Lecture 2:

1) How can the *high viscosity* sQGP
flow like a “perfect fluid”?

** Because of deconfinement ! ** (T.Hirano, MG)

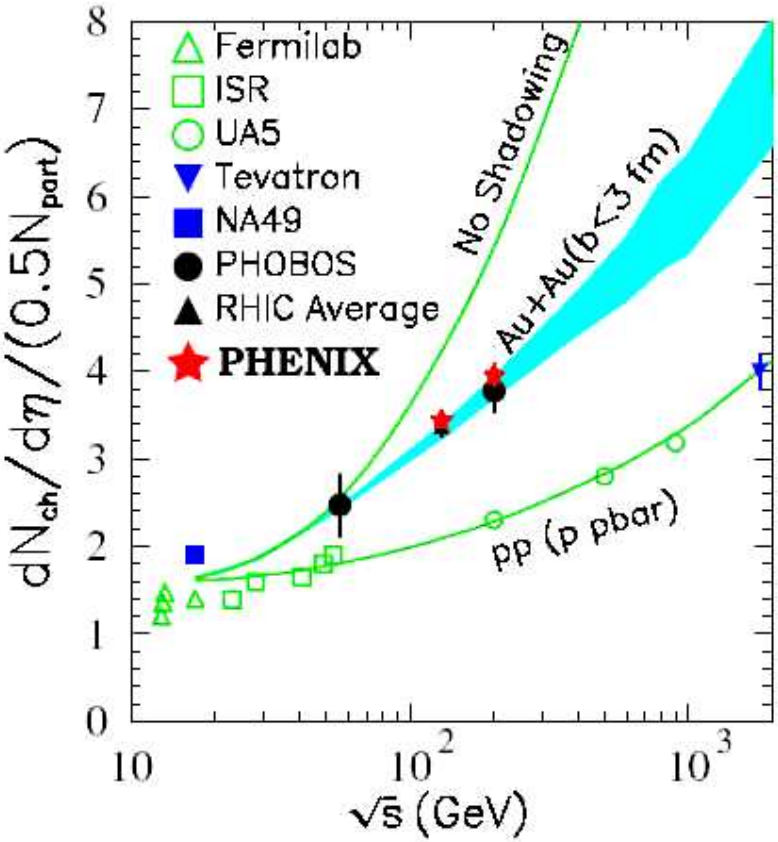
2) *Intrinsic* violations of Bjorken boost invariance:
The p+A Triangle and B+A Trapezoid

**Octupole twist of RHIC Initial Conditions (A. Adil, MG)

Refined test of CGC Initial Conditions at RHIC and LHC

Global Evidence for saturating CGC initial state at RHIC

$$Q_s^2(x,A) \approx 2\ \mathrm{GeV}^2 \left(\frac{10^{-2}}{x} \frac{A}{200} \right)^{0.3}$$



Gluon Saturation below $k_T < Q_s$

Limits Rapid Growth of pQCD mini-jets

Corresponds to Deep Gluon Shadowing in x<0.01 region

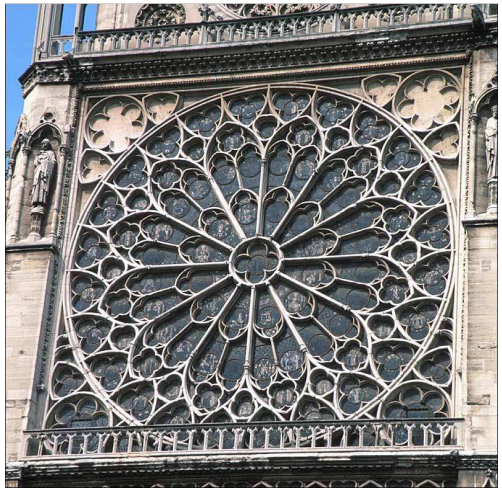
$$N_{ch} = \rho_s(Q_s)\, A + \rho_H(Q_s)\, A^{4/3}$$

RHIC data prove Q_s varies with *s and A*

Nuclear Glue Structure RHIC vs LHC

RHIC is a sQGP machine with a partial view of CGC

CGC at RHIC



$$\frac{dN_g}{dy} \approx c\,\frac{Q_s^2 R^2}{\alpha_s(Q_s^2)}\left(1-\frac{2Q_s}{\sqrt{s}}e^{|y|}\right)^4$$

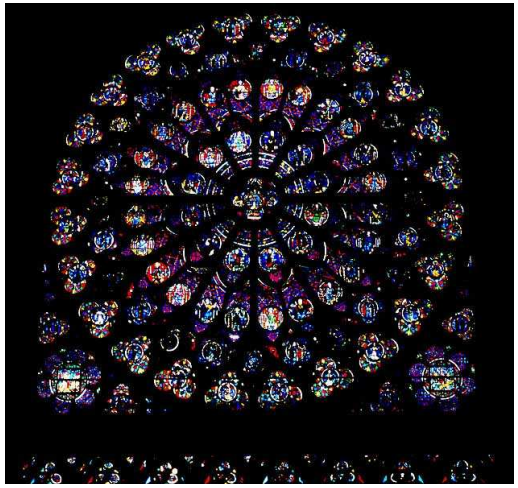
Kinematically Limited at y = 3

$$Q_s^2(x,A) \approx 2\,\mathrm{GeV}^2\left(\frac{10^{-2}}{x}\frac{A}{200}\right)^{0.3} \Rightarrow \begin{cases} \xrightarrow[y=3\ p_T=2]{\text{RHIC}} 5\,\mathrm{GeV}^2 \\ \xrightarrow[y=0\ p_T=2]{\text{LHC}} 5\,\mathrm{GeV}^2 \\ \xrightarrow[y=3\ p_T=2]{\text{LHC}} 14\,\mathrm{GeV}^2 \end{cases}$$

$$\frac{dN_g}{dy} \approx c\,\frac{Q_s^2 R^2}{\alpha_s(Q_s^2)}$$

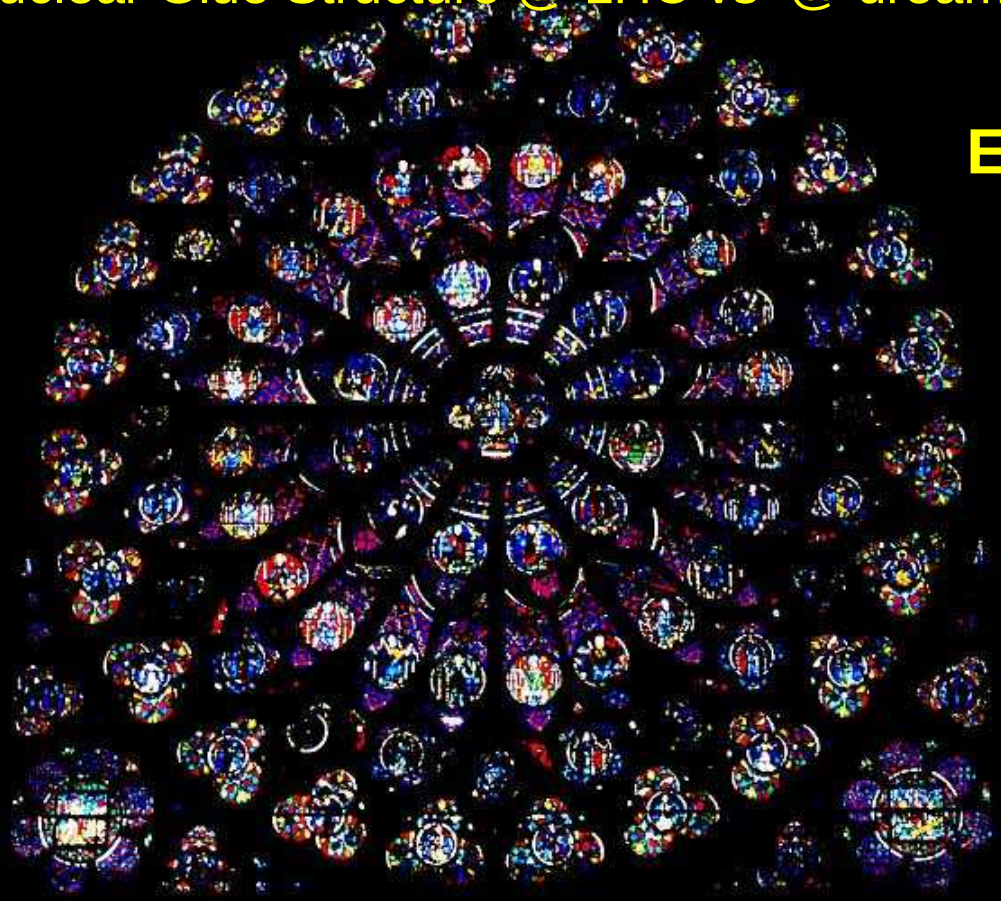
Kinematics Free at y ≤ 3

CGC at LHC



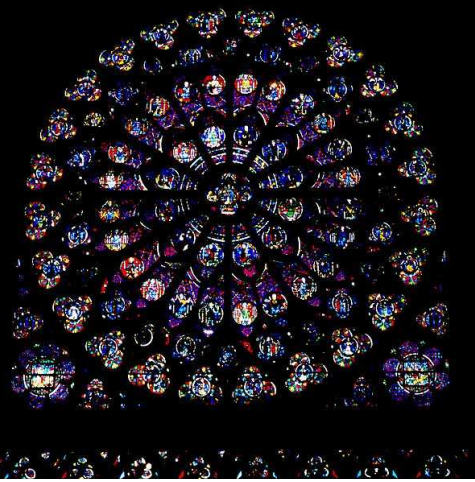
LHC is a CGC machine with a partial view of sQGP

Nuclear Glue Structure @ LHC vs @ dream machine EIC (ELIC or eRHIC)



EIC ~ 2020

LHC ~2010



Bulk Collective Flow of QCD matter

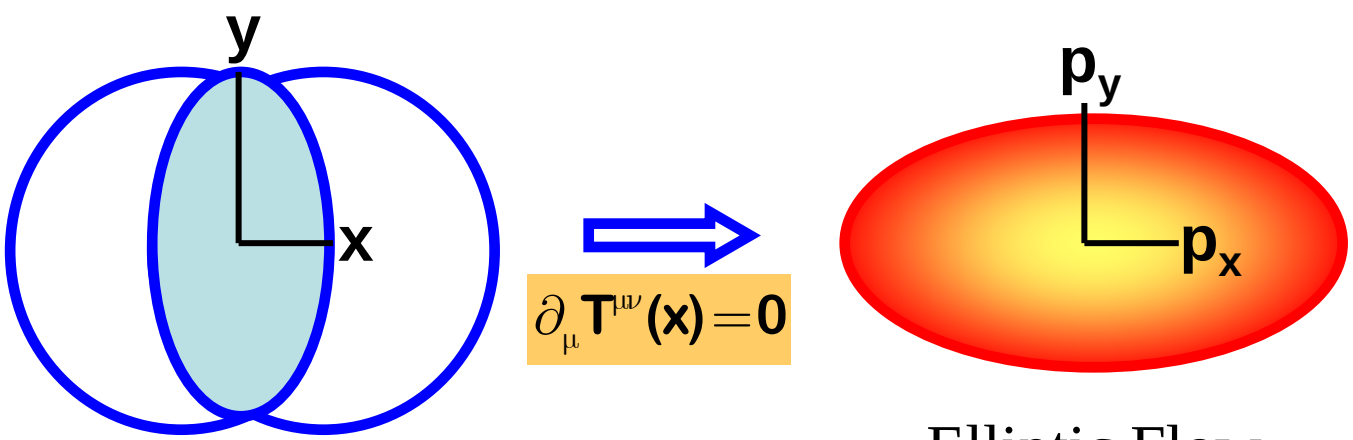
$$\partial_\mu T^{\mu\nu} = \partial_\mu \left\{ u^\mu u^\nu (\varepsilon(T) + P(T)) - g^{\mu\nu} P(T) \right\} = 0$$

