so that, as in the experiments already mentioned, the difference in the length of the two light-paths from a point $A$ in the photocathode to two points $B$ and $C$ in the source remains constant, to within a small fraction of a wave-length, as $A$ is moved over the photocathode surface. (This difference need not be small, nor need the path-lengths themselves remain constant.) Now it will be found, even with the steadiest source possible, that the fluctuations in the counting-rate are slightly greater than one would expect in a random sequence of independent events occurring at the same average rate. There is a tendency for the counts to 'clump'. From the quantum point of view this is not surprising. It is typical of fluctuations in a system of bosons. I shall show presently that this extra fluctuation in the single-channel rate necessarily implies the cross-correlation found by Brown and Twiss. But first I propose to examine its origin and calculate its magnitude.

Turning now to the split-beam experiment, let $n_1$ be the number of counts of one photomultiplier in an interval $T$, and let $n_2$ be the number of counts in the other in the same interval. As regards the fluctuations in $n_1$ alone, from interval to interval, we face the situation already analysed, except that we shall now assume both polarizations present. The fluctuations in orthogonal polarizations are independent, and we have, instead of (2),

$$\overline{\Delta n_1^2} = \overline{n_1^2} - \overline{n}_1^2 = \overline{n}_1(1 + \tfrac{1}{2}\overline{n}_1\tau_0/T) \qquad (3)$$

where $n_1/T$ has been written for the average counting-rate in channel 1. A similar relation holds for $n_2$. Now if we should connect the two photomultiplier outputs together, we would clearly revert to a single-channel experiment with a count $n = n_1 + n_2$. We must then find:

$$\overline{\Delta n^2} = \overline{n}\,(1 + \tfrac{1}{2}\overline{n}\tau_0/T) \qquad (4)$$

But $\overline{\Delta n^2} = \overline{(\Delta n_1 + \Delta n_2)^2}$

$$= \overline{n}_1(1+\tfrac{1}{2}\overline{n}_1\tau_0/T) + \overline{n}_2(1+\tfrac{1}{2}\overline{n}_2\tau_0/T) + \overline{2\Delta n_1\Delta n_2} \qquad (5)$$

From (4) and (5) it follows that:

$$\overline{\Delta n_1 \Delta n_2} = \tfrac{1}{2}\overline{n}_1^2\tau_0/T \qquad (6)$$

This is the positive cross-correlation effect of Brown and Twiss, although they express it in a slightly different way. It is merely another consequence of the 'clumping' of the photons. Note that if we had separated the branches by a polarizing filter, rather than a half-silvered mirror, the factor 1/2 would be lacking in (4), and (5) would have led to $\overline{\Delta n_1\Delta n_2} = 0$, which is as it should be.

If we were to split a beam of electrons by a non-polarizing mirror, allowing the beams to fall on separate electron multipliers, the outputs of the latter would show a negative cross-correlation. A split beam of classical particles would, of course, show zero cross-correlation. As usual in fluctuation phenomena, the behaviour of fermions and the behaviour of bosons deviate in opposite directions from that of classical particles. The Brown–Twiss effect is thus, from a *particle* point of view, a characteristic quantum effect.

# GGLP ('59-'60)

- ⊳⊲ **1959** $\mapsto$ **empirical observation (Goldhaber, Goldhaber, Lee & Pais) [Phys. Rev. 120 (1960) 300]**
- ⊳⊲ $\overline{p}\,p$ **propane bubble chamber exp. at 1.05 GeV/c Bevatron (LBL): search for $\rho^o \to \pi^+\pi^-$ comparing $\pi^+\pi^-$ mass-distribution with $\pi^\pm\pi^\pm$**
- ⊳⊲ **Not enough statistics to establish the existence of $\rho^o$ but observed unexpected <u>angular correlation among identical $\pi$'s</u>!!**
- ⊳⊲ **1960** $\mapsto$ **reproduced angular distrib. by multi-$\pi$ phase-space calculation using symmetrized wave-function for like-particles**

$$\Updownarrow$$

- ⊳⊲ **Effect: consequence of <u>Bose-Einstein</u> nature of $\pi^+\pi^+$ and $\pi^-\pi^-$**
- ⊳⊲ **Parameterized the correlation as:**

$$\boxed{C(Q^2) = 1 + e^{Q^2 r^2}; Q^2 = -q^2 = -(k_1 - k_2)^2 = M_{12}^2 - (m_1 - m_2)^2}$$

- ⊳⊲ **GGLP** $\mapsto$ **NOT aware of the Hanbury-Brown and Twiss experiment**

# HBT at intermediate scale

- **HBT for the sonoluminescence bubble**

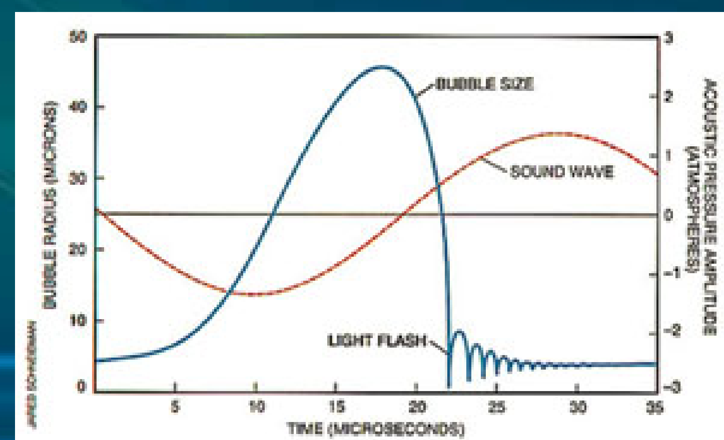
**[Trentalange and Pandey, J. Acoust. Soc. Am. 99 ('96) 2439 , Hama, Kodama & SSP, P.R. A56 ('97) 2233; Slotta & Heinz, P.R. E58 ('98) 526]** ✷

**Sonoluminescence → emission of light by bubbles in liquid excited by sound waves**

**Univv. Cologne,1934: Gaitan et al., 1988 → single bubble of gas (air) formed in water is trapped in standing acoustic wave emitted light with each pulsation**

**Each bubble formed grows experiencing vibrations, contracting each 10-12 psec., when emits light: bubble is so small & elapsed time between emissions is so brief ⇔ only estimates available → photon HBT for determining source size and life-time (✷ ).**

starfire.ne.uiuc.edu/~ne201/1995/levinson/sonolum.html




**Intermediate size scale ($R$ ~$10^{-5}m$), in between stars ($10^{10}m$), and HIC ($10^{-15}m$)**

## - Hama, Kodama and SSP

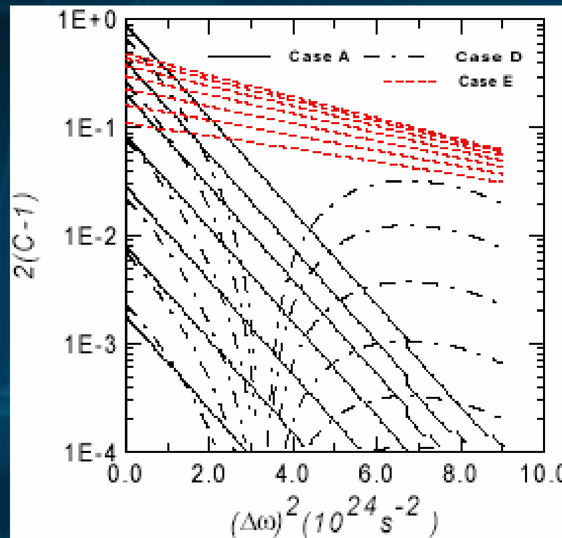
$X = q\sqrt{(-½)[d^2\Phi(0)\,/\,dq^2]}$

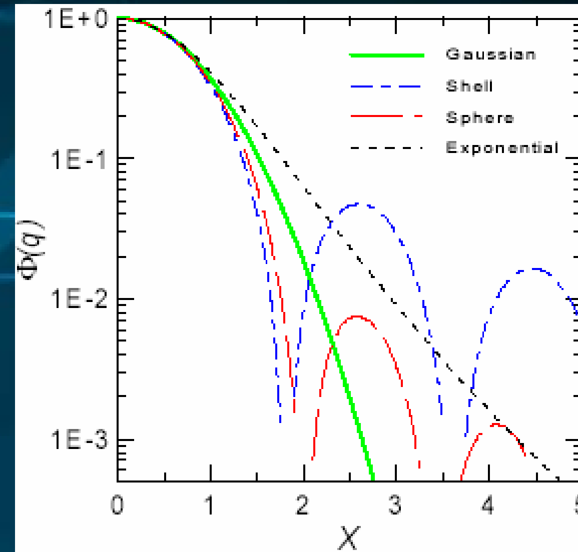
- Negligible phase-space correlations
- Space & time factorizes:

$$C(k_1,k_2) = 1 + \tfrac{1}{2}T(\Delta\omega)\Phi(q)$$

$$\Delta\omega = \omega_1 - \omega_2 \,\|\, q = k_1 - k_2$$

$2$-$\gamma$ **HBT from SBSL → select thermal models (chaotic) from Casimir (coherent) based ones**




– **Slotta & Heinz:**

» **Supplement previous suggestions with the variances approach, focusing in experimental limited range of accessible wave-lengths**

» **Measurements could be sensitive to sizes $\in (10$ nm – few $\mu$m)**

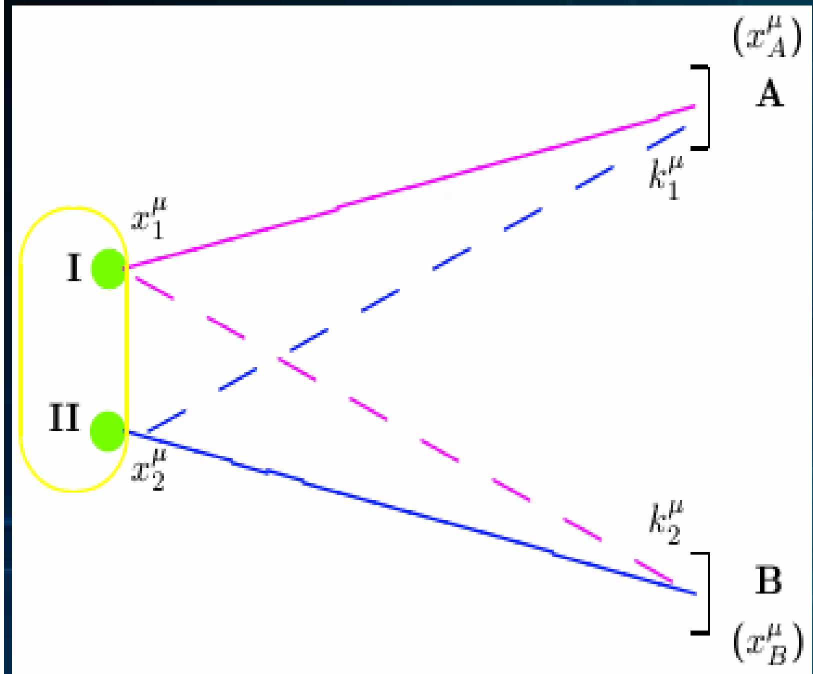
» **Technological limitations on frequency resolution: access of flash duration to pulse length < $0.1$ psec.**

» **Dynamics of the bubble not accessible (as considered in ✷)**

**•HBT sonoluminescence experiment: one in course [PRL 92(04)114301] but no published results yet (auto-correlations bigger than crossed ones)**

# Two-Particle Correlation or Interferometry or Second Order Interference

## ↔ Adequate Quantum Statistics ⊕ source chaoticity




<u>Simple Illustration</u> (2 sources):

emitted quanta ↔ plane waves

Amplitude for the process (sources **I** & **II** ):

$$A(k_1,k_2) = \frac{1}{2}[e^{-ik_1.(x_A-x_1)}e^{i\phi_1}e^{-ik_2.(x_A-x_2)}e^{i\phi_2} \pm e^{-ik_1.(x_A-x_2)}e^{i\phi_2'}e^{-ik_1.(x_A-x_1)}e^{i\phi_1'}]$$

- (+) ⇔ bosons; (−) ⇔ fermions
- $\phi_i$ ⇔ phases at emission (indep. on $k$)

(chaotic sources ⇔ different $\phi_i$ in each emission)

(average over phases ⇔ probability)

$$\left\langle e^{\pm i(\phi_1+\phi_2-\phi_1'-\phi_2')} \right\rangle = \delta_{\phi_1\phi_1'}\delta_{\phi_2\phi_2'} + \delta_{\phi_1\phi_2'}\delta_{\phi_2\phi_1'}$$