Initial Conditions + QCD EOS

W. Greiner, H. Stocker(1974)
P.Kolb, U. Heinz et al (2000)
D.Teany, E. Shuryak
T. Hirano, Y. Nara

Initial *spatial* anisotropy

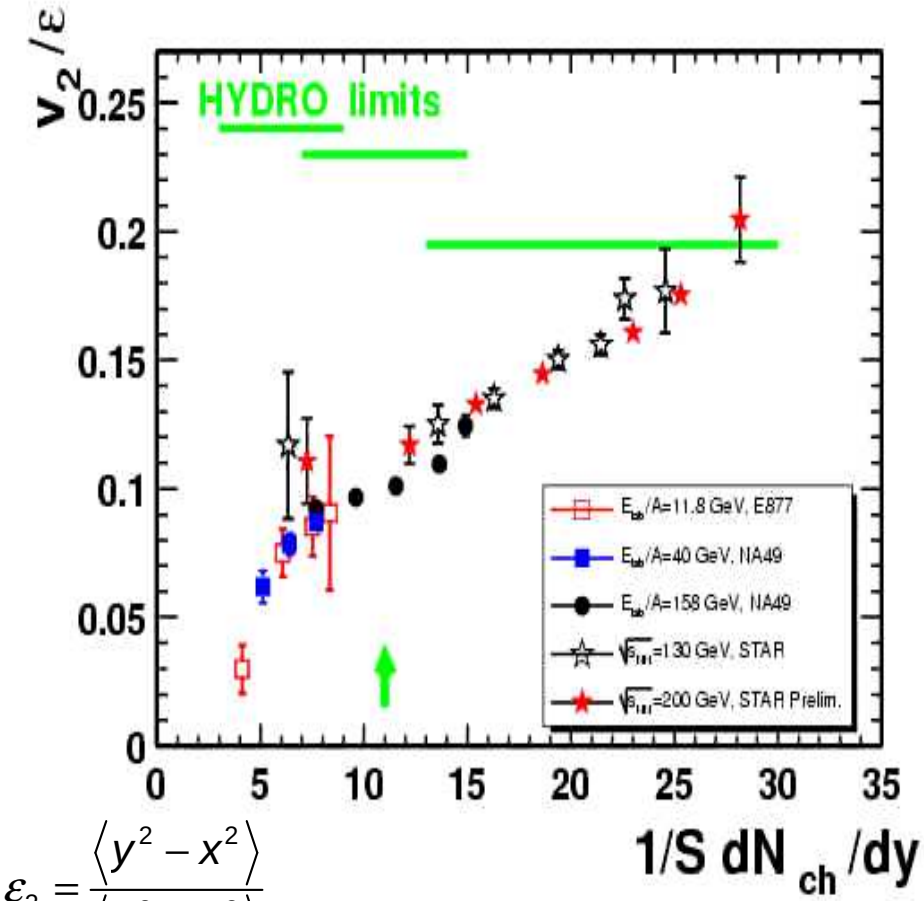
Final *momentum* anisotropy



Elliptic Flow

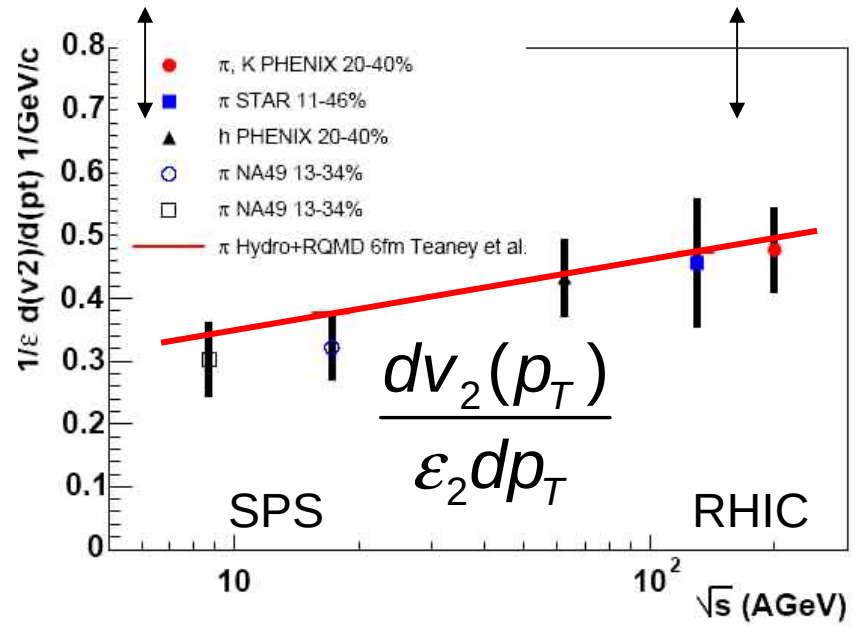
$$\frac{dN}{dy\,dp_T^2\,d\phi} = \rho(y, p_T)\left\{1 + 2v_2(p_T)\cos(2\phi) + \cdots\right\}$$

"Lousy fluid" observed at lower SPS, AGS energies due to Ordinary highly *dissipative* Hadronic Corona



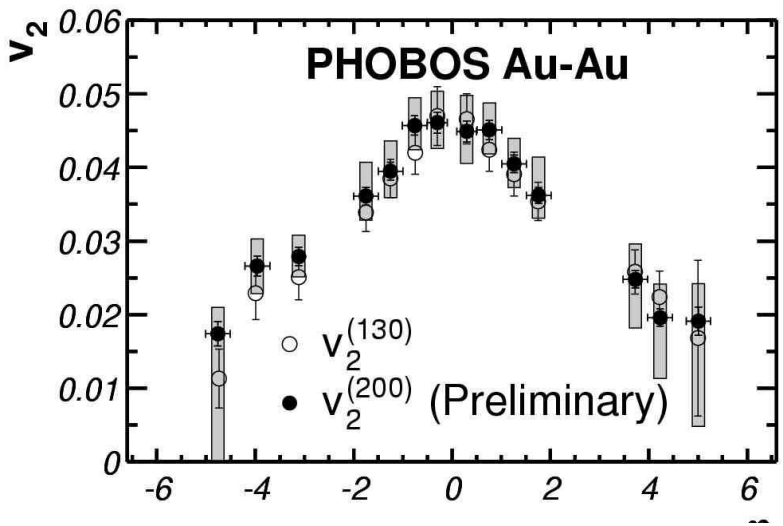
$$\varepsilon_2 = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

Gyulassy LBL 05.26.05



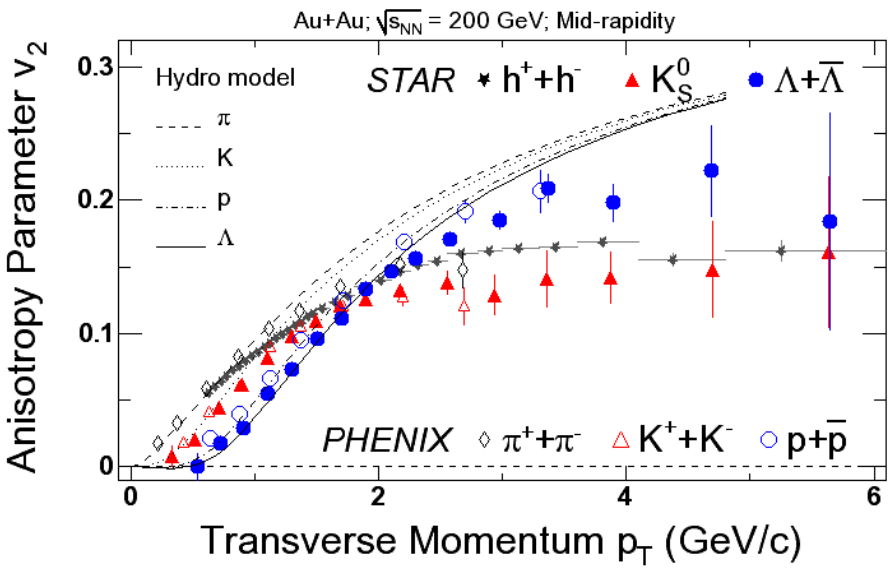
Breakdown of Hydrodynamics in **Hadronic Phase** explains excitation function of $v_2(p_T,\sqrt{s})$

Breakdown of Bulk "Perfect Fluid" Hydrodynamics



Away from mid-rapidity

Due to increasing viscous hadron corona

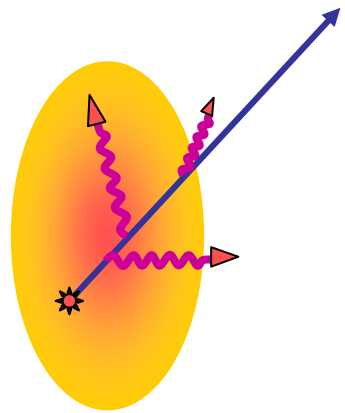


Short wavelengths

due to asymptotic freedom

Azimuthal $v_2(p_t)$ Tomography

$$\rho_{QGP}(\tau, x_0 + \hat{n}\tau)$$



STAR

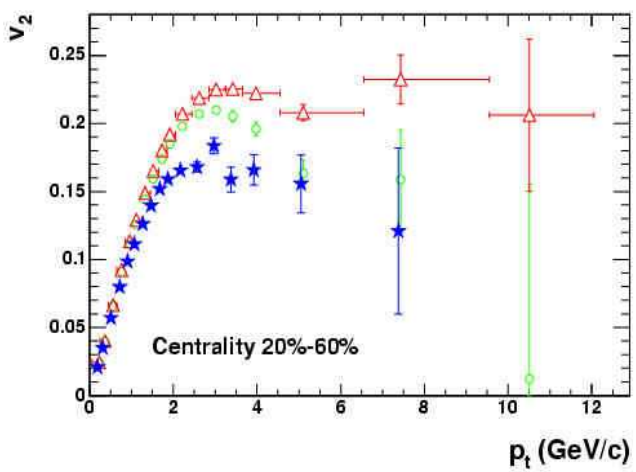
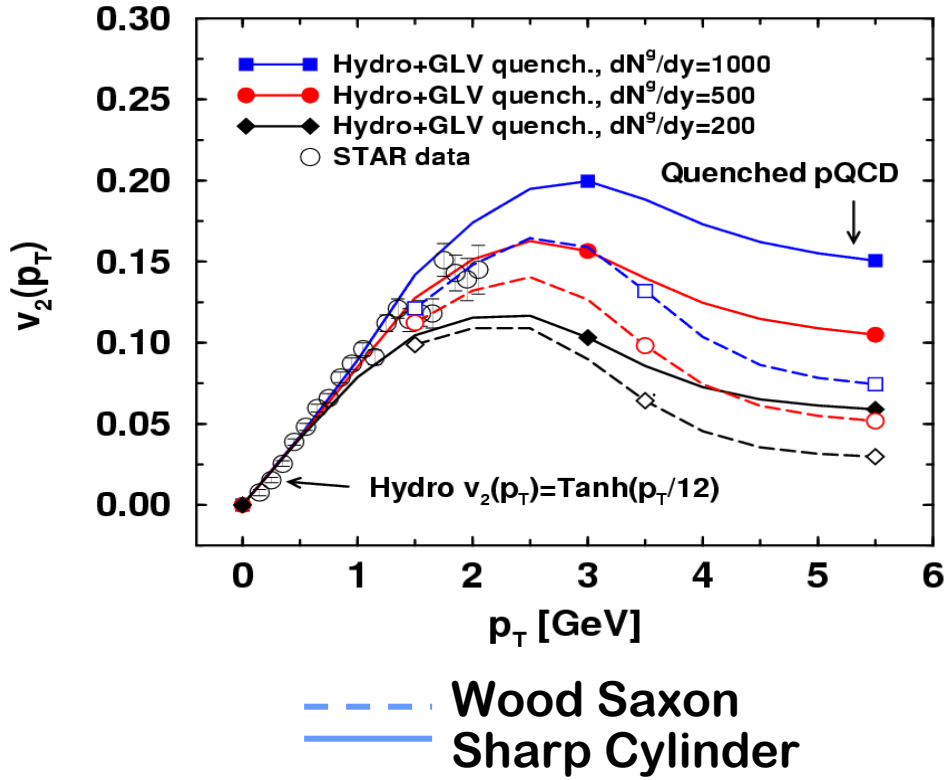


FIG. 2: (color online) v_2 of charged particles as a function of transverse momentum from the two-particle cumulant method (triangles) and four-particle cumulant method (stars). See text for explanation of the open circles. Only statistical errors are shown.