**Probability:**

$$P_2 = \left\langle |A(k_1,k_2)|^2 \right\rangle =$$

$$= \frac{1}{2}\left\{2 \pm \left[e^{i(k_1-k_2).(x_1-x_2)} \left\langle e^{\pm i(\phi_1+\phi_2-\phi_1^{'}-\phi_2^{'})} \right\rangle + c.c.\right]\right\} =$$

**Extended Sources**

$$= 1 \pm \cos[(k_1 - k_2).(x_1 - x_2)]$$

$$\boxed{q^\mu = k_1^\mu - k_2^\mu \;\; ; \;\; K^\mu = \frac{1}{2}(k_1^\mu + k_2^\mu)}$$

More generally: $\rho(x)$ is the normalized phase-space distribution

$$P_2(k_1,k_2) = P_1(k_1)P_1(k_2)\int d^4x_1 \int d^4x_2 \, |A(k_1,k_2)|^2 \, \rho(x_1)\rho(x_2)$$

$$= P_1(k_1)P_1(k_2)\left[1 + |\tilde{\rho}(q)|^2\right]$$

* $\tilde{\rho}(q) = \int d^4x \, e^{iq^\mu x_\mu} \rho(x)$ $\leftrightarrow$ Fourier transform of $\rho(x)$

Two-particle Correlation function →

$$\boxed{C(k_1,k_2) = \frac{P_2(k_1,k_2)}{P_1(k_1)P_1(k_2)} = 1 \pm \lambda |\tilde{\rho}(q)|^2}$$

* $\lambda(x)$ → incoherence or chaoticity parameter

Deutschmann et al., 1978) – for reducing sistematic errors (Gaussian fits)

# Simplest example

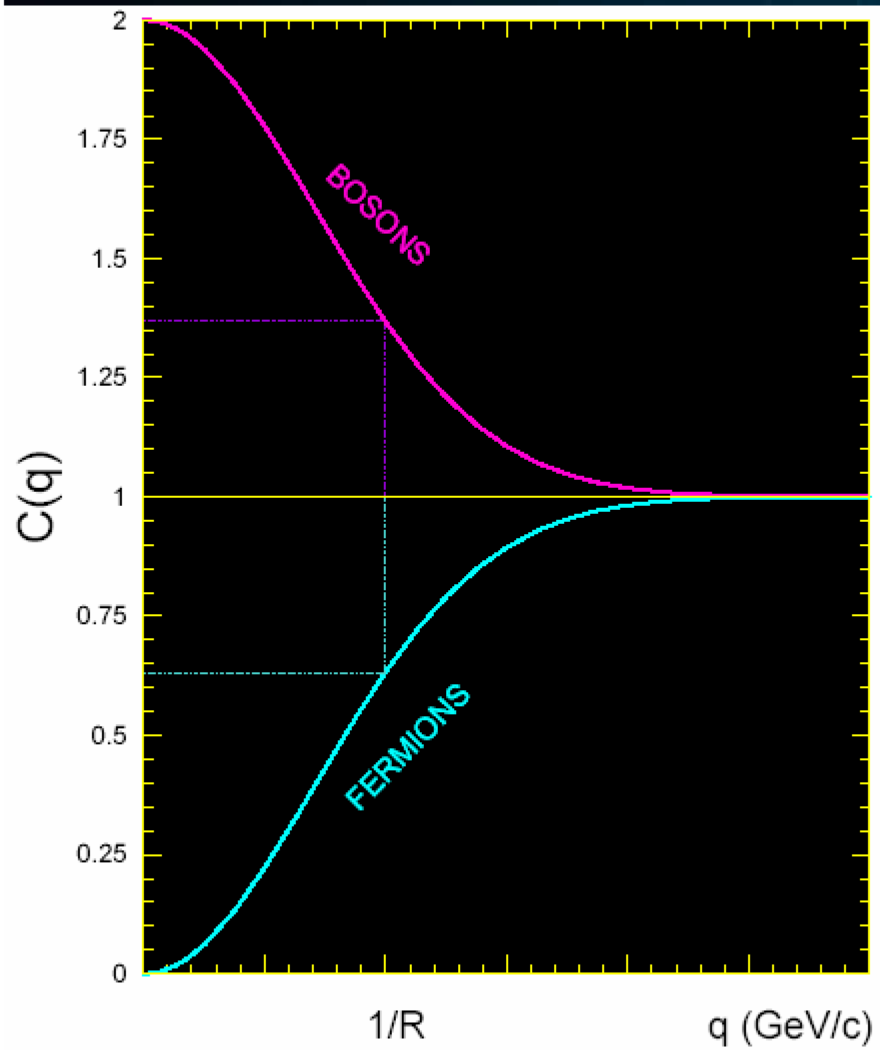
(decopled) Phase-space distribution:

$$f(x,p) = \rho(x)\ g(p)$$

$$\rho(x) \sim \exp[-x^2/(2R)^2]$$

$$\Updownarrow$$

2-$\pi$ Correlation Function

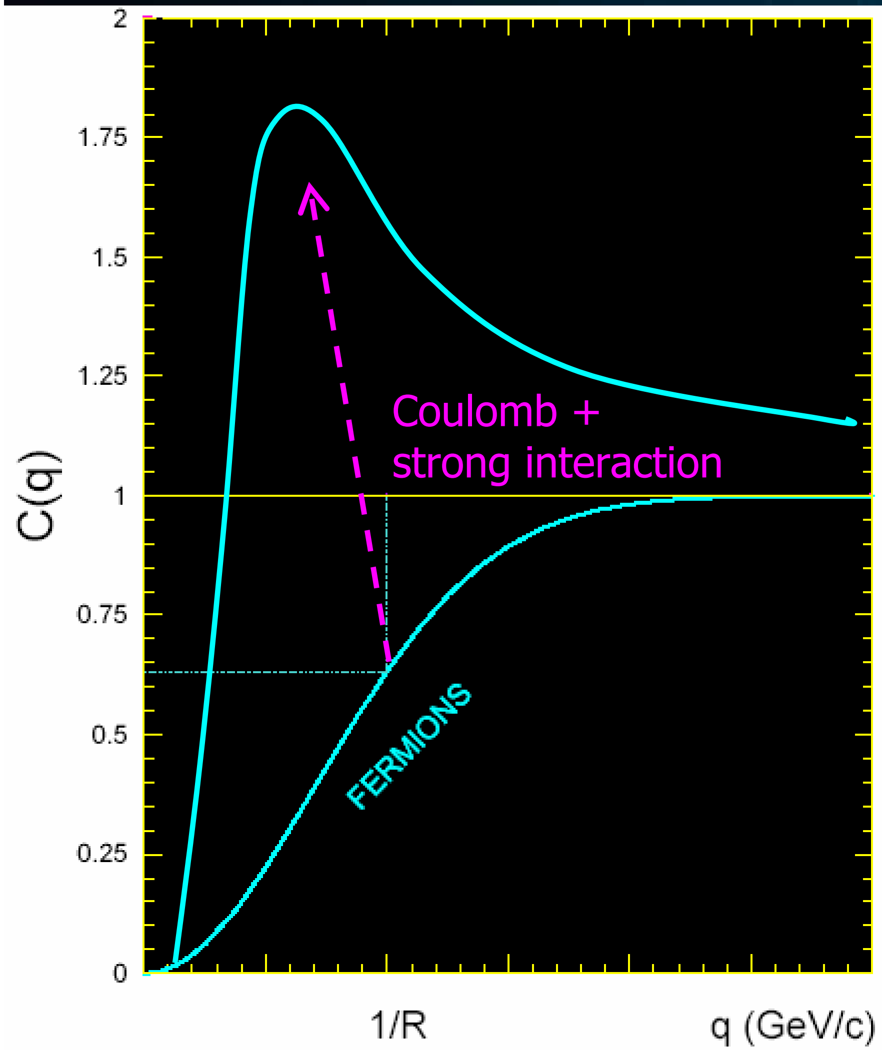



$$\boxed{C(k_1,k_2) = 1 \pm |\tilde{\rho}(q)|^2 = 1 \pm \exp(-q^2R^2)}$$

$R$= radius of the emitting source

(symmetrization but no interactions!)

# Simplest example




**(decopled) Phase-space distribution:**

$$f(x,p) = \rho(x)\ g(p)$$

$$\rho(x) \sim \exp[-x^2/(2R)^2]$$

$$\updownarrow$$

**2-$\pi$ Correlation Function**

$$\boxed{C(k_1,k_2) = 1 \pm |\tilde{\rho}(q)|^2 = 1 \pm \exp(-q^2R^2)}$$

$R$= **radius of the emitting source**

**In general** $\mapsto f(x,p) \neq \rho(x)\ g(p)$

$$\updownarrow$$

☹ **Simple Picture breaks down (*$K$* dep.)**

*** Model dependent analysis**

**(sensitive to underlying dynamics)**

***Recquires more general formalism**

**(Wigner, Cov. Current Ensenble, wave-packets around classical trajectory, etc.)**

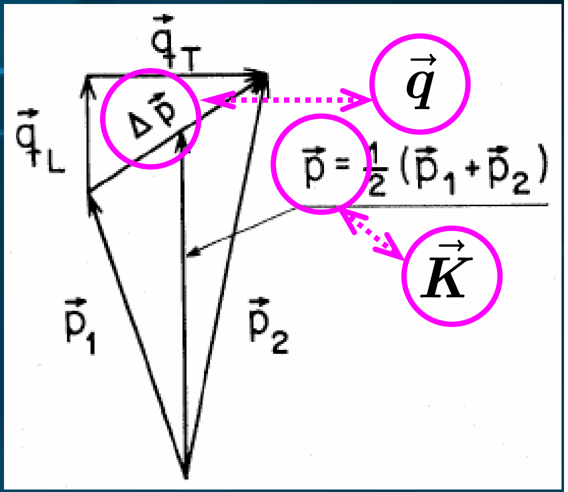
# History in the 70's: Models & aplications

♣ **Kopylov & Podgoretiskii**

**Example: emission from the surface of hard sphere with radius R**

$$\boxed{C(k_1,k_2) = 1 \pm \left[\frac{2J_1(q_T R)}{q_T R}\right]^2 [1 + (q_0 \tau)^2]^{-1}}$$

$$\approx 1 \pm \lambda \exp(-\tfrac{1}{2} q_T^2 R_T^2)\exp(-q_0^2 \tau^2)$$




$$|q_L| = q \cdot \frac{K}{|K|}; \; q_T = q - q_L$$

$$|q_0| = E_1 - E_1 \approx \frac{1}{2m}(p_1^2 - p_2^2) = \frac{1}{m}(p_1 - p_2)\frac{1}{2}(p_1 + p_2) \propto |q_L|$$

⊙ **Similar forms for studying:**

- **Lifetime of excited nuclei ⇔ interferometry of evaporated neutrons**
- **Shape and size of multiproduction region ⇔ correlations of $\pi^\pm\pi^\pm$**
  - **applied to CERN/ISR data on $pp$ & $\bar{p}p$**

## ♣ Many others:

**Shuryak; Cocconi; Fowler & Weiner; Giovannini & Veneziano; Grassberger; Yano & Koonin; Gyulassy, Kauffmann & Wilson, ... (+ exp. @ ISR,Bevalac...)**

⇕

**Models & formalisms, final state interactions, relation of resonances ⇔ λ**

**♣ Experimental fit ``Preference´´ ⇔ Gaussians (easier!)**

$$C\,(k_1,\ k_2) = 1 \pm \lambda\ \exp(-Q^2_{\mathrm{inv}} R^2) \qquad (pp,\ e^+e^-,\ \bar{p}\,p)$$

**OR**

$$C\,(k_1,\ k_2) = 1 \pm \lambda\ \exp(-\ q^2{}_0 \tau^2/2 - q^2{}_T R^2{}_T/2 - q^2{}_L R^2{}_L/2\ )$$

**Adapted to ↕ relativistic heavy ion collisions as**

- $q_L$ ⇔ along the beam direction
- $q_T$ ⇔ transversal to beam direction
- $q_0$ ⇔ time component

**Experimental definition of CF**

$$\boxed{C(k_1, k_2) = \frac{A(q)}{B(q)}}$$

**Signal (particles from same event)**

**Background (particles from ≠ events)**

# The 80's: further achievements

♣ Applications $\mapsto$ High Energy Collisions ($\bar{p}p$, $pp$, $e^+e^-$, heavy ions):

(1) ☀

Andersson & Hoffmann, Bowler, Byiajima, Suzuki, Pratt, Hama & SSP,

Makhlin & Sinyukov, Csörgö & Zimányi, Ornik, Plümer & Weiner,...