I. Vitev, X.N. Wang, MG, PRL86(01)



High v_2(10 GeV) Still Open Problem

Crash Course on Dissipative Hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = \left\{(\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}\right\} + \tau^{\mu\nu}$$

Flow velocity field $u^\mu(x,t)$ and temperature field $T(x,t)$

1+1D Hubble

$$\frac{d\epsilon}{d\tau} + \frac{1}{\tau}(\epsilon + p) = \left(\tfrac{4}{3}\eta + \xi\right)/\tau^2$$

Bjorken

sound

$$\omega = u_s q - \frac{i}{2}\frac{1}{\epsilon + P}\underbrace{\left(\zeta + \frac{4}{3}\eta\right)}_{\text{Damping}} q^2, \qquad u_s^2 = \frac{\partial P}{\partial \epsilon}$$

Shear viscosity

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\boldsymbol{x}\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

Kubo

$$\eta = \tfrac{1}{5}(\varepsilon + P)\,\lambda = \sigma_{\text{entropy}} \left(\frac{T\lambda}{5}\right)$$

Gas kinetic theory

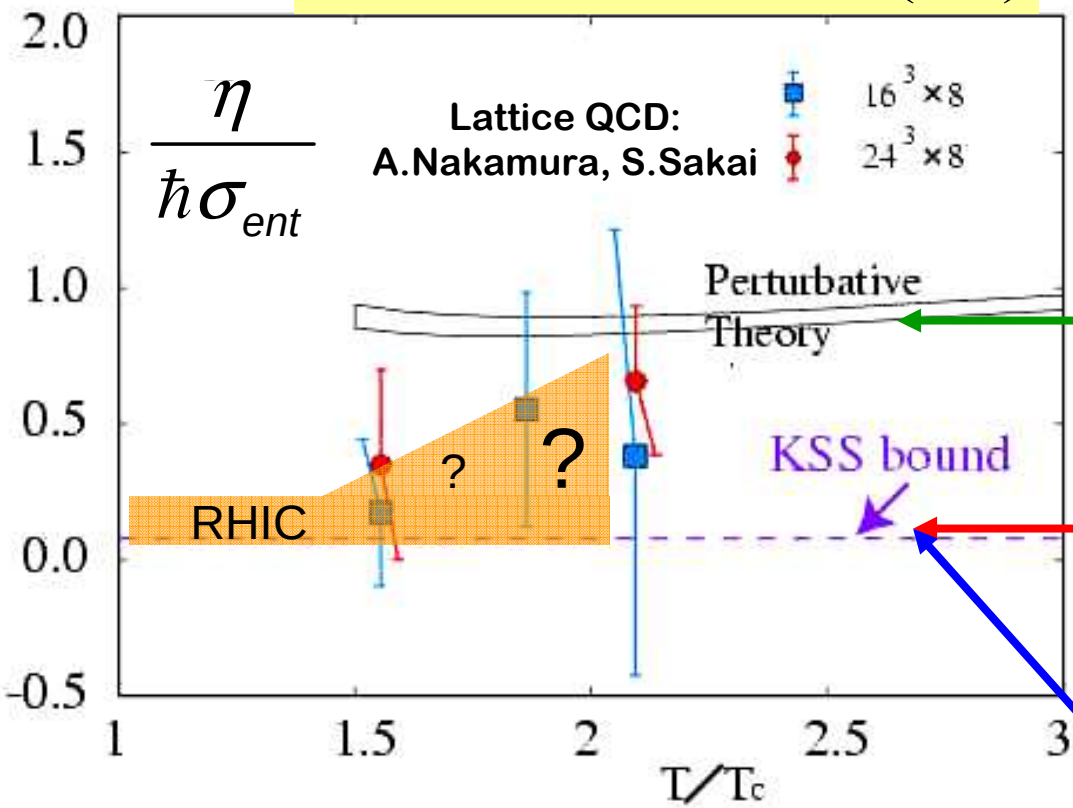
Is Mystery of sQGP superfluidity ⇔ small Viscosity ??

Lattice QCD *vs* pQCD *vs* AdS/CFT *vs* $\Delta x \Delta p \geq \hbar$

Shear Viscosity

$$\eta = \tfrac{1}{5}(\varepsilon + P)\,\lambda = \sigma_{entropy}\left(\frac{T\lambda}{5}\right)$$

Kinetic Theory



pQCD

$$\frac{T\,\lambda_{pQCD}}{5} \approx \hbar\left(\frac{1}{3\alpha}\right)^2$$

$$\left(\frac{\eta}{\sigma}\right) \geq \frac{\hbar}{4\pi}$$

AdS / CFT

$N = 4$ *SUSY*

$g^2 N_c \to \infty$

$$\frac{T\lambda}{5} \geq \frac{\hbar}{15}$$

$\Delta x = \lambda$

$\Delta p \approx 3T$

N=4 SUSY: Kovtun,Son,Starinets (2004)

Danielewicz, MG (1985)

A.Buchel, J.T.Liu and A.O.Starinets,
``Coupling constant dependence of the shear viscosity in N = 4 supersymmetric Yang-Mills theory,"
arXiv:hep-th/0406264.

Thus for large 't Hooft coupling $g^2_{YM} N_c \gg 1$ the correction to the ratio of shear viscosity to the entropy density in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is *positive*,

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{135}{8}\zeta(3)\,(2g^2_{YM} N_c)^{-3/2} + \cdots\right) . \qquad (4.7)$$

But for g^2N_c=10-20 correction is < 20% to 1/4π !

$$\frac{4\pi\eta}{\hbar\sigma}$$

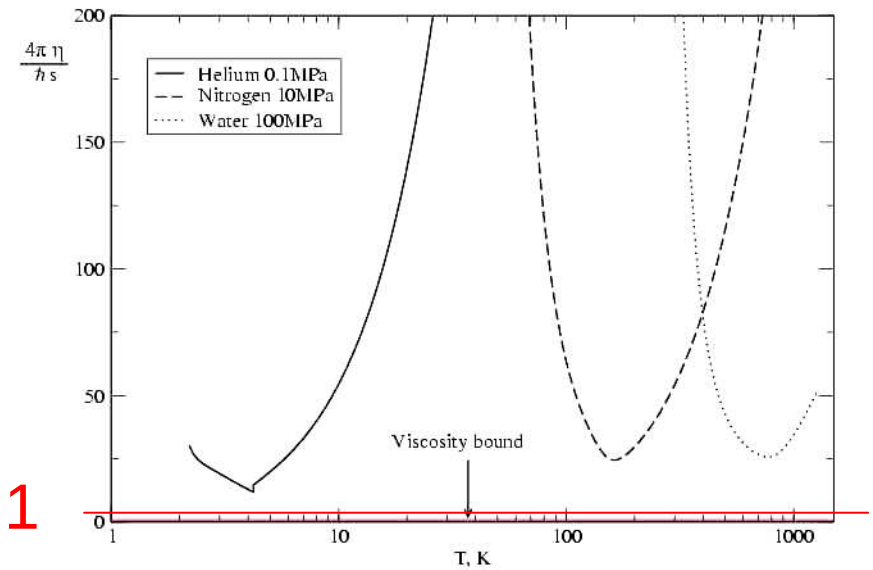


Figure 1: The viscosity-entropy ratio for some common substances.

Kovtun,Son,Starinets 04

adS/CFT
Universal lower bound conjecture

Relaxation time of strongly coupled Yukawa systems

Tomoyasu Saigo, August Wierling and Satoshi Hamaguchi

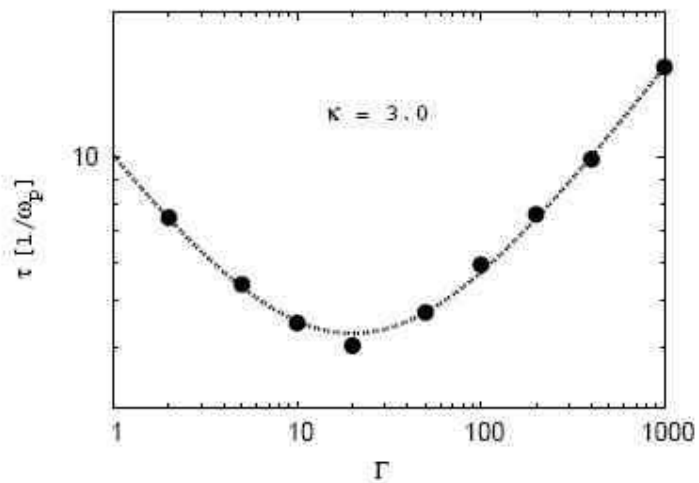


Figure 2: The relaxation time τ as a function of Γ for $\kappa = 3.0$. The fitting curve is based on a form $\tau = a\sqrt{\Gamma} + b/\sqrt{\Gamma} + c$.

We find that the flows retains the initial information for a long time in both weakly and strongly coupled regimes. In the weakly coupled regime, the initial information is retained due to the low collision frequencies. In the strongly coupled regime, it is also retained due to the relatively rigid structure of the system, analogous to the dynamics of ideal crystals.

S. Ichimaru et al., Statis

Viscosity in EM Plasmas

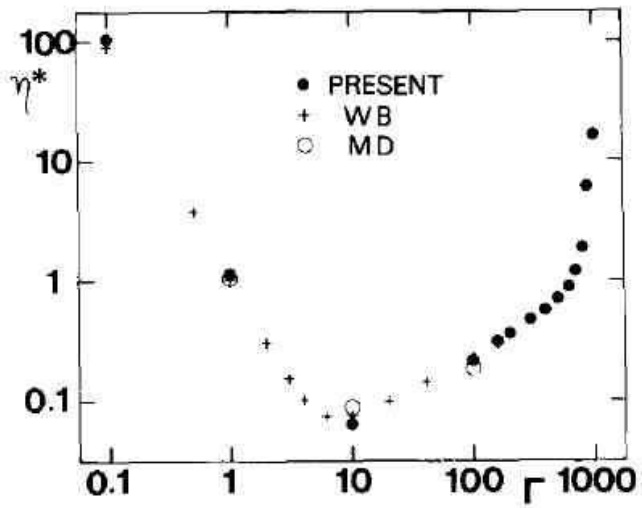


Fig. 33. The reduced shear viscosity $\eta^* \equiv \eta/Mn\omega_p a^2$ calculated on the basis of eqs. (4.15)–(4.17) (solid circles, ref. [77]). The crosses refer to the calculation by Wallenborn and Baus [72], the open circles to the MD simulation result [160].

Viscosity of HeII and CFL via Linearized Boltzmann of Phonons $f(p,t)=f_{eq}(p)+\delta f_p$

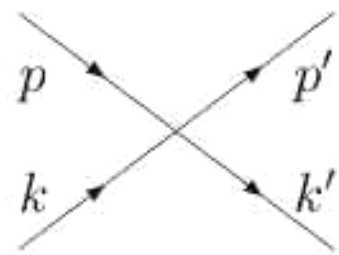
The shear viscosity η enters as a dissipative term in the energy-momentum tensor as follows

$$\tau^{ij} = -\eta\left(\partial_i V_j - \partial_j V_i - \frac{2}{3}\nabla\cdot\mathbf{V}\right) , \qquad \tau_{ij} = \int \frac{d^3p}{(2\pi)^3} p_i p_j \, \delta f$$

Boltzmann equation in local rest frame

$$df_p/dt = C[f_{\mathbf{p}}] = \frac{1}{2}\int_{\mathbf{p},\mathbf{k},\mathbf{p}',\mathbf{k}'} (2\pi)^4 \delta^{(4)}(P+K-P'-K')\,|\mathcal{T}|^2 D$$

$$D = f_{\mathbf{p}'} f_{\mathbf{k}'}(1+f_{\mathbf{p}})(1+f_{\mathbf{k}}) - f_{\mathbf{p}} f_{\mathbf{k}}(1+f_{\mathbf{p}'})(1+f_{\mathbf{k}'}) .$$



$$\delta f = -\frac{f^{\text{eq.}}}{T} g(p) p_x p_y \frac{dV_x}{dy}$$

$$\eta = \frac{4\pi}{15Tv}\int \frac{dp}{(2\pi)^3} p^5 g(p) f^{\text{eq.}}$$

$$\eta = 1.15\,\frac{\mu^4}{T}$$

High viscosity of cold diquark "Neutron" Stars makes Them stable against gravitation damping

SHEAR VISCOSITY IN A CFL QUARK STAR.
Cristina Manuel Antonio Dobado, Felipe J. Llanes-Estrada **hep-ph/0406058**

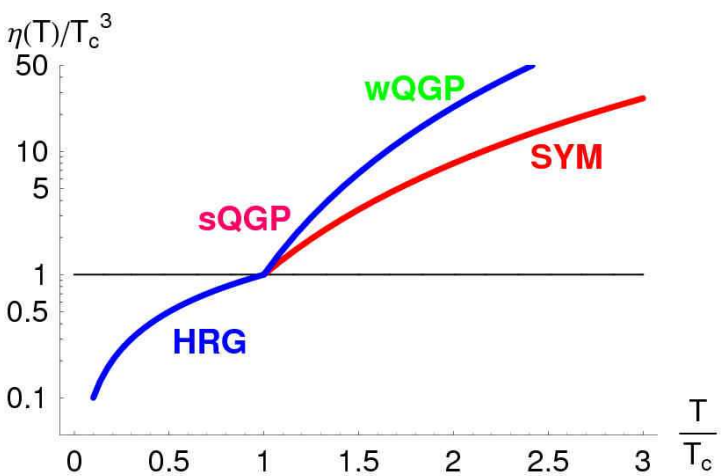
HeII: I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **44**, 769 (1963)

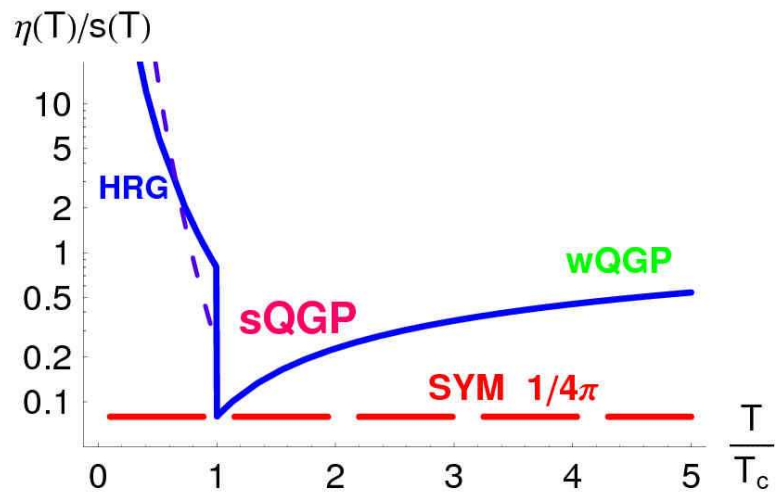
The Dissipative Hadronic Corona and the Higher Viscosity of the "Perfect Fluid" Quark Gluon Plasma

Tetsufumi Hirano and Miklos Gyulassy

In order that all the data on (1) hadronic ratios, (2) radial flow, as well as (3) differential elliptic flow be reproduced, the sQGP must expand with a "minimal viscosity", $\eta_{sQGP} \approx T^3$, that is however numerically comparable near T_c to the viscosity, $\eta_H \approx T/\sigma_H$, of the ordinary highly dissipative hadronic corona.

The "perfect fluid" property of the sQGP is not due to a sudden reduction of the viscosity at the critical temperature T_c, but to the sudden increase of the entropy density and is therefore a signal of deconfinement.





The viscosity of a weakly coupled wQGP based on pQCD kinetic theory [P. Danielewicz, MG, PRD41 (85)]

$$\eta_{\rm wQGP} = \frac{4}{15}\epsilon_{\rm SB}(T)\lambda_{\rm tr} = \frac{T s_{\rm SB}(T)}{5\sigma_{\rm tr}\rho_{\rm SB}(T)}$$

$$\frac{\eta_{\rm wQGP}}{s_{\rm SB}} = \frac{T\lambda_{\rm tr}}{5} \approx 0.4\left(\frac{0.3}{\alpha_s}\right)^2 .$$

$$\epsilon_{\rm SB}(T) = 3P_{\rm SB}(T) = \frac{3}{4}T s_{SB}(T) \approx 3T\rho_{\rm SB}(T) \approx K_{\rm SB}T^4$$

The pQCD transport mean free path $\lambda_{\rm tr} = 1/(\rho_{\rm SB}\sigma_{\rm tr})$ is controlled by the Debye screened transport cross section

$$\sigma_{\rm tr} = \int d\sigma_{\rm el}\,\sin^2\theta_{\rm cm} \approx \frac{4\alpha_s^2}{3T^2}$$ ~ few mb

How small can η/s get with increased coupling?

$$\frac{\eta_{\rm wQGP}}{s_{\rm SB}} = \frac{T\lambda_{\rm tr}}{5} \approx 0.4 \left(\frac{0.3}{\alpha_s}\right)^2 .$$

II. HEISENBERG UNCERTAINTY PRINCIPLE LOWER BOUND.

$$\lambda_{tr} \gtrsim \frac{1}{\langle p \rangle} \approx \frac{1}{3T} \quad \longrightarrow \quad \frac{\eta}{s_{\rm SB}} \gtrsim \frac{1}{15}$$

III. $G^2N = \infty$ STRING THEORY BOUND

Infinitely coupled $\mathcal{N} = 4$ supersymmetric conformal Yang-Mills (SYM) gauge theory using the AdS/CFT duality conjecture [Son, et al]

$$\left(\frac{\eta}{s}\right)_{\rm SYM} = \frac{1}{4\pi}$$

The SYM entropy density in the $N_c = \infty, g^2 N_c \gg 1$ limits is given by [Gubser et al (98)]

$$s_{\rm SYM} = \left\{\frac{3}{4} + \frac{0.6}{(g^2 N_c)^{3/2}} + O(\frac{1}{N_c^2})\right\} \frac{2\pi^2}{3} N_c^2 T^3$$

$$\eta_{\rm sQGP}(T) = \frac{K_{\rm SB} T^3}{4\pi} \approx T_c^3 \left(\frac{T}{T_c}\right)^3$$

Effective sQGP transport cross section

$$\sigma_{\rm tr}^c \approx \frac{4}{5} \frac{T}{\eta_{sQGP}(T)} \approx \frac{16\pi}{5 K_{SB}} \frac{1}{T^2} \sim (12 \text{ mb}) \frac{T_c^2}{T^2}$$

IV. HADRONIC RESONANCE GAS

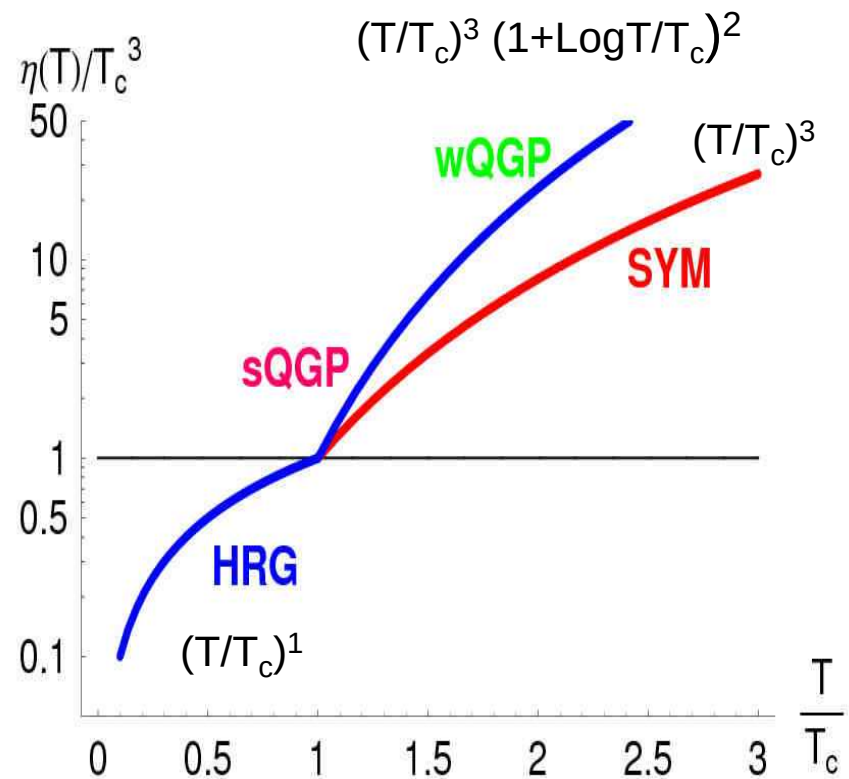
$$s_{HRG}(T) = s_H \left(\frac{T}{T_c}\right)^{\frac{1}{c_H^2}} \qquad c_H^2 \approx 1/6 - 1/3$$

$$\eta_H(T) \approx \frac{T}{\sigma_H} \approx T_c^3 \frac{T}{T_c}$$

Because the hadronic transport cross sections are $\sigma_H \approx 10$ mb , we should not expect a large variation of the absolute value of the matter viscosity across T_c if the minimal η/σ holds above T_c.

$$\eta(T) \approx T_c^3 \begin{cases} (T/T_c)^1, & T < T_c \\ (T/T_c)^3, & T > T_c \end{cases}$$

What changes rapidly at T_c is therefore not the viscosity but rather its entropy density due to the deconfinement of the quark and gluon degrees of freedom.



"perfect fluid" sQGP has higher viscosity than the viscous HRG

Eventually, asymptotic freedom should slowly convert the sQGP into a wQGP for $T \gg T_c$. The following interpolation formula for $T > T_c$ may describe the slow sQGP$\rightarrow$ wQGP transition

$$\boxed{\frac{\eta(T)}{s(T)} \approx \frac{1}{4\pi}\{1 + c\log(T/T_c)\}^2}$$

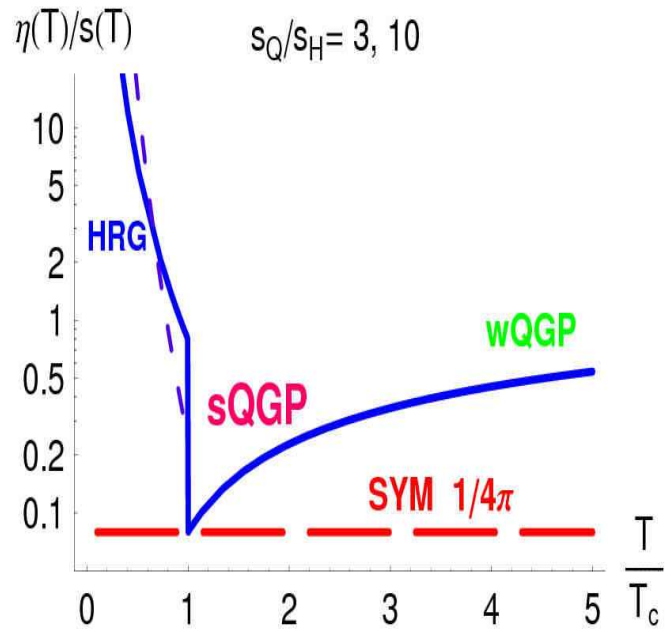
$T>T_c$

$$c^2 = \frac{9\beta_0^2}{20\pi K_{\rm SB}} \approx 0.9 - 1.1$$

With $K_{SB} = 12 - 15$ and $\beta_0 = 11 - 2N_f/3 \approx 9.67 - 9.0$

Summary: $\eta(T)$ varies smoothly near T_c but the ratio η/s has a rapid drop due to the rapid onset of deconfinement.

$$\frac{\eta(T)}{s(T)} \approx \frac{1}{4\pi} \begin{cases} \left(\frac{s_Q}{s_H}\right)\left(\frac{T_c}{T}\right)^{1/c_H^2}, & T < T_c \\ \log^2(eT/T_c), & T > T_c \end{cases}$$



with the negative discontinuity (within $\Delta T_c / T_c \approx 0.1$)

$$\left[\frac{\eta}{s}\right]_{T_c} = -\frac{1}{4\pi}\left(\frac{s_Q}{s_H} - 1\right)$$

It is the entropy jump $s_Q/s_H \sim 3 - 10$ that causes a drop of η/s near T_c.