Static Gaussian can be misleading ⇔ HBT sensitive to geometry⊕dynamics

☀ (2) Gyulassy & SSP ⇔ inclusion of resonances ⊕ dynamical effects

(& general formalism to treat these cases)

Bertsch

↕

convenient names

to knwon quantities

$q_L$ ⇔ along the beam direction

$q_{out}$ ⇔ $\perp$ to beam // $K_T = (k_{1T}+k_{2T})/2$

$q_{sid}$ ⇔ $\perp$ to beam direction but $\perp$ to $K_T$

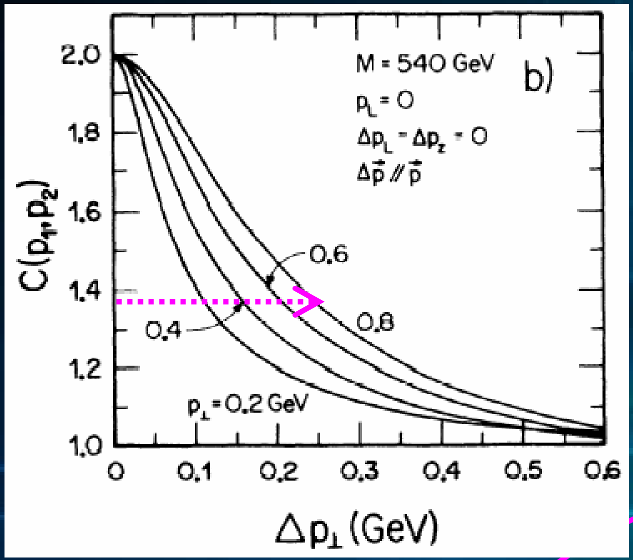
$q^0$ ⇔ temporal component

# Some findings...

(1) Y. Hama & SSP: P.R. D37 ('88) 3237

**Main hypotheses:**

- QGP formation ⊕ 1st order phase-transition
- Expanding system (1-D hydrodynamics)

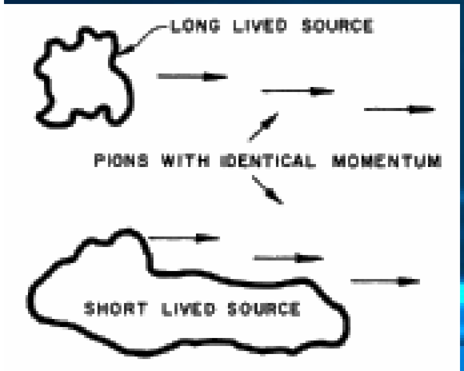



➡ Strong distortions in the correlation functions (departing from Gaussian shape):

✶ On the left: effects of ≠ emission times ( "depth effects" ) coming from dep. on $(p_1+p_2)/2$

also claimed by Pratt [PRD33 ('86) 72]

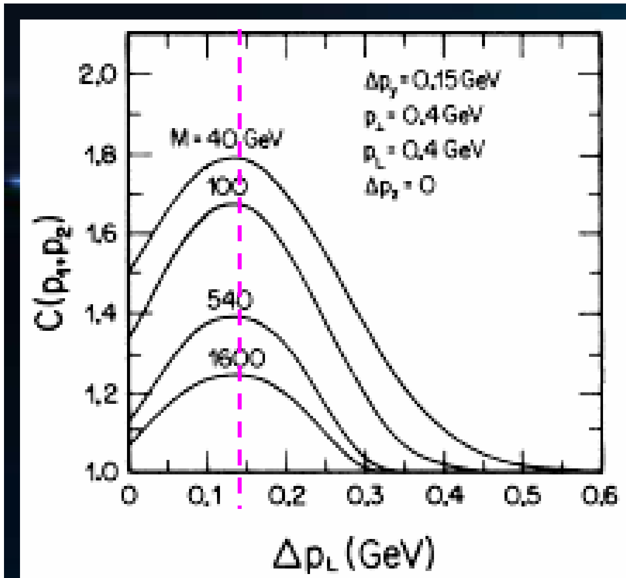
small & long-lived $\simeq$ large & short-lived




➡ Effective $R$ sizes probed by HBT correlation of particles emitted at large (⊥) angles, in the case of expanding sources, decreased with source rapidity

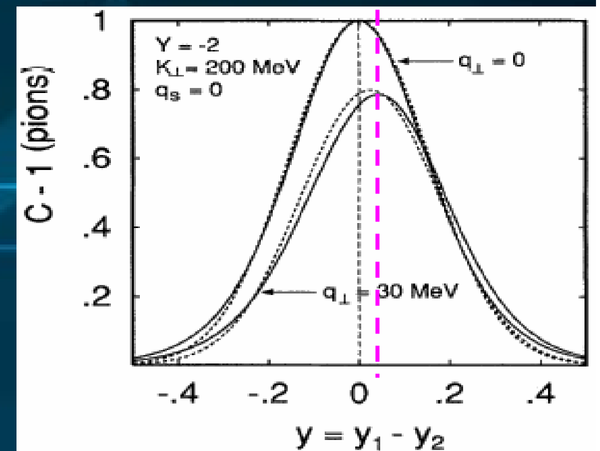
➡ Chaotic sources : maximum of $C(p_1,p_2) < 2$ at $q_T(\Delta p)=0$ trivially → experimentally $q_L \neq 0$ always (**next**)

➡ **Directional dependence of the measurement emphasized**




- $C_2(k_1,k_2)_{max}$ vs. $\Delta p_L < 2\ (\Delta p_\perp > 0)$
- $P_L = ½\,(p_1 + p_2)_L \neq 0$ → maximum shifts from $q_L = 0$

**Chapman, Scotto & Heinz: rediscovered cross-term ('95)**



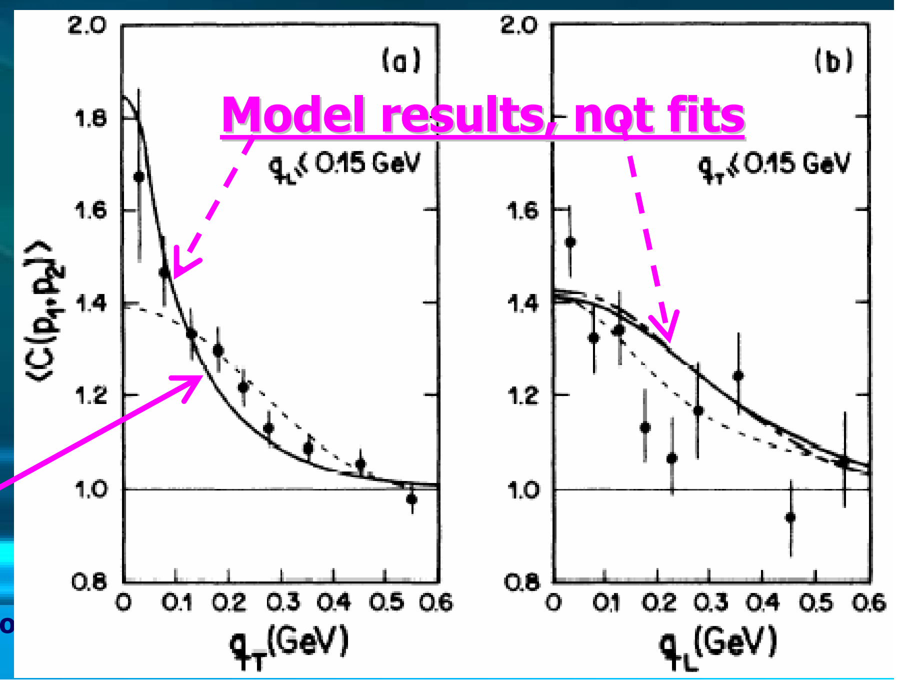

**Makhlin & Sinyukov: ('89)**
$$R_{L\,eff} = \frac{R_* \cosh(y_{pair} - Y)}{\cosh(y_{pair})}$$

$$(V = \tanh Y;\ P_L \approx m\ u_L)$$

| | |
|---|---|
| $Y = 0 \rightarrow R_L = R_*$ | $y_{pair} = 0 \rightarrow R_L = R_* \cosh Y$ |
| $y_{pair} = Y \rightarrow R_L = R_* / \cosh Y$ | |

- Comparison with exp. Data on $pp$ & $\bar{p}p$ collisions - CERN/ISR ($\sqrt{s} = 53$ GeV)

* **Sole model able of describing data trend: evidencing expansion effects (clear non-Gaussin behavior)**



(2) M. Gyulassy & SSP: N.P. B339 ('90) 378

## * Departing from Ideal Bjorken Inside-Outside Cascade Picture:

Correlation Function reflects dynamical and geometrical param.

→ **Momentum space:**

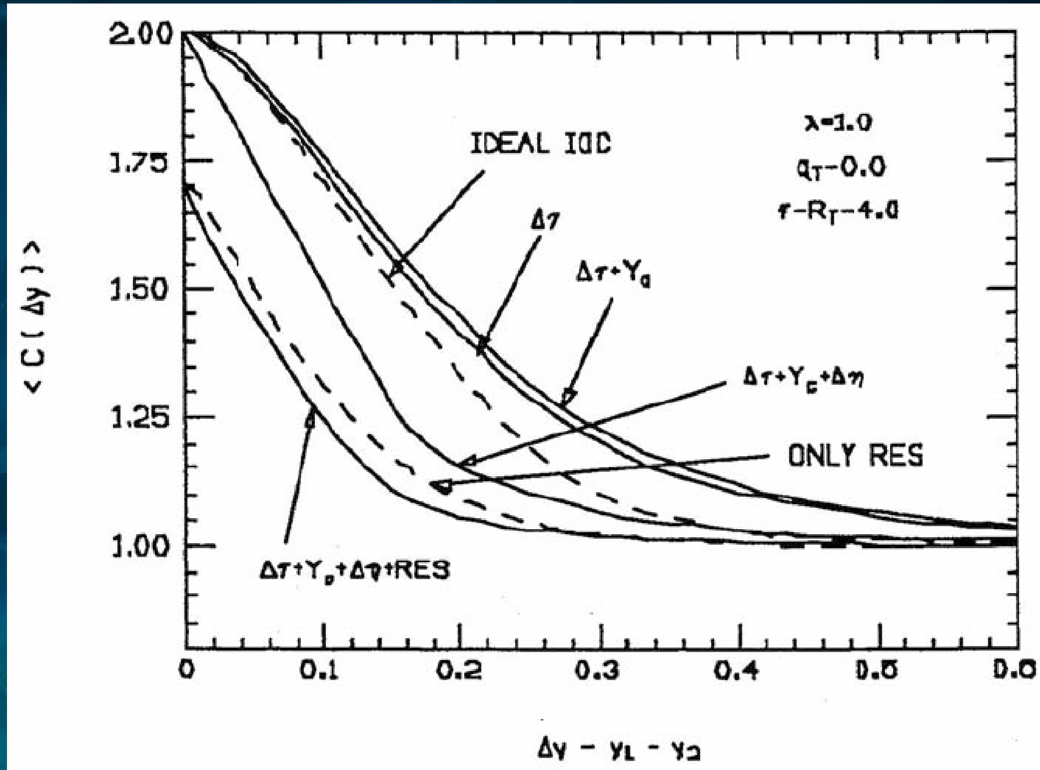
* width of rapidity distribution $Y_c$
* Resonances: $f_{\pi/r}$ → $r=\pi,\rho,\omega,K,\eta,\eta'$

→ **Coordinate space:**

* $\tau_0, \Delta\tau$ (average, width proper-time)
* $R_T$ (transverse radius)

→ **Phase-space:**

* $<(\eta-y)^2> = \Delta\eta^2$
* [ $<x_\perp . p_\perp>$ (transverse flow) ]