Continuity of viscosity from sQGP -> HRG

Is also important for simplifying the Hydro -> Cascade

Boundary Conditions

Self-Consistent Relativistic Hydrodynamics

Kyrill Bugaev

Lawrence Berkeley National Laboratory
Nuclear Science Division

It is an illusion that Hydro Equations are of the form:

$$\partial_\mu T_f^{\mu\nu}(x,t) = 0 , \qquad T_f^{\mu\nu}(x,t) = (\epsilon_f + p_f)\, u_f^\mu u_f^\nu - p_f g^{\mu\nu} ,$$
$$\partial_\mu N_f^\mu(x,t) = 0 , \qquad N_f^\nu(x,t) = n_f u_f^\nu ,$$

Without boundary conditions (=**Freeze-out Procedure**)

Hydro Equations **do not make any sense** at all!

No term can be NEGLECTED!!!

For switch criterion $F(x.t) = 0$ between ideal fluid F and $non-ideal$ Hadronic Gas G described by cascade:

$$T^{\mu\nu}_{tot}(x,t) = \Theta^*_f\, T^{\mu\nu}_f(x,t) + \Theta^*_g\, [T^{\mu\nu}_g(x,t) + \tau^{\mu\nu}_g(x,t)]\,,$$

with

$$\Theta^*_f = 1 - \Theta^*_g\,, \quad \Theta^*_g = \Theta(F(x,t)): \quad \Theta^*_g = 1 \;\; for \;\; gas \;\; only!$$

If at the switch hypersurface

$$d\sigma_\mu T^{\mu\nu}_f(x,t^*) \neq d\sigma_\mu [T^{\mu\nu}_g(x,t^*) + \tau^{\mu\nu}_g(x,t^*)]\,,$$

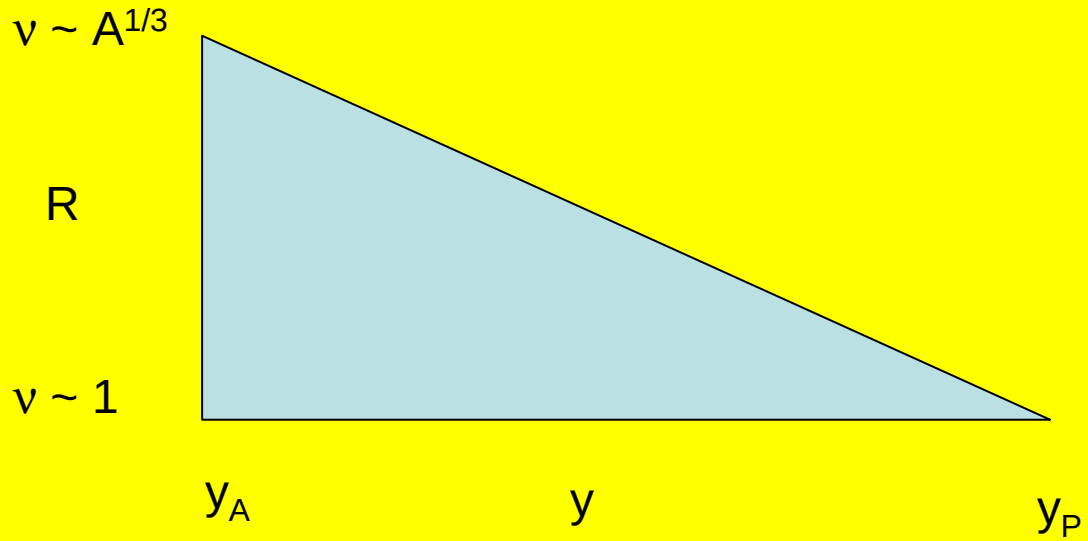
Then equations of motion are as follows:

$$\begin{aligned} \Theta^*_f\, \partial_\mu T^{\mu\nu}_f(x,t) &= -\Theta^*_g\, \partial_\mu [T^{\mu\nu}_f(x,t) + \tau^{\mu\nu}_g(x,t^*)] \\ &- \sigma_\mu [T^{\mu\nu}_f(x,t^*) - T^{\mu\nu}_g(x,t^*) - \tau^{\mu\nu}_g(x,t^*)]\, \underline{\delta(t - t^*(x))}\,, \end{aligned}$$

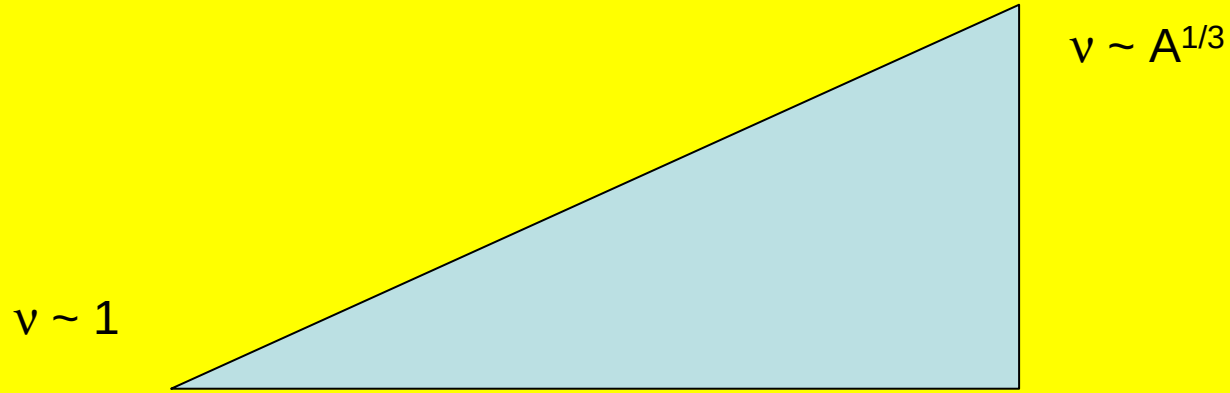
No matter how small is coefficient in front of δ-function
IT CANNOT BE NEGLECTED!!!

But viscosity and $\tau^{\mu\nu}$ are continuous across Tc, so this problem may be solved

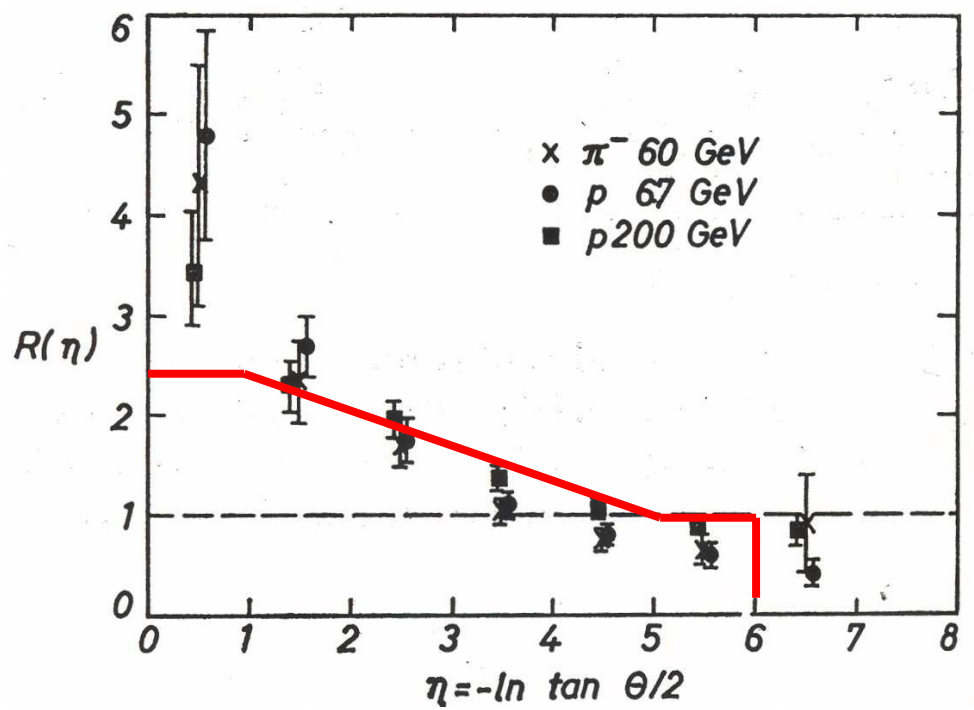
Part II: The P+A Triangle for R= dN^{pA}/dy / dNpA/dy



The A+P Triangle for R= dN^{Ap}/dy / dNpA/dy



Proton+Emulsion data
W. Busza review 1976
(Acta Phys. Pol. B8, 333)

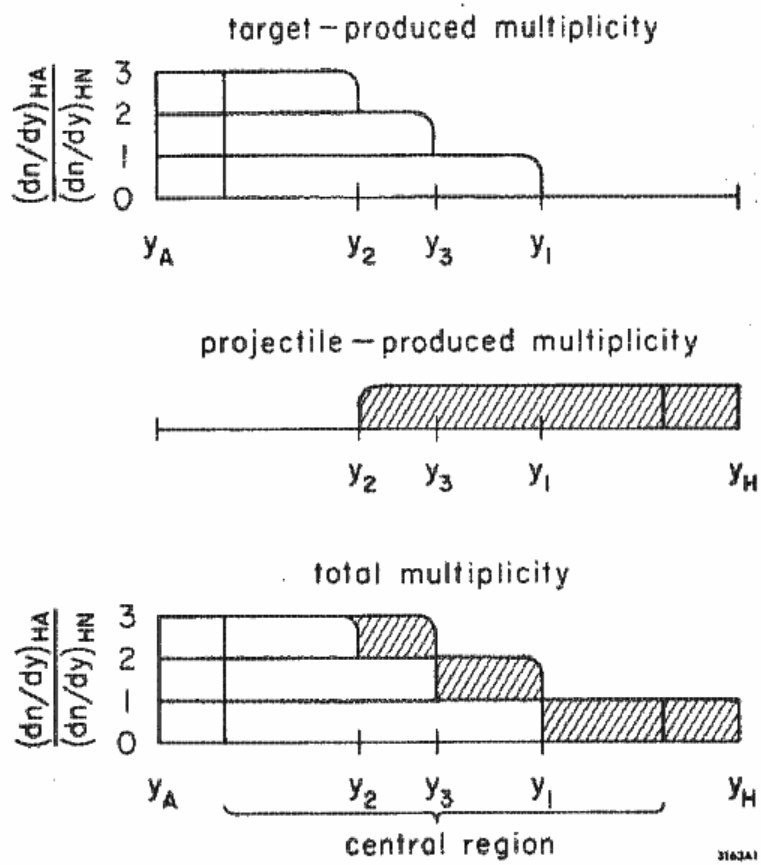


Brodsky, Gunion, Kuhn

PRL39(77)1120

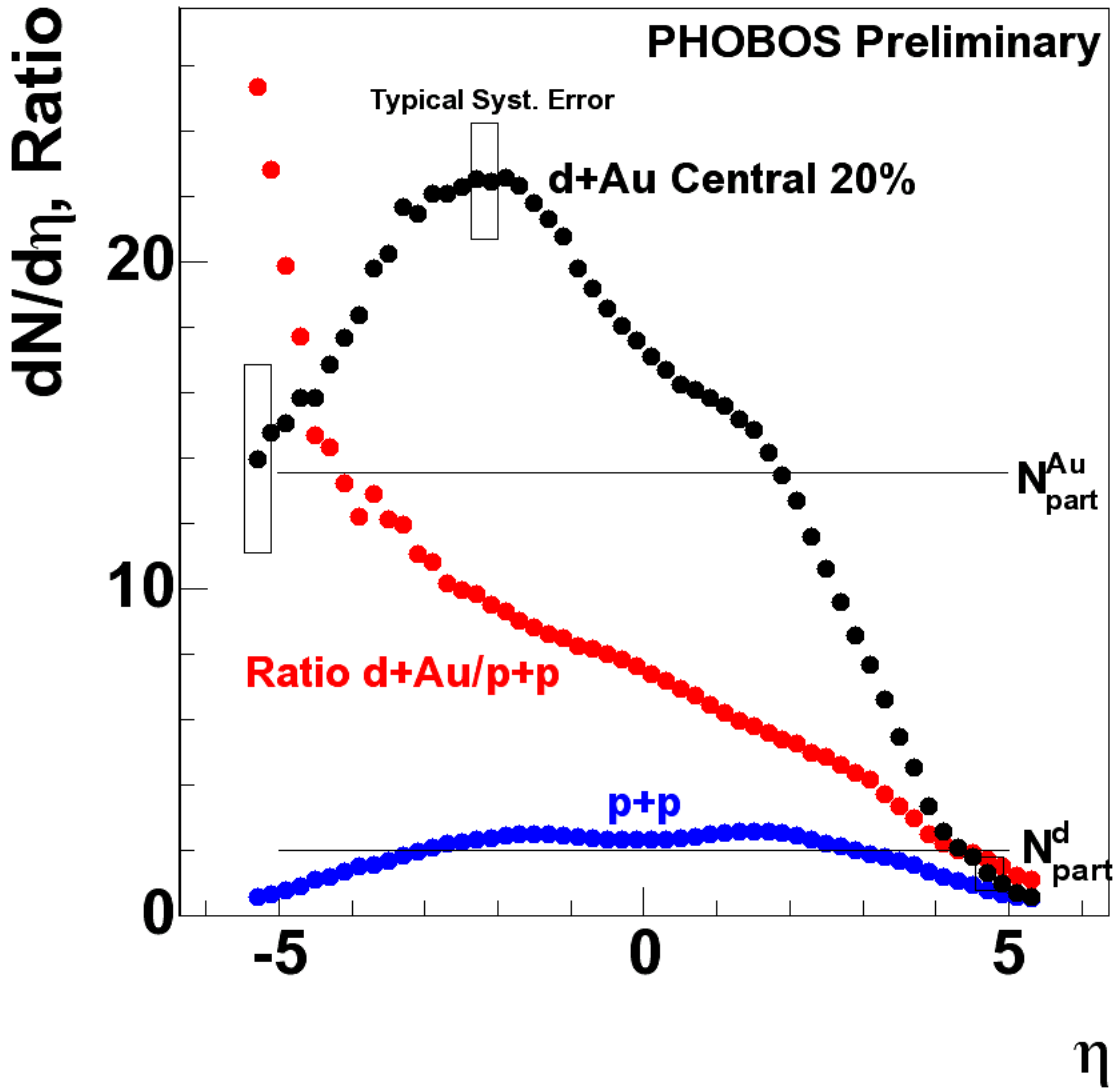
Color Neutralization Model

Feynman gas dy=dx/x= dM/M



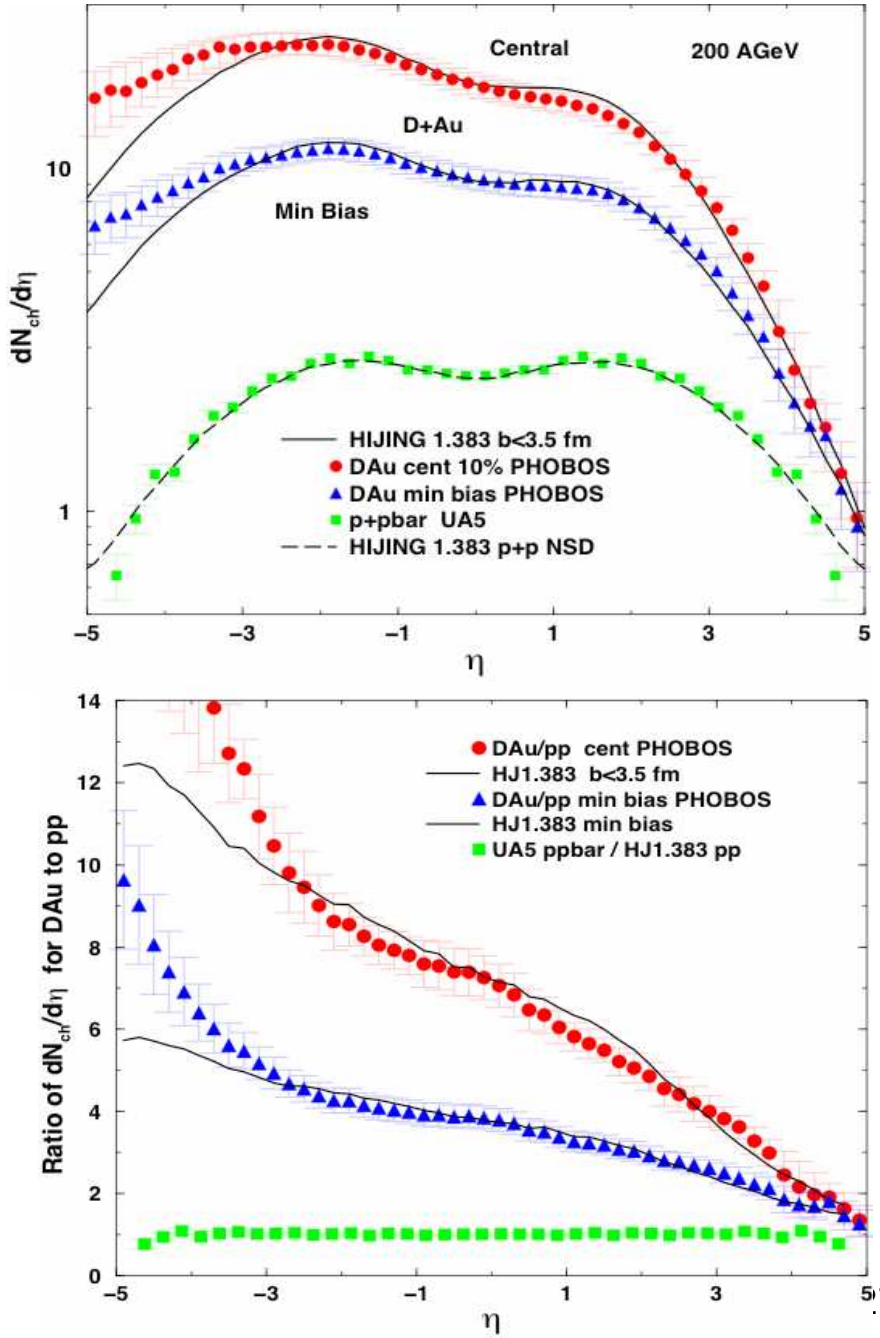
The low pT *Triangle* Boundary Condition on $R_{pA}(y,p_T<1)$

Triangle rapidity distribution found at RHIC



HIJING includes this basic Triangular or Trapezoidal

A+B multiplicity enhancement through Lund strings



BGK Saddle in (η,x_T) in Au+Au

INTRINSIC LOCAL BJORKEN SCALING VIOLATION O($A^{1/3}$/log(s))

- Local participant density

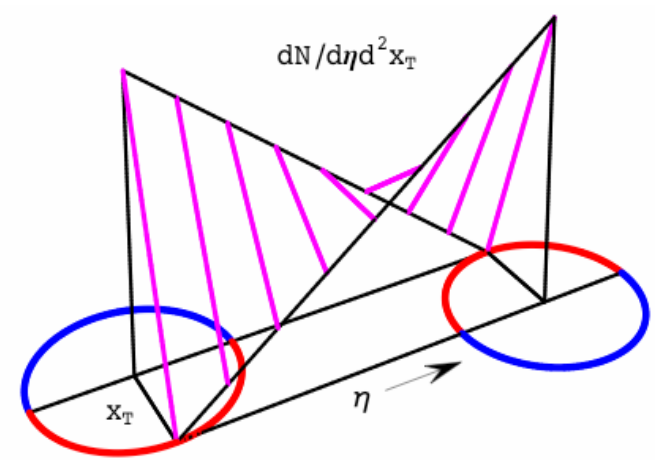
$$\frac{dN_{AA\to g}}{d\eta d^2\vec{x}_T} / \frac{dN_{pp\to g}}{d\eta d^2\vec{x}_T}$$

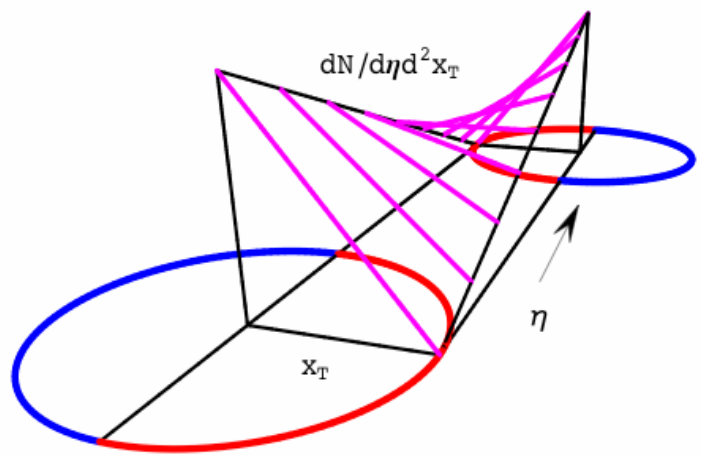
$$\approx \nu_A(\mathbf{x}_\perp - \mathbf{b}/2)\frac{Y-\eta}{2} + \nu_B(\mathbf{x}_\perp + \mathbf{b}/2)\frac{Y+\eta}{2}$$

- global multiplicity AA/pp

$$\approx \frac{1}{2}(N_A + N_B) + \frac{\eta}{2Y}(N_B - N_A)$$

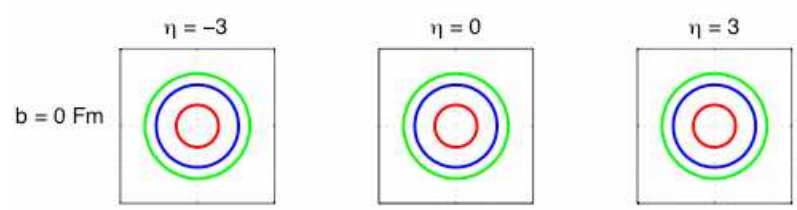
- Note global multiplicity is boost invariant for A = B but local density is Not!