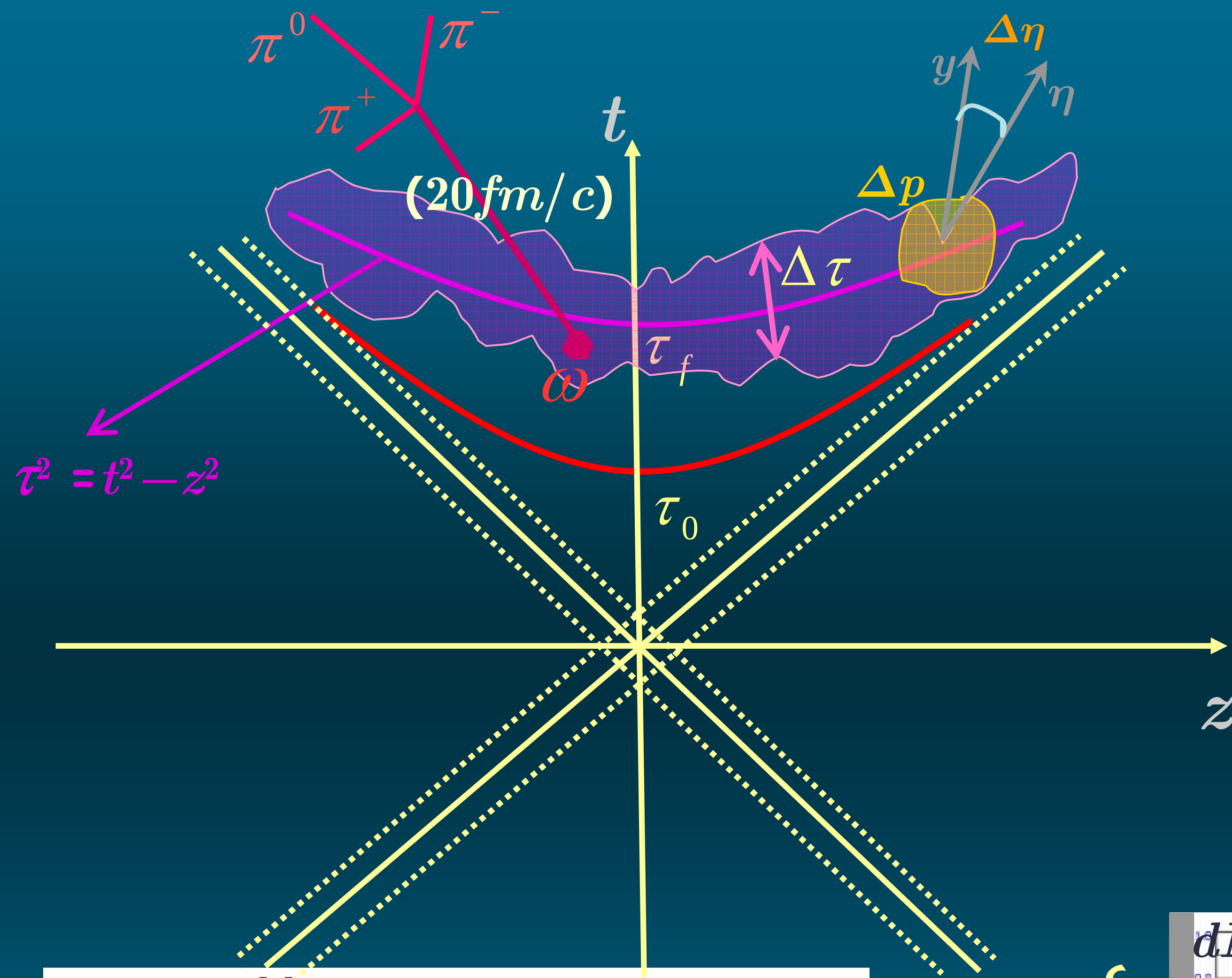



N.P. B339 ('90) 378

$$\left(\frac{dN}{dy}\right)\frac{1}{\tau}\delta(\tau_f-\tau)\delta(\eta-y) \;\rightarrow\; \frac{2}{\Delta\tau^2}\exp\left(-\frac{\tau_f}{\Delta\tau^2}\right)\exp\left[-\frac{(y-y^*)^2}{Y_c^2}\right]\frac{1}{\sqrt{2\pi}\Delta\eta}\exp\left[-\frac{(\eta-y)^2}{\Delta\eta^2}\right]$$

(Bjorken plateau)

+ resonances (Lund Model: $f_{\pi/\omega}=0.16$; $f_{\pi/\rho}=0.40$; $f_{\pi/K^*}=f_{\pi/(\eta+\eta')}=0.09$; $f_{\pi/dir}=0.19$)

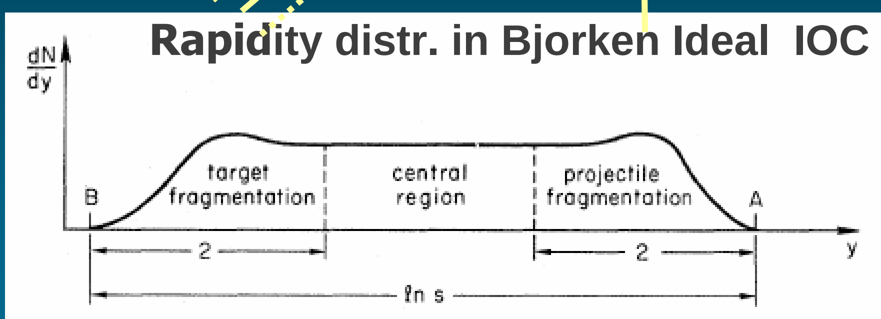



$$\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

(space-time rapidity)

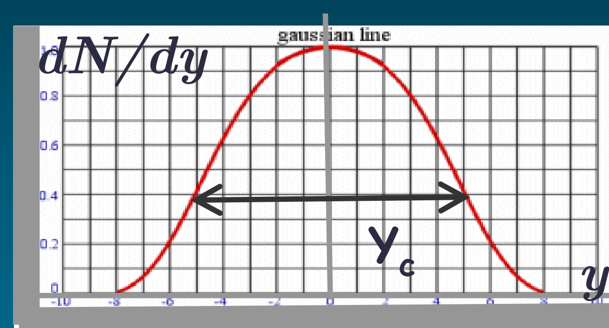
$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$$

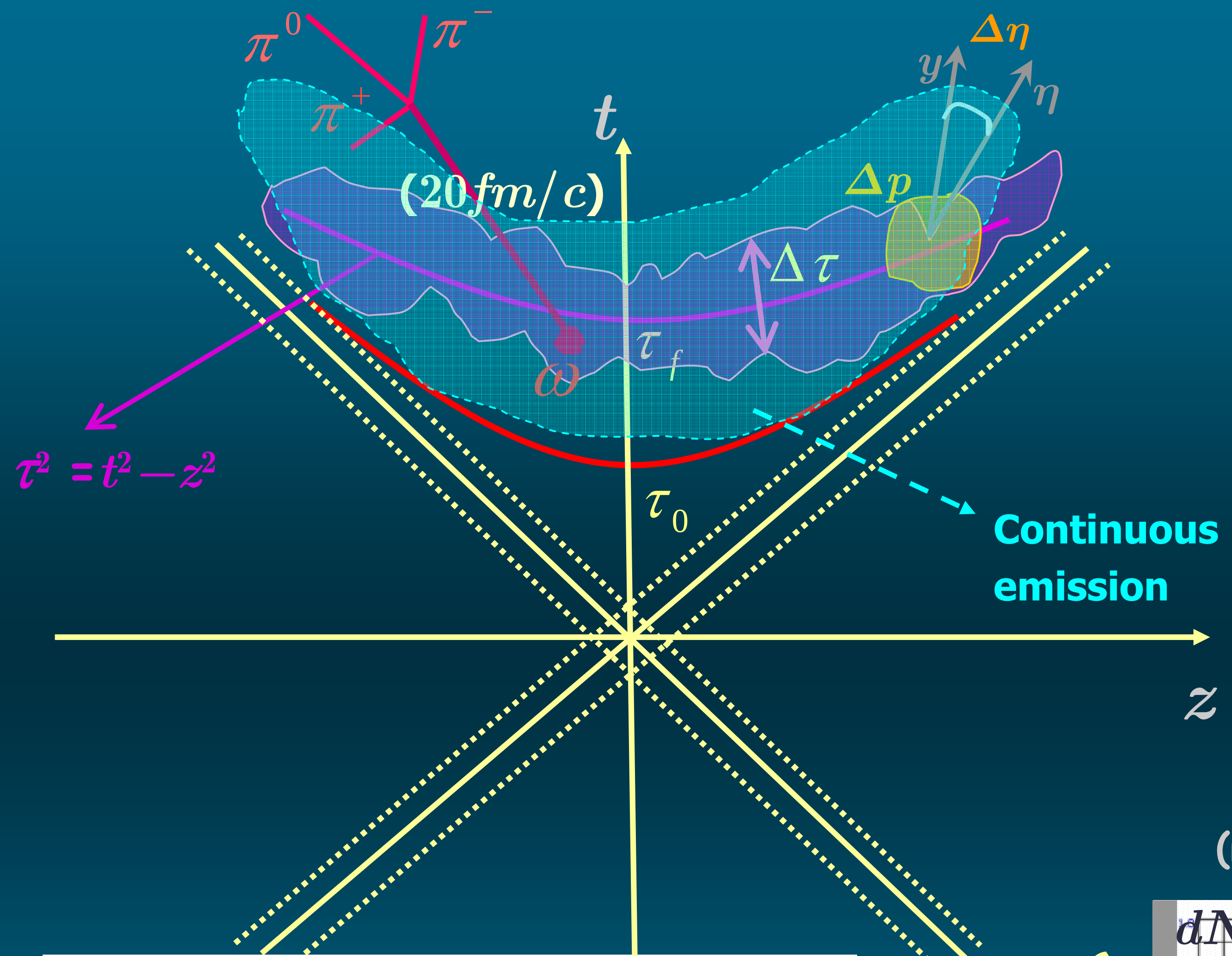
(rapidity)




Non-ideal IOC

(rapidity distribution)





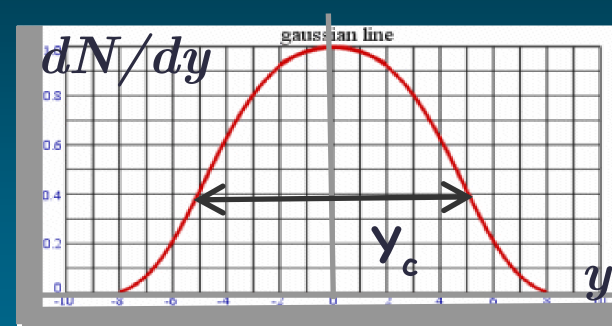

$$\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

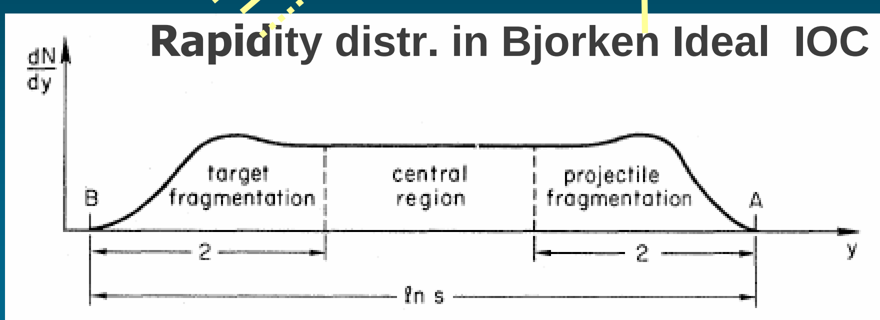
(space-time rapidity)

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$$

(rapidity)

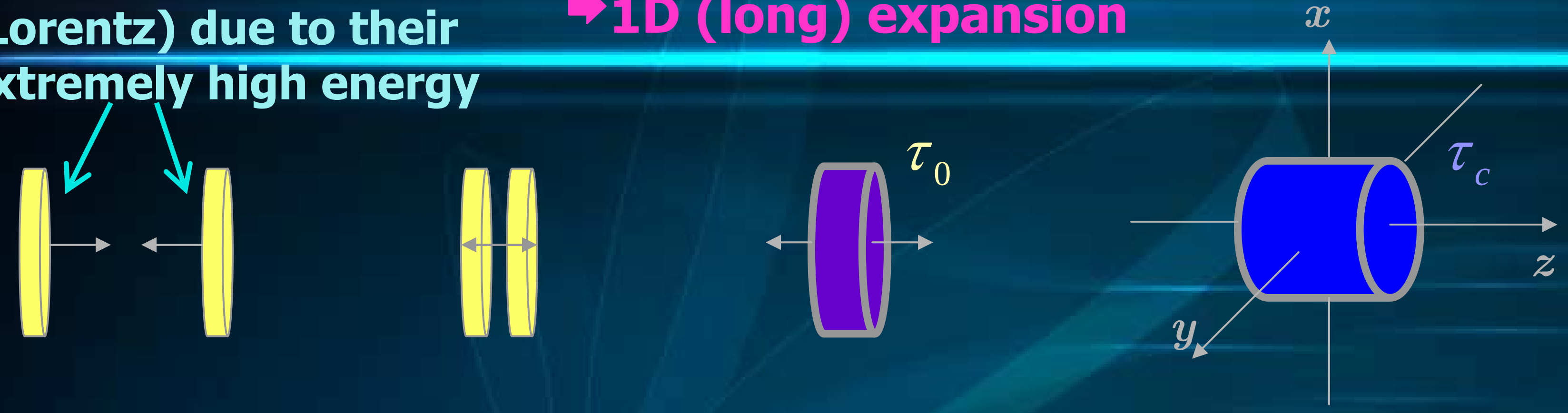
(rapidity distribution)