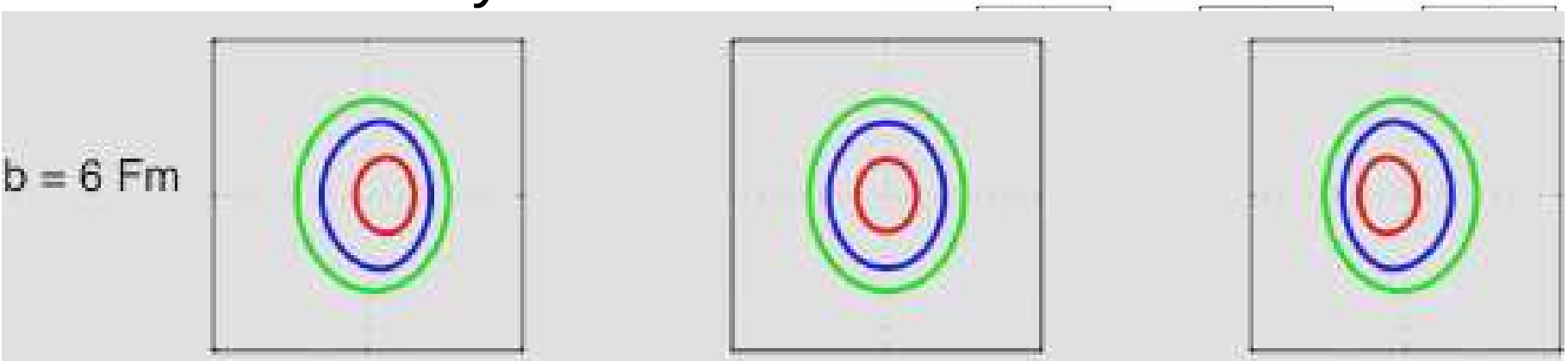


Forward-Backward Rapidity Asymmetry

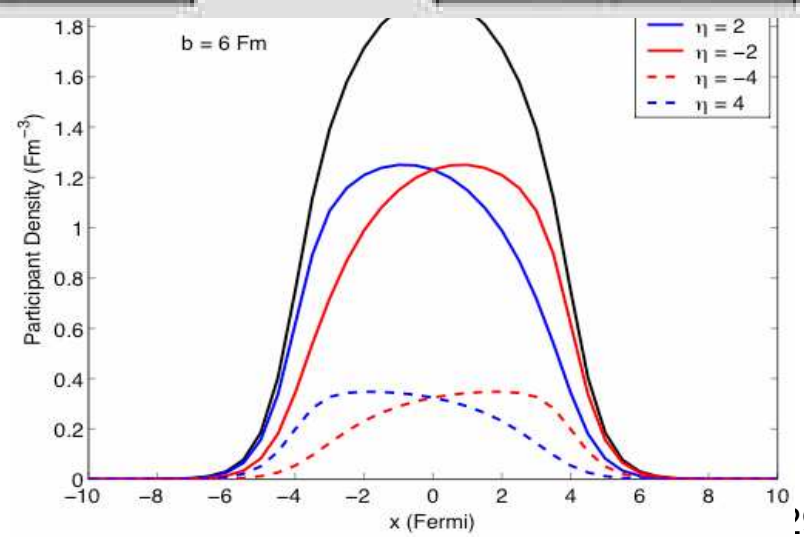
- Contour Plots of the local density



Green - 10% , Blue - 50% , Red - 90%

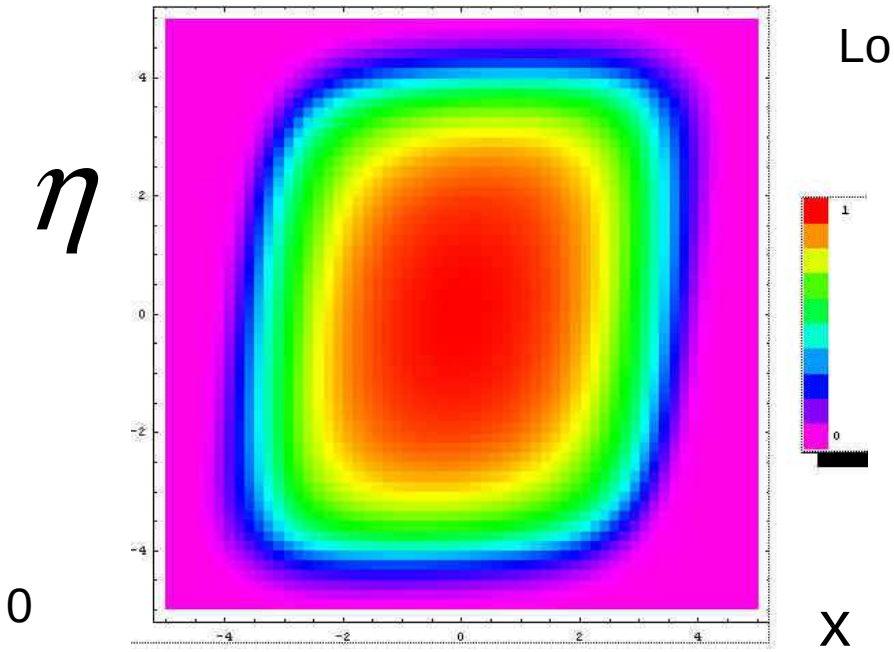


- Similar geometries studied by Hirano/Nara



Long Range Rapidity Twist

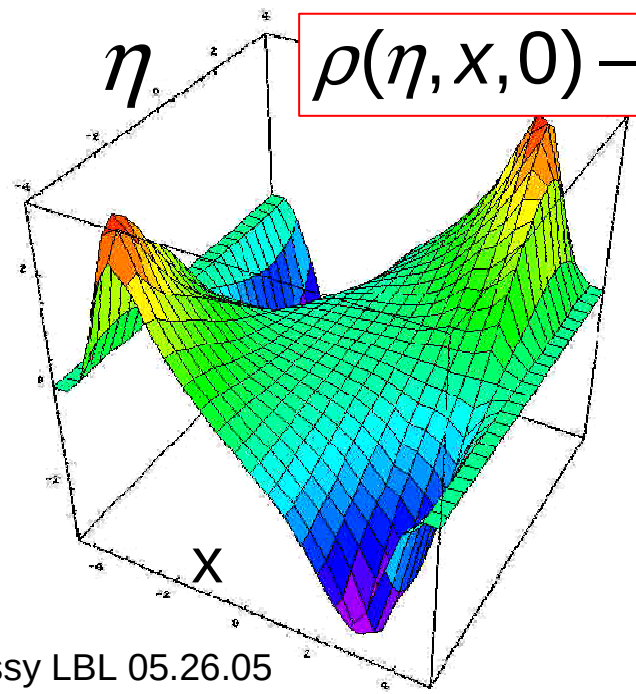
$$\rho(\eta, x, 0) = \frac{dN_{AA\to g}}{d\eta d^2\vec{x}_T} / \frac{dN_{pp\to g}}{d\eta d^2\vec{x}_T}$$



For CGC Initial Conditions

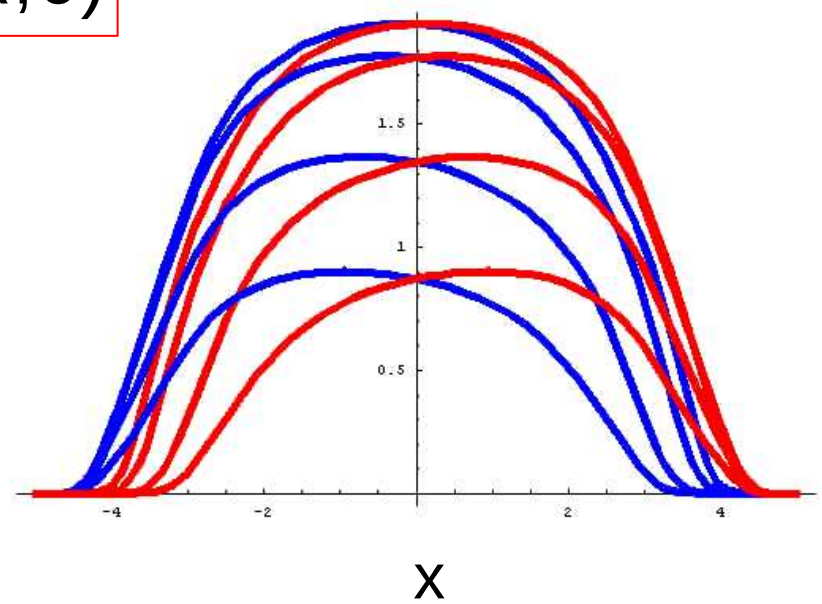
(T. Hirano, MG)

$$\rho(\eta, x, 0) - \rho(-\eta, x, 0)$$

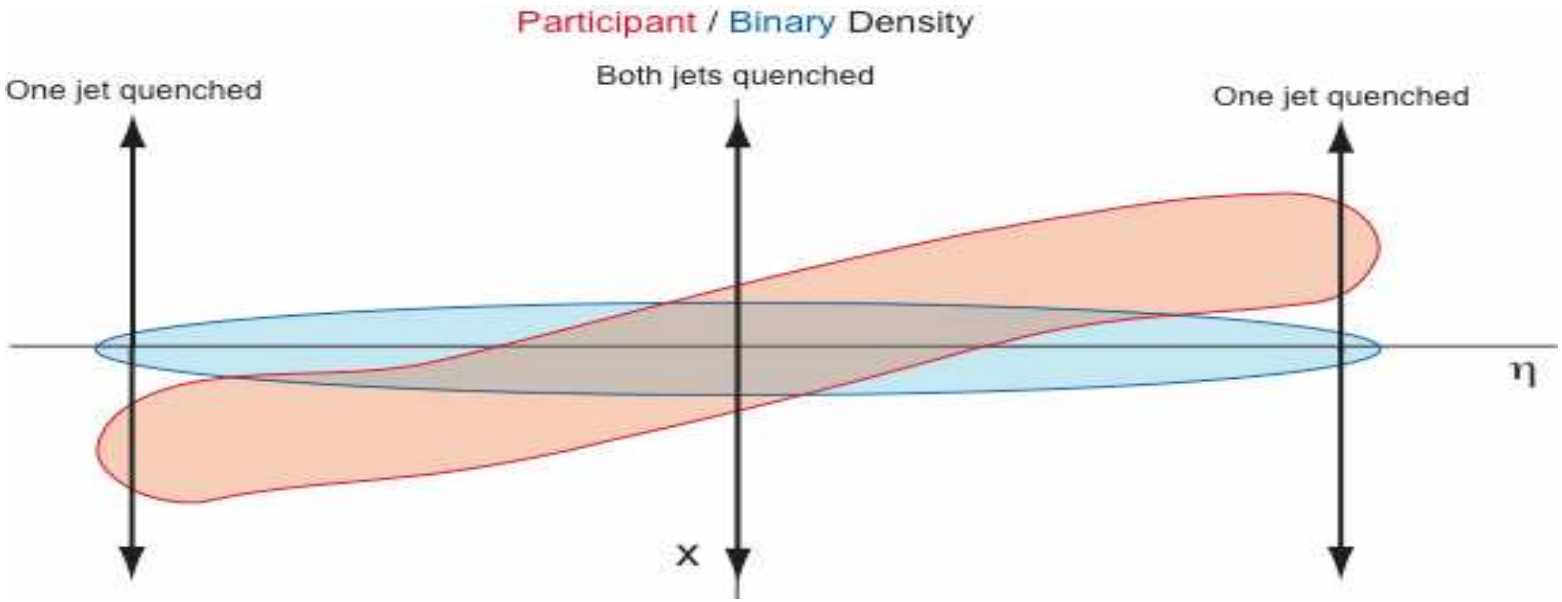


Saddle

$$\eta = \pm 1.5, \pm 2.5, \pm 3.5$$



Twisted sQGP Initial Conditions



-
  - Asymmetry apparent in Participant density (rotation around y-axis)
  - Binary collision density is symmetric for A+A
- Asymmetry can be probed via jet quenching
  - Long range rapidity anti correlations can be recorded.
  - * CGC has more complex twist pattern (in progress)