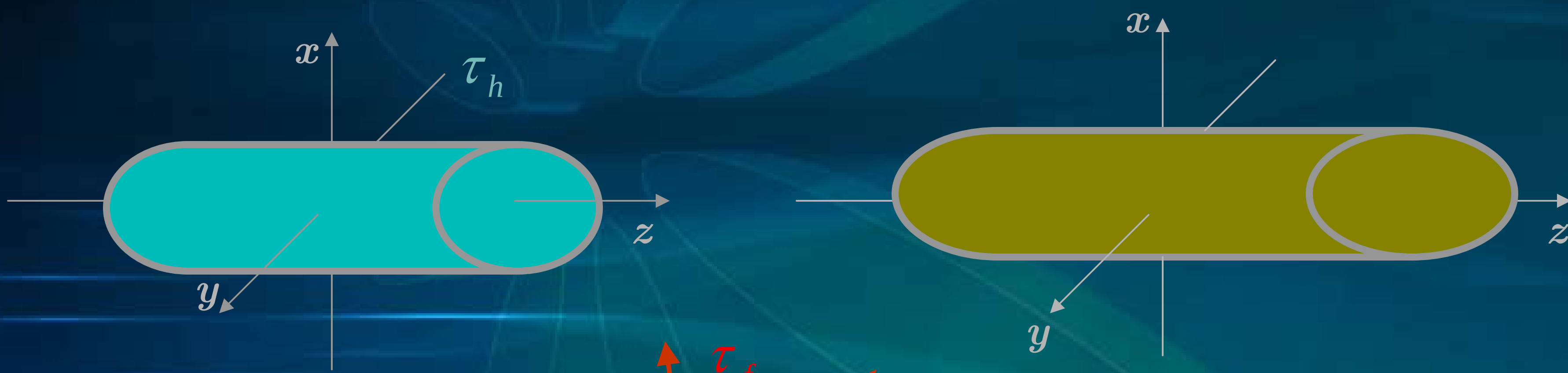


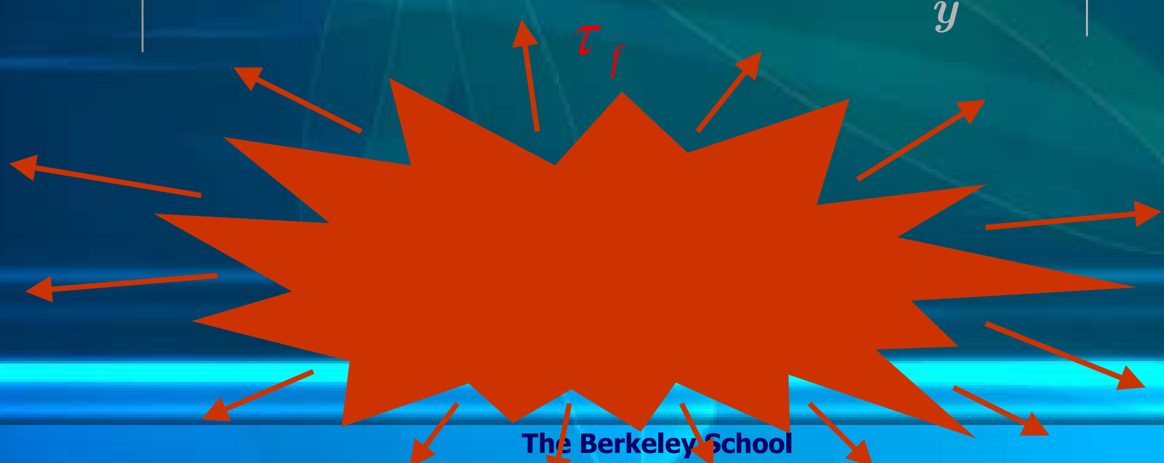



Contracted nuclei (Lorentz) due to their extremely high energy

➡1D (long) expansion



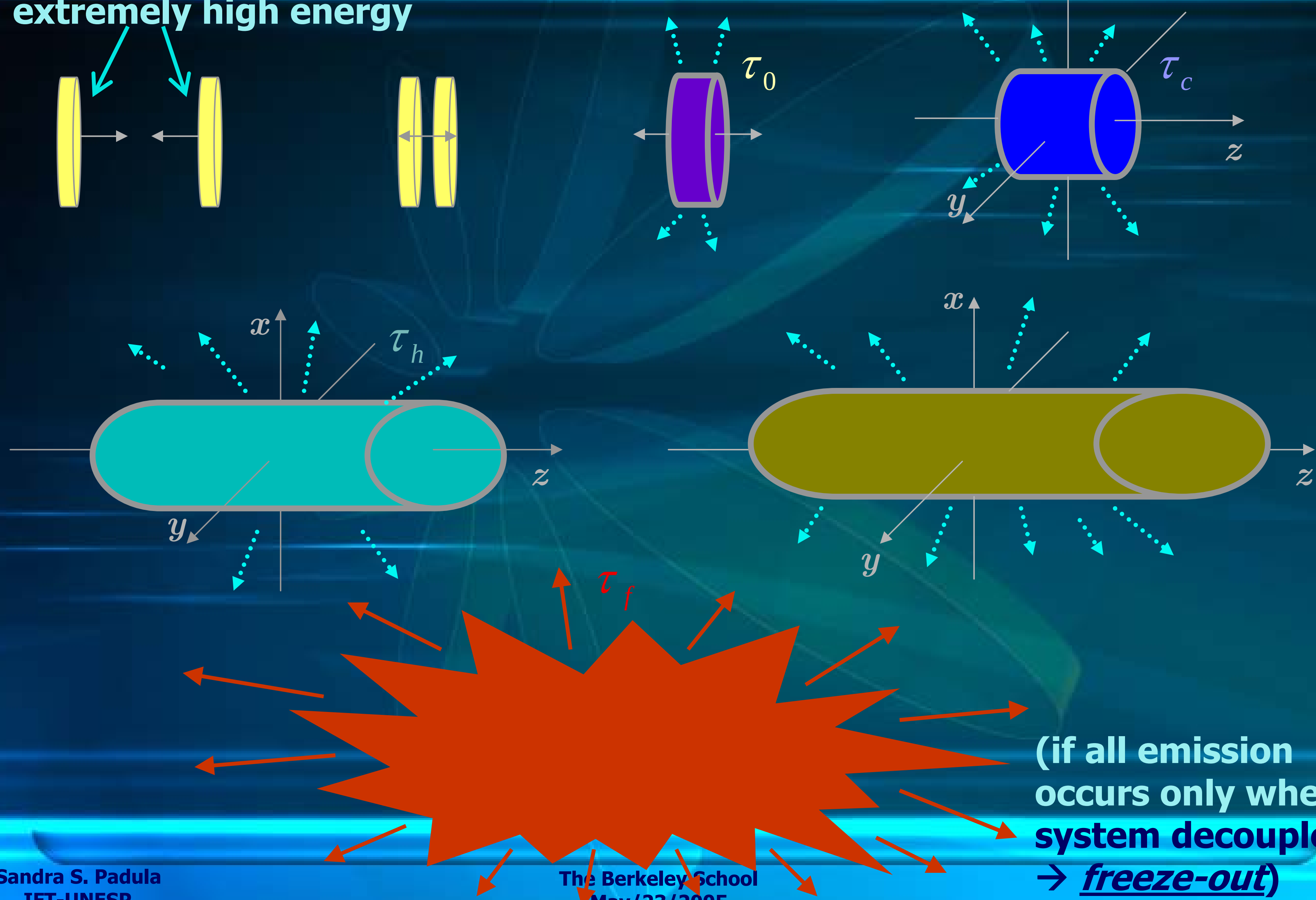




(if all emission occurs only when **system decouples** → ***freeze-out***)

( ····➔ =continuous emission mechanisms)

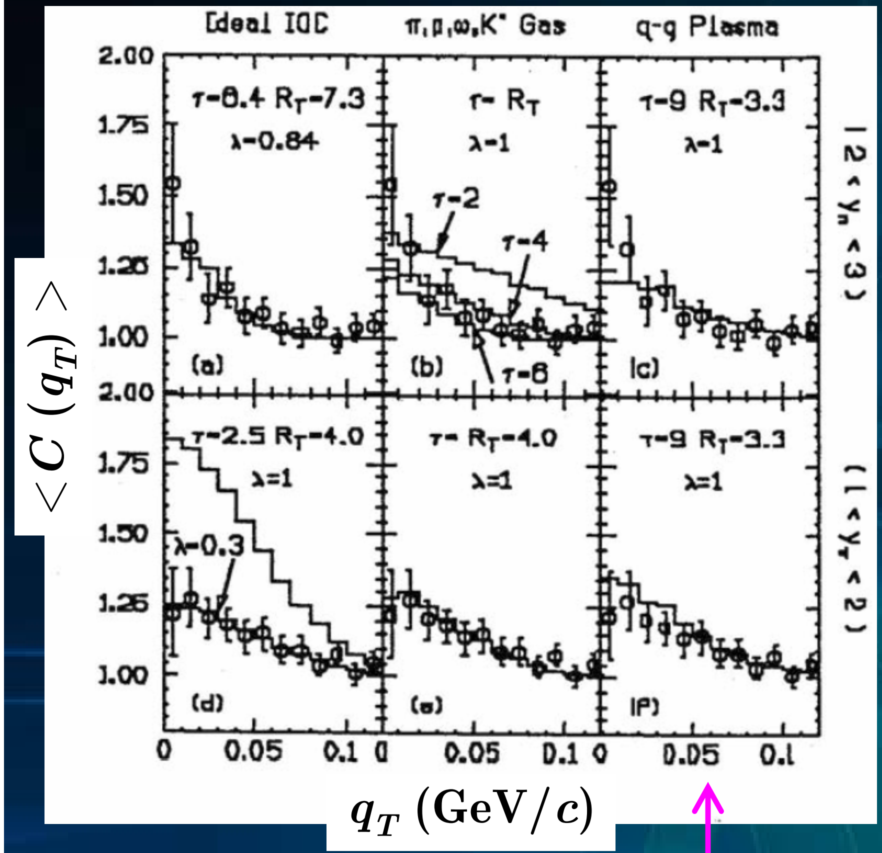
**Contracted nuclei (Lorentz) due to their extremely high energy**

➡**1D (long) expansion**




(if all emission occurs only when **system decouples** → ***freeze-out***)

# Testing CERN/NA35 x (non) ideal IOC




M.Gyulassy & SSP: P.L. B217 ('89) 181

- 3 distinct scenarios:
  - Ideal IOC but $\lambda$ < 1
  - Non-ideal + resonances
  - Quasi-ideal & QGP

Equivalently good description of data

**Challenge:**

- How can we separate distinct scenarios?
  - » Required quantitative analysis

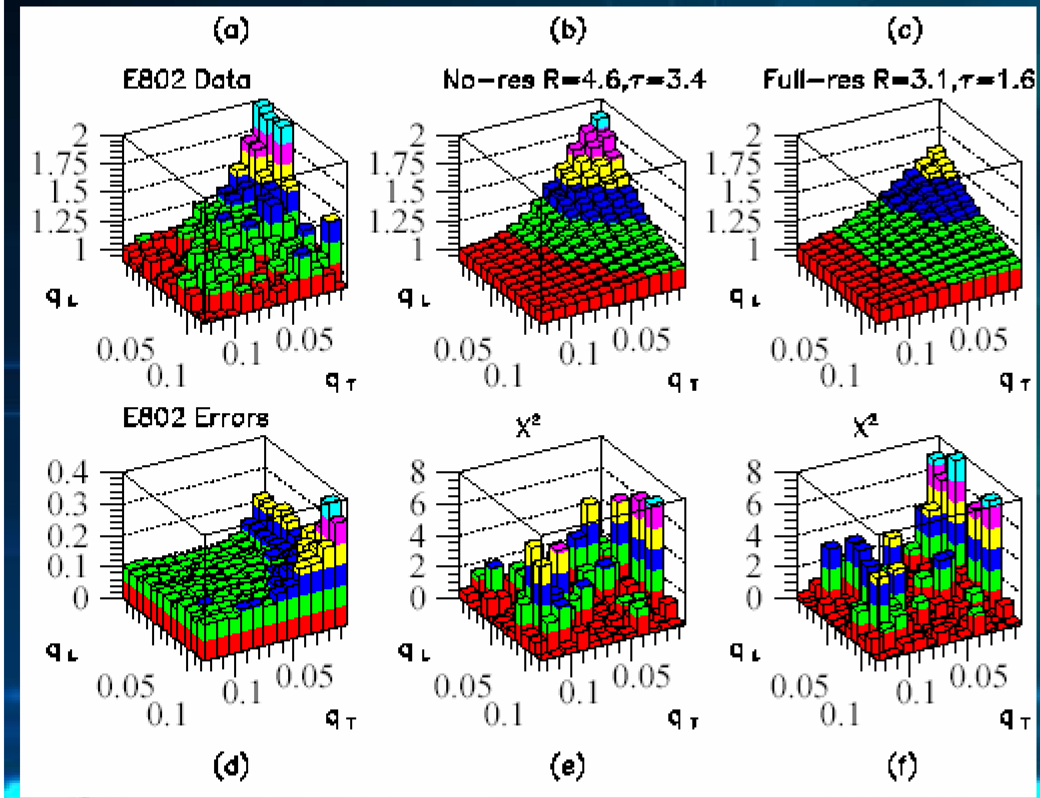
Solution later:

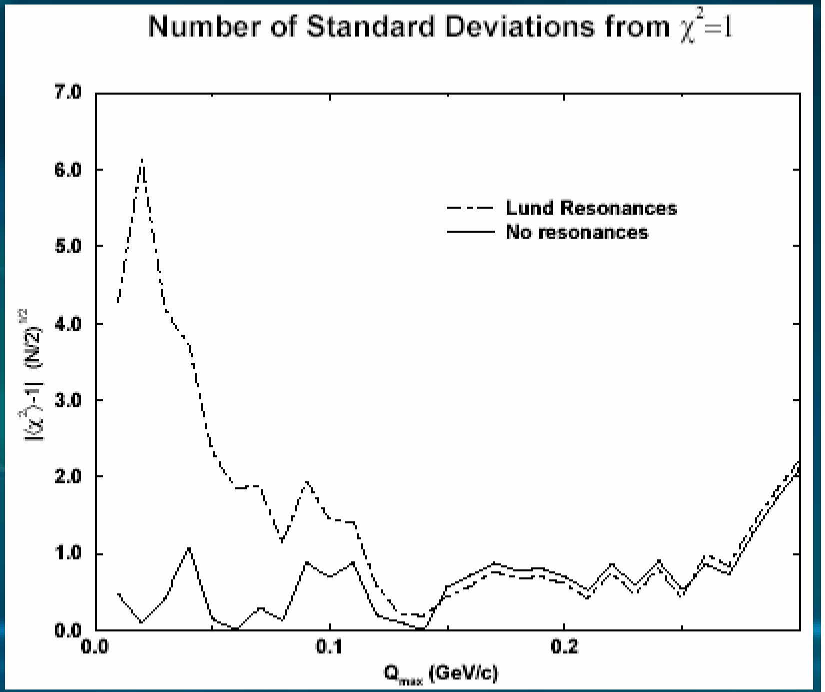
- » 2-D $\chi^2$ analysis

Large proper-times expected for QGP

## 2-D $\chi^2$-analysis of $\pi\pi$ HBT BNL/AGS (Si+Au @ 14.6 GeV/c) data

- method for disentangling ≠ dynamical scenarios ⇔ w/similar HBT results
  - 2-D $\chi^2$-analysis compares
    - ▹ no-resonance scenario vs Lund reson. ($\omega$, $K^*$, $\eta+\eta' \to \pi^-$)
  - Resolving power increased by studying variation of $|\langle\chi^2\rangle/\mathrm{dof} - 1|$ in $(q_T, q_L)$ plane




M.Gyulassy & SSP: P.L. B348 ('95) 303

(shown to work also for kaons, case not favored by long-lived resonances)

# Non-ideal effects → general formalism

M. Gyulassy, S. Gavin & SSP: N.P. B329 ('90) 357

- Semi-classical generalization of Wigner density formalism ⊕ wave-packet spread with respect to classical *n-particle phase-space distribution*

- Treats complex systems by allowing *arbitrary phase-space correlations* [i.e., $f(x,p) \neq \rho(x)\, g(p)$] and *multi-particle* B-E (F-D) *correlations*

- For **minimal packets** [$\Delta x = \Delta p = 1/2$] → general form. reduces to the Cov. Current Ensemble [*GKW,* PRC20('79)2267] w/ momentum spread: $\Delta p = \sqrt{mT}$

➔ time dependence of $R_T$ → $R^2_{T_{eff}} = 2R^2_T + \tau^2 (K_T / E_K)^2$

⇕ direct consequence of the *on-shell approximation* ($k_1, k_2$ on-shell but $K$, $q$ off-shell, by definition):

$$K^0 = \tfrac{1}{2}(k_1 + k_2) = \tfrac{1}{2}\sqrt{(k_1^2 + m^2) + (k_2^2 + m^2)} \approx \sqrt{K^2 + m^2} = E_k$$

(on-shell approx.)

$$k_1 = K + \frac{q}{2}\ ;\ k_2 = K - \frac{q}{2} \rightarrow q.K = q^\mu K_\mu \equiv 0 \Leftrightarrow q^0 = \frac{\vec{q}.\vec{K}}{E_K}$$

# 2-particle correlation in generalized Wigner

$$C(k_1,k_2) = 1 + \frac{e^{q^2\Delta x^2}\left|\int d^4p \int d^4x\, e^{iq^\mu x_\mu} D(x,p)\, e^{(K-p)^2/(2\Delta p^2)}\right|^2}{\int d^4p_1 \int d^4x_1\; D(x_1,p_1)\, e^{\frac{(p_1-k_1)^2}{(2\Delta p^2)}} \int d^4p_2 \int d^4x_2\; D(x_2,p_2)\, e^{\frac{(p_2-k_2)^2}{(2\Delta p^2)}}}$$

**In the limit $\Delta x = \Delta p = 0$ →**

**extended (4-D) Wigner formulation, by Pratt**

$$\left.\begin{array}{l}(\lim \Delta x, \Delta p \to 0) \\ D(x_i,k_i) \to S(x_i,k_i)\end{array}\right\} \Rightarrow\; C(k_1,k_2) = 1 + \frac{\left|\int d^4x\, e^{iq^\mu x_\mu} S(x,K)\right|^2}{\int d^4x\, S(x,k_1) \int d^4x\, S(x_2,k_2)}$$

# Smoothness approximation

$$\left.\begin{aligned} k_1 &= K + \tfrac{1}{2}q \\ k_2 &= K - \tfrac{1}{2}q \end{aligned}\right\} \Rightarrow C(q,K) = 1 + \frac{\left|\int d^4x\, e^{iq^\mu x_\mu} S(x,K)\right|^2}{\int d^4x\, S(x, K+q/2) \int d^4x\, S(x, K-q/2)}$$

– **Smoothness approximation**

$$S(x_i, k_i) \equiv S(x_i, K \pm q/2) \approx S(x_i, K)$$

- **good for big systems ($R \geq 2$ fm) [Pratt, PRC 56 (1997) 1095]**

$$\therefore C(q,K) \approx 1 + \left| \frac{\int d^4x\, e^{iq^\mu x_\mu} S(x,K)}{\int d^4x\, S(x,K)} \right|^2 \longrightarrow$$

**(popularized by U. Heinz and collaborators → mid to late 90's)**

# The Gaussian approximation

– Heinz and collaborators $\rightarrow$ assuming validity of

- Smoothness approximation

$$C(q,K) \approx 1 + \left| \frac{\int d^4x\, e^{iq^\mu x_\mu} S(x,K)}{\int d^4x\, S(x,K)} \right|^2$$

- On-shell approximation

$$K^0 = \tfrac{1}{2}(k_1 + k_2) \approx \sqrt{K^2 + m^2} = E_k$$

- On-shell constraint

$$q^0 = \frac{\vec{q}.\vec{K}}{K^0} \approx \frac{\vec{q}.\vec{K}}{E_K}$$

- Focused in *half-widths of the correlation function*

➔ 3 x 3 *tensor* describing its curvature near $q=0$

Heinz et al.:

## Arbitrary emission function → in terms of variances

$$S(x,K) \propto S\,(\bar{x}(K),K\,)\, e^{-\frac{1}{2}\tilde{x}^{\mu}(K)B_{\mu\nu}(K)\tilde{x}^{\nu}(K)} + \delta S(x,K)$$

**with vanishing 0th, 1st and 2nd order space-time moments**

$$\int d^4x\,(x^{\mu})^n\,\delta S(x,K) = 0 \qquad (n = 0,1,2)$$

$$\begin{cases} \langle\tilde{x}^{\mu}\rangle(K) = \langle x^{\mu}\rangle \\ (B^{-1})_{\mu\nu} = \langle\tilde{x}^{\mu}\tilde{x}^{\nu}\rangle \equiv \langle(x-\bar{x})_{\mu}(x-\bar{x})_{\nu}\rangle \end{cases}$$

– **Resulting 2-particle correlation function (reflecting curvature near $q=0$)**

$$\underbrace{C(q,K) = 1 + \exp[-\tfrac{1}{2}q^{\mu}q^{\nu}\langle\tilde{x}^{\mu}\tilde{x}^{\nu}\rangle(K)]}\, + \delta C(q,K)$$

**Receives contribution from $\delta S$ (*neglected!*)**

- **Assumed to be essential part for heavy ion collisions**
- **Applied to different parameterizations**
- **Dependence in $q$ from $\delta C(q)$ only when including resonances**

# Cartesian parameterization

- **Illustration for *azimuthally symmetric* event samples**
  - Longitudinal beam direction ($z$) $\rightarrow$ L
  - $\vec{K}$ in $x$-$z$ plane so that $\vec{K} = (K_x, K_y, K_z) = (K_\perp, 0, K_L)$
  - $C(q,K)$ symmetric for $q_s \rightarrow -q_s$
  - $q^0 = \frac{\vec{q}.\vec{K}}{K^0} \approx \frac{\vec{q}.\vec{K}}{E_K} = \beta_\perp q_o + \beta_L q_L$

Illustration: Heinz, NATO ASI ('96)

$$C(k_1,k_2) = 1 + \lambda(K)\exp\{-R_s^2(K)q_s^2\}$$

$$\exp\{-R_o^2(K)q_o^2 - R_l^2(K)q_l^2 - 2R_{ol}^2(K)q_o q_l\}$$

$$R_s^2(K) = \langle \tilde{y}^2 \rangle;$$

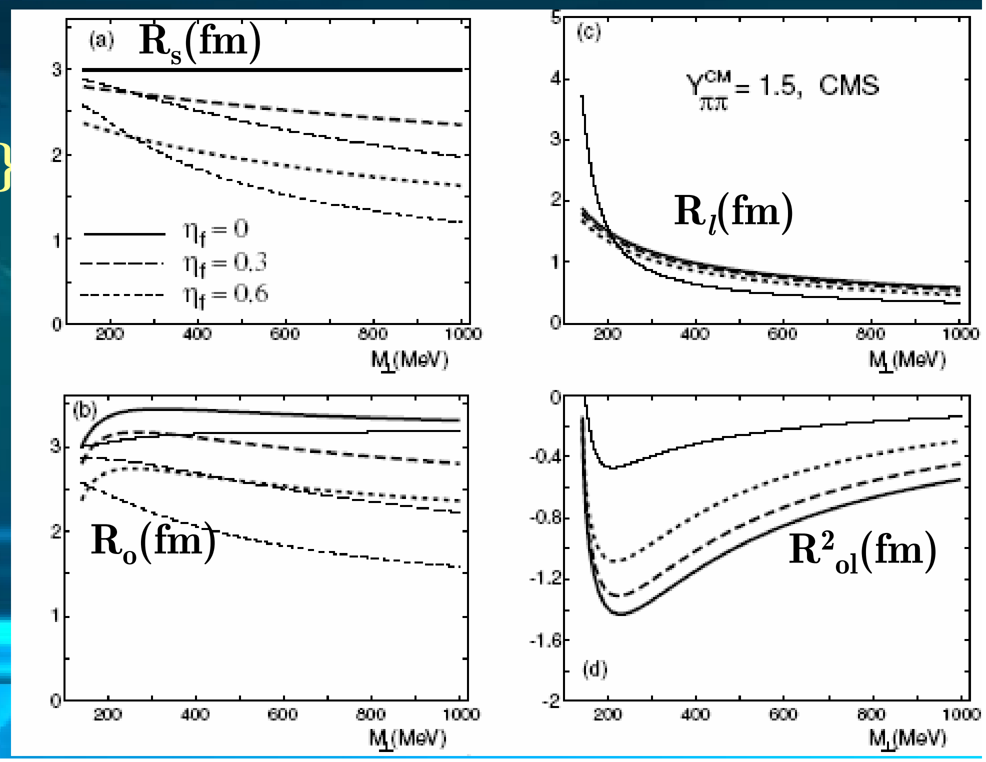
$$R_o^2(K) = \langle (\tilde{x} - \beta_\perp \tilde{t})^2 \rangle;$$

$$R_l^2(K) = \langle (\tilde{z} - \beta_l \tilde{t})^2 \rangle;$$

$$\underbrace{R_{ol}^2(K)}_{\downarrow\ \text{Bertsch-Pratt parameterization}} = \langle (\tilde{x} - \beta_\perp \tilde{t})(\tilde{z} - \beta_l \tilde{t}) \rangle$$

Bertsch-Pratt parameterization

"cross-term"



# Modeling expanding sources

- Popular formulation (hydro-based): Csörgő & Lörstad [N. P. A590('95)468] ; Chapman, Nix & Heinz [P.R. C52 ('95) 2694]

$$S(x,K) \propto \frac{m_T \cosh(y-\eta)}{(2\pi)^3} \exp\left[-\frac{K.u(x)-\mu_0(x)}{T(x)}\right]$$
$$\exp\left[-\frac{r^2}{2R_G^2}-\frac{(\eta-y)^2}{2\Delta\eta^2}-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right]$$

- **Model widely applied at SPS**
- **Good qualitative results, undeniably useful**
- **Concern → Reinforced the "Gaussian appeal" to treat data**
  **→ Considerably reduced or eliminated exhibition of correlation function curves (tendency now reverting?...)**
- We should <u>remember</u>: estimate of variances assumes that curves are Gaussians ($\delta C(q)=0$) **a**nd what matters is behavior near q=0!