Azimuthal asymmetric R_{AA}

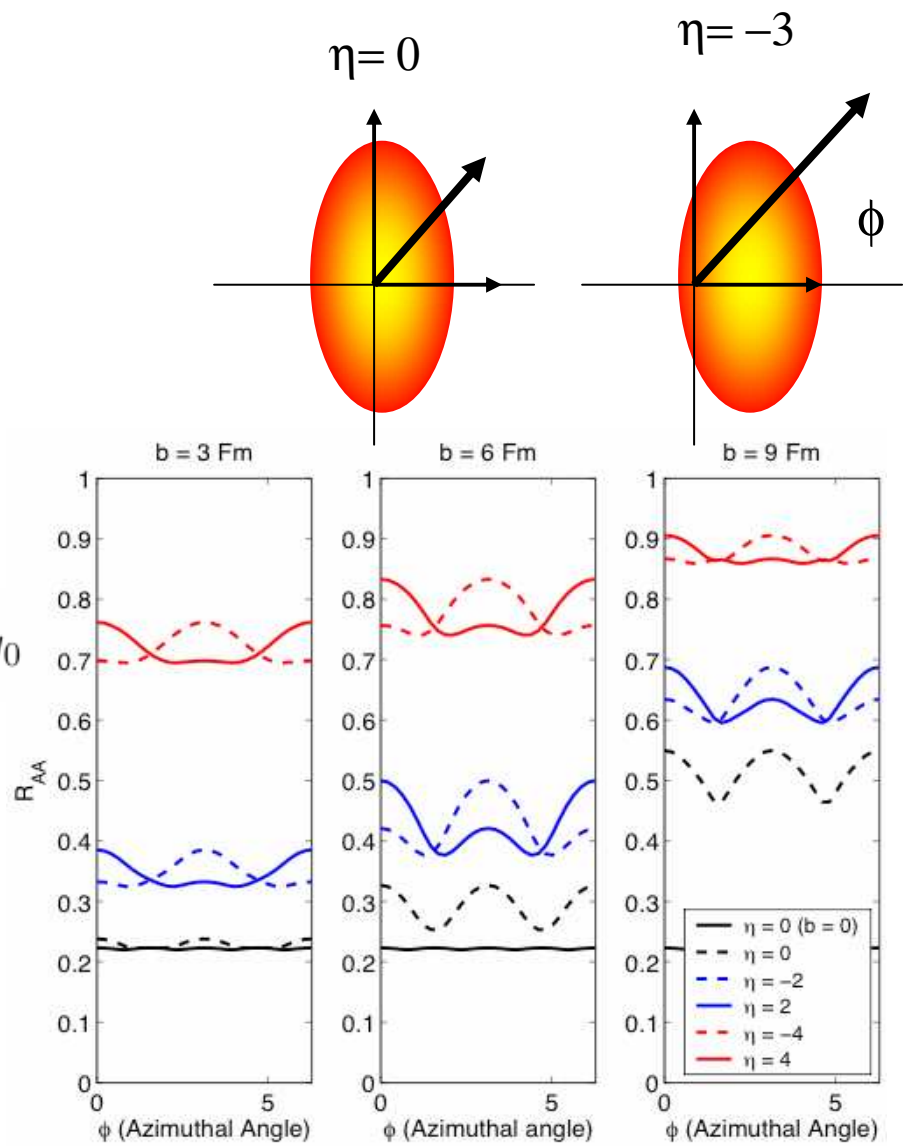
- Nuclear Modification

$$R_{AA} = \frac{dN_{AA}/d\eta d^2p_T}{T_{AA}(d\sigma_{pp}/d\eta d^2p_T)}$$

- Calculated using Drees, et al. absorption model

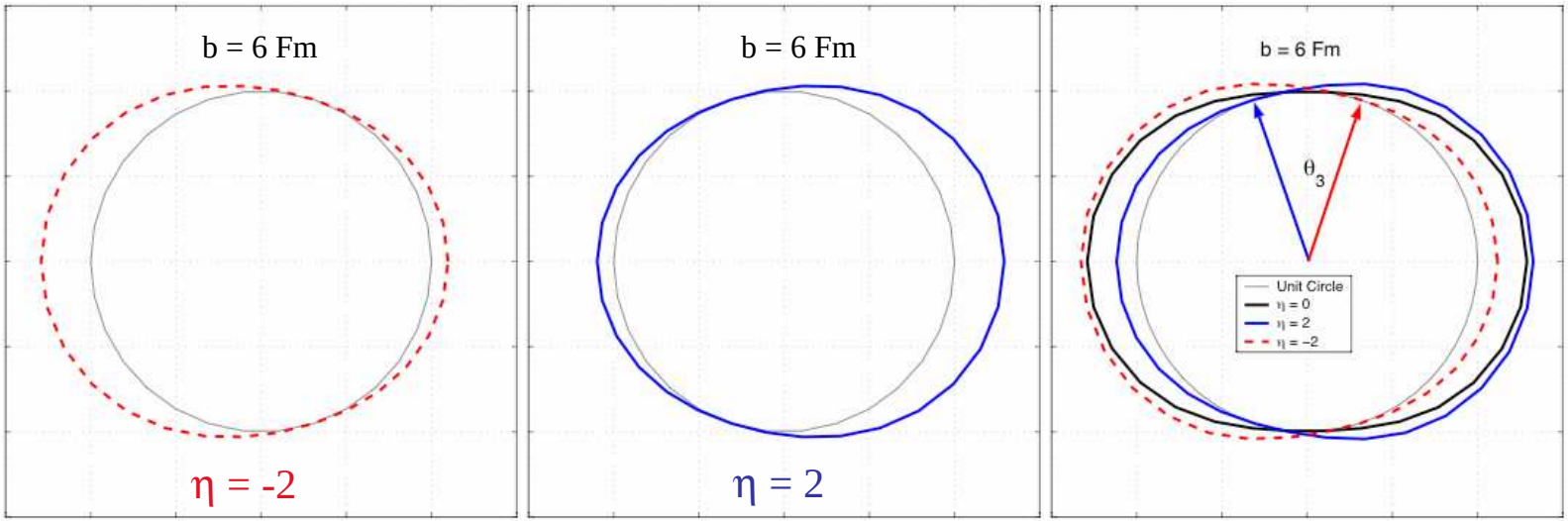
$$R_{AA} = \frac{1}{N_{Bin}} \int \frac{dN_{Bin}}{dxdy}(x_0, y_0, b) e^{-\kappa\chi} dx_0 dy_0$$

  – $\kappa \sim 0.25$



Polar Plot of R_{AA}/R_{min}

$$\frac{R_{AA}(\phi,\eta,b)}{R_{AA}^{min}(\eta,b)}$$

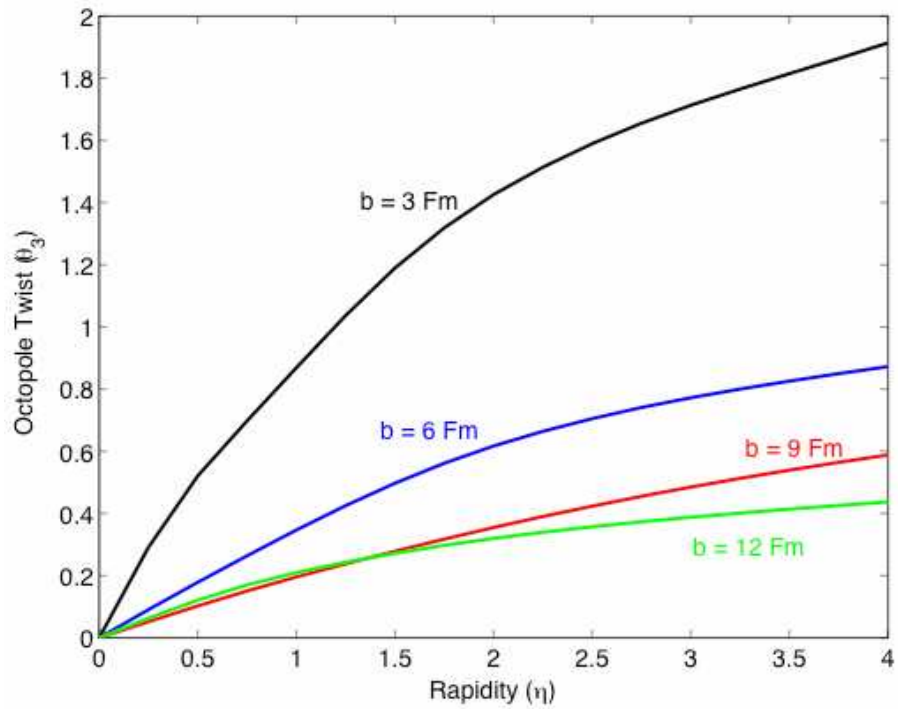


Pear shape due to twisted sQGP initial conditions

- Measure using Octupole Twist 'θ_3'
- Long range anti-correlation over rapidity

Octupole Twist Evolution

- Evolution with rapidity and impact parameter
- At higher rapidity there is an increasing Octupole Twist



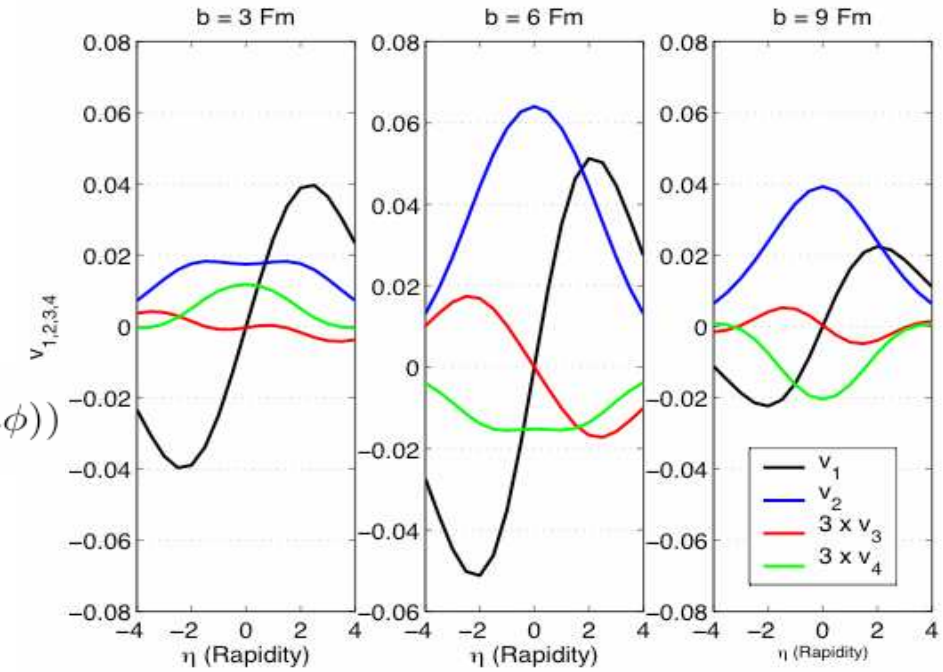
Twisted Harmonic Decomposition

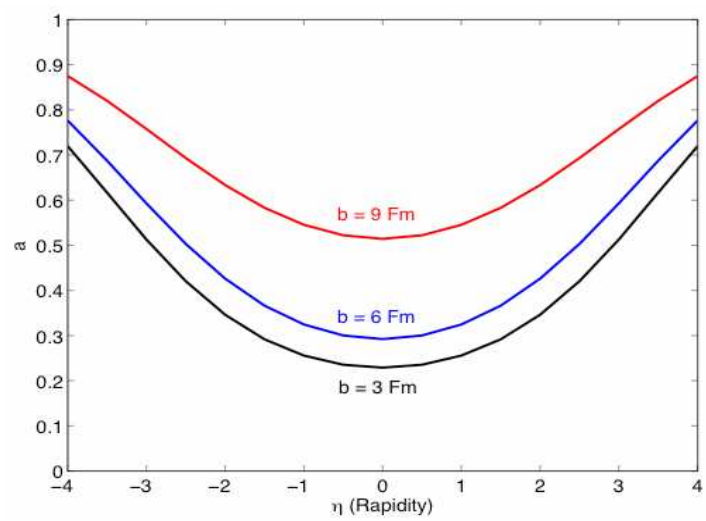
(A.Adil, MG (05))

- Decompose R_{AA} into fourier moments

$$R_{AA}(\phi,\eta,b) = a(\eta,b)(1 + 2\sum_{n=1}^{4} v_n(\eta,b)\cos(n\phi))$$

- Moments increase in magnitude with increasing asymmetry
- Higher moments increase in significance with larger b and η





Part II Conclusions

- Bjorken Scaling Violation is an intrinsic feature of A-A and leads to twisted sQGP initial conditions
  - Inherent but differ in detail in (BGK , HIJING, CGC)
  - Leads to qualitatively novel jet