**→ Why the concern? Some models predict distorted correlation curves**

# PHENIX and STAR correlation curves

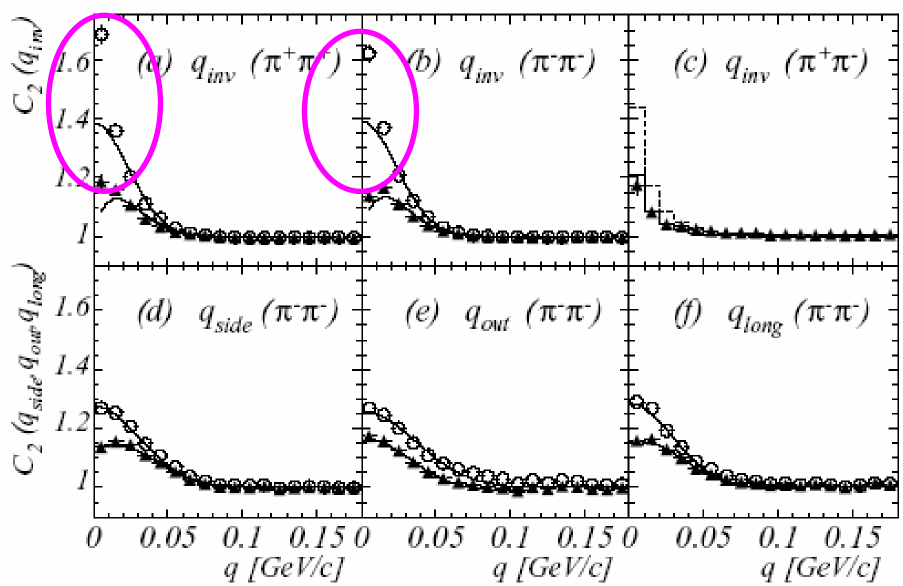



FIG. 1: Panels (a) and (b) show one-dimensional correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$. The bottom figures show the three-dimensional correlation function for $\pi^-\pi^-$ with the *full* Coulomb (opened circle) and without Coulomb (filled triangle) corrections for 0.2 GeV/c $< k_T <$ 2.0 GeV/c for 0-30% centrality. The projection of the 3-D correlation functions are averaged over the lowest 40 MeV in the orthogonal directions. The error bars are statistical only. The lines overlaid on the open circles (filled triangles) correspond to fits to Eq. 1 (Eq. 2) over the entire distribution. Panel (c) shows the one-dimensional correlation function of unlike-signed pions for 0.2 $< k_T <$ 2.0 GeV/c. The two overlaid histograms show calculations for the *full* (dashed) and the 50% partial (solid) Coulomb corrections.

**PHENIX, PRL 93 (04) 152302**

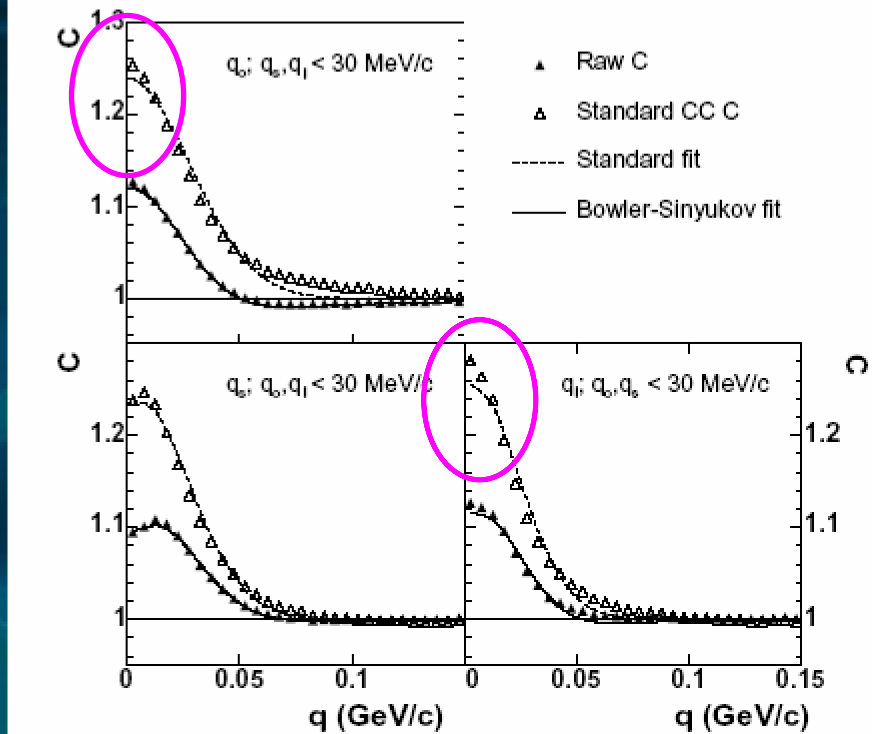


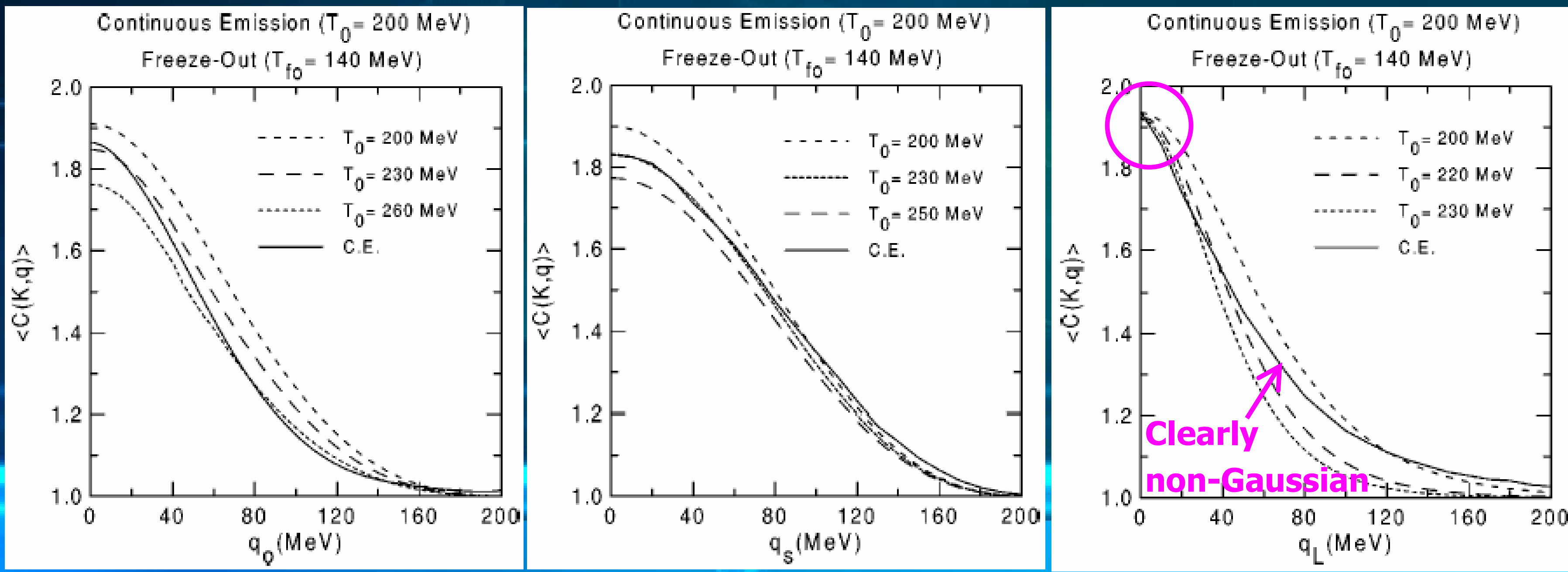

FIG. 4: Projections of the 3 dimensional correlation function and corresponding fits for negative pions from the 0–5% most central events and $k_T$ = [150,250] MeV/c according to the *standard* and *Bowler-Sinyukov* procedures.

**STAR, nucl-ex/0411036**

**only pairs with B-E interaction are considered to Coulomb interact**

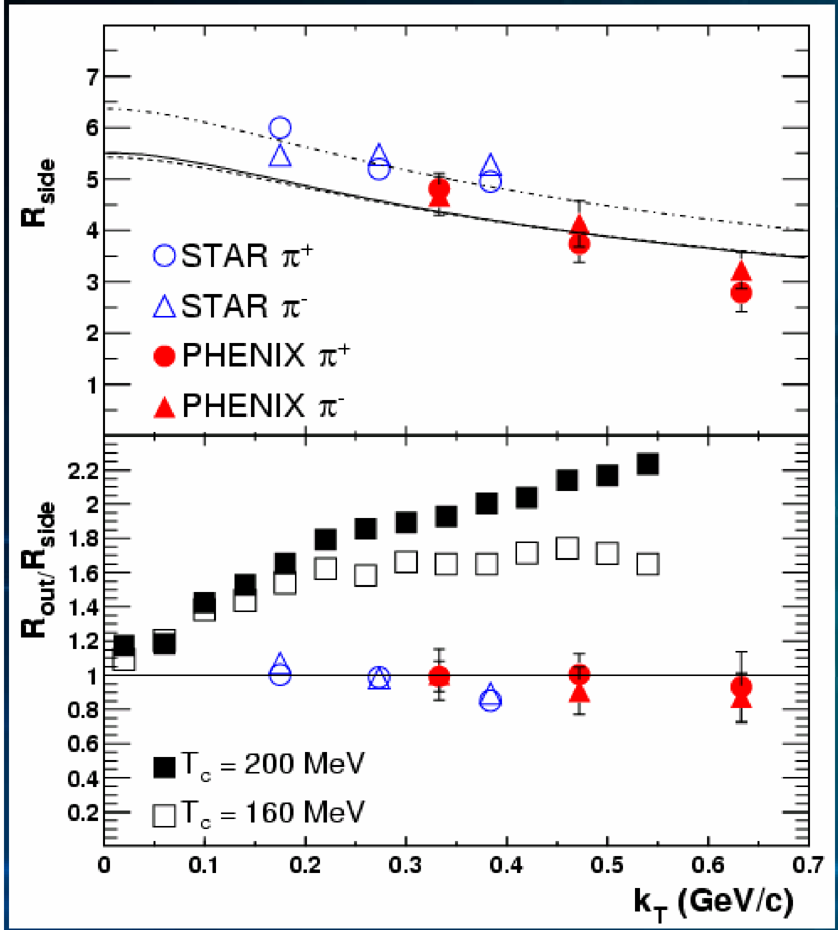
# Continuous Emission

- Grassi, Hama and Kodama [P.L. B355('95)9;Z.P. 73('96)153]
  - Alternative version of extended freeze-out
  - Difference: emission has finite probability of occurring since $\tau_0$
- HBT for Continuous Emission [PRC62 ('00)44940]
  - Introduce slightly but equivalent formalism
  - Expression from CCE are recovered for instant freeze-out
- Predict highly distorted correlation functions (also near $q=0$), reflecting large emission time interval



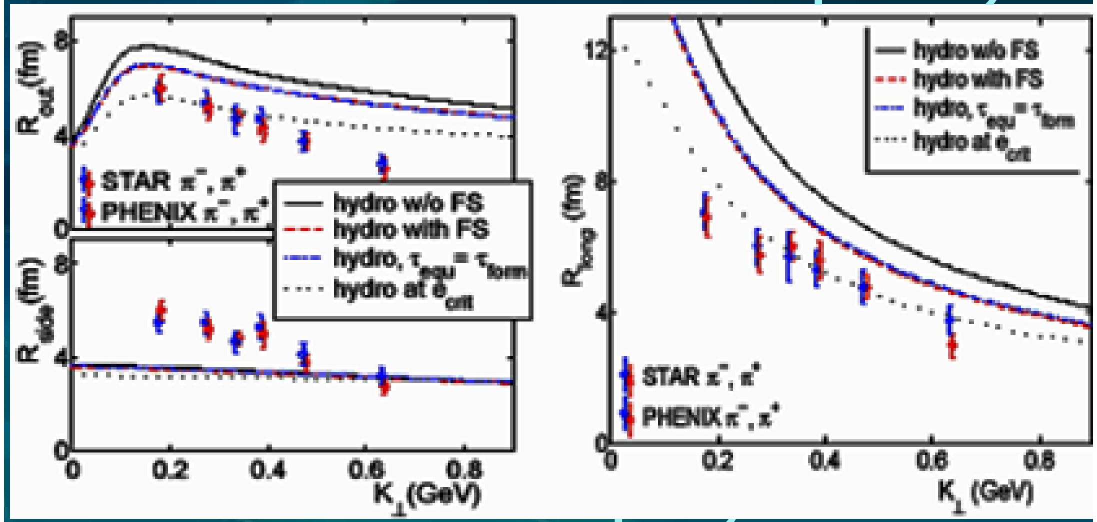
# RHIC PUZZLE → Challenge

**Hydro+uRQMD**
**Soff, Bass et al. NPA 715 (03) 801**




**Hydrodynamics**
**Heinz, Kolb, NPA 702 (02) 269**

**(best results for freeze-out at hadronization point ↓ )**

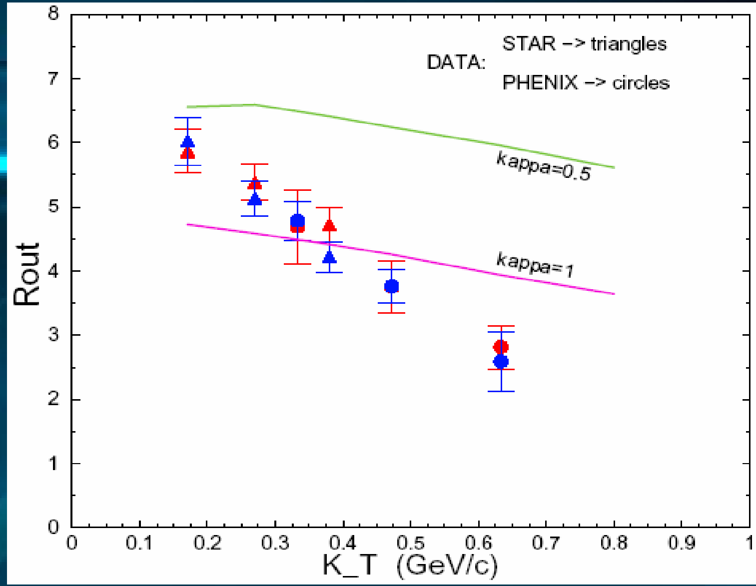


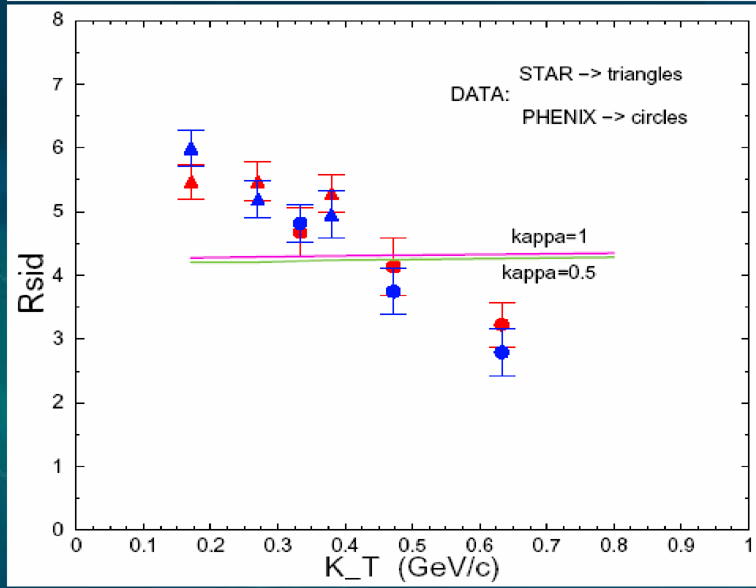

**Both overestimate evolution time & emission duration**

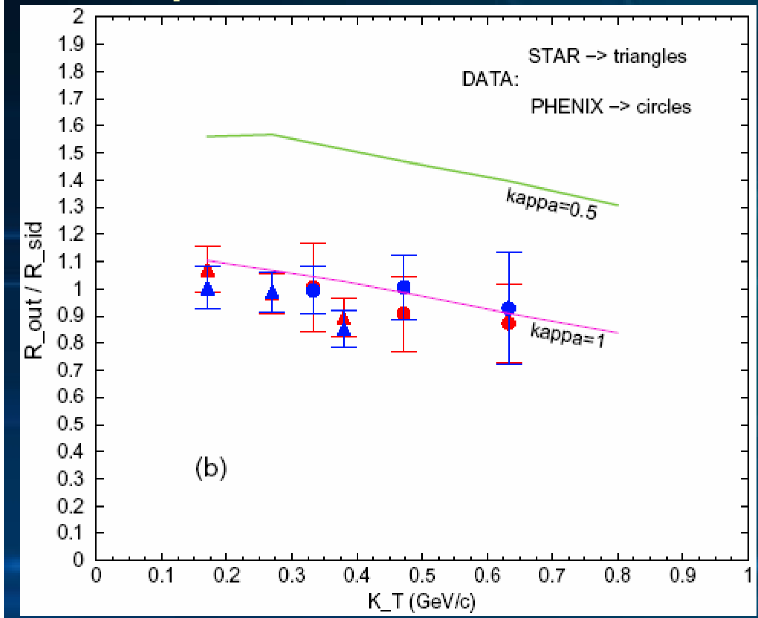
# Challenge…

**L.McLerran & SSP → opaque source ⊕**
**black-body radiation with emissivity $\kappa$ ($\pi$'s only) ⊕**

- **Initial formation of QGP @ RHIC**
- **Ideal Bjorken +hydro (1+1) (No ⊥ flow)**
- **Phase-trans. starts: $\tau_c$ ($T_c$); ends $\tau_h$($T_c$)**
- **Hadron expandes further till $\tau_f(T_f)$**
- **At $T_f$ → system decouples (vol. emission)**




$T_0 \approx 411$ MeV
$T_c = 175$ MeV
$T_f = 150$ MeV

**$\kappa_{\text{calc}}=4\ \kappa_{\text{blackbody}}$**
**∴ requerires ⊥ flow**

| $\kappa$ | $\tau_0$ (fm/c) | $\tau_c$ (fm/c) | $\tau_h$ (fm/c) | $\tau_f$ (fm/c) | $\mathcal{S}/\mathcal{N}$ ($\tau_0 \leq \tau \leq \tau_f$) | $\mathcal{V}/\mathcal{N}$ (at $\tau_f$) |
|---|---|---|---|---|---|---|
| 1 | 0.160 | 1.54 | 5.73 | 6.97 | 0.844 | 0.156 |
| 0.5 | 0.160 | 1.75 | 8.37 | 10.5 | 0.758 | 0.242 |

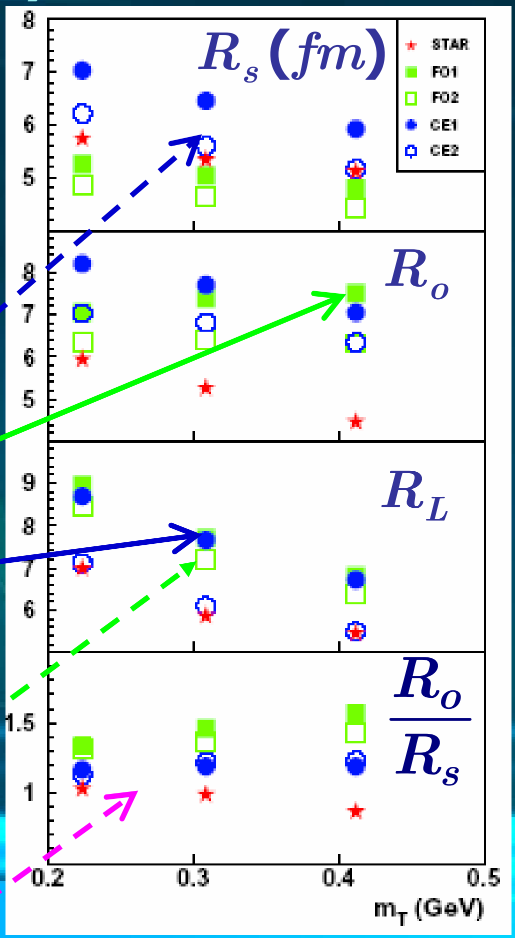
# Challenge...

- **RHIC data vs. fluctuating IC and CE**
  - [Socolowski, Grassi, Hama & Kodama, PRL 93('04)409001]

Initial Conditions { • Average of several (15) events (smooth); • Fluctuating IC (event-by-event) - NeXus }

Particle emission { • Instant freeze-out ($\Delta\tau=0$); • Continuous Emission Model }

System Evolution → SPheRIO (based on 3+1 Smoothed Particle Hydro)



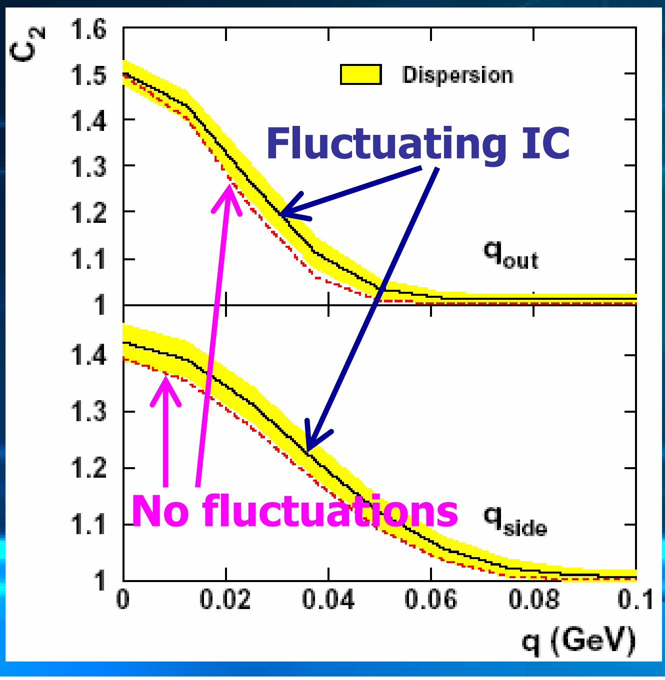

CE + fluct. IC

FO but no fluct. IC

CE but no fluct. IC

FO + Fluct. IC

Ratio still above 1



# Achievements…

- **Blast wave model, Buda-Lund model, etc.**
  **i.e., models that succeed to explain data trend → to follow (next talk, by Mike Lisa)**

**I end my talk with challenges…**
**further accomplishments later**

**still → some comments and alerts (to follow)**

# Summary and ...

- Overview of history, some challenges and achievements along $\approx$ 50 years of HBT
- Number of applications, models, experiments (energy and realm, macro to micro-cosmos) has constantly increased
- In HIC, vast quantity of good quality data, from AGS, SPS to RHIC along last 15-20 years (event-by-event more recently)
- Accordingly, impressive quantity of models, formalisms and improved parameterizations
- Only a tiny fraction of contributions could be shown here
- Constant interplay of challenge and enormous effort to achieve knowledge of $S(x,p)$ through HBT

# …some comments → Why the concern?

– **Although undeniably useful, the radii variance technique may not tell all the story …**

- We should not take for granted that HBT curves are Gaussians, even if they seem to be (so far?) in high energy heavy ion collisions
- True correlation functions should be compared to models (even after determining radii variances)
- Estimate goodness of fit (3-D $\chi^2$-analysis) model vs. data
- WHY? * HBT for HIC is model-dependent interferometry
  * Some models predict highly distorted correlation functions reflecting large time interval (e.g., models with continuous emission)
- *May help to disentangle different models/scenarios that successfully explain current RHIC data*
- **Without it: interpretation of the source "appearance" through Gaussian variances can be incomplete or even misleading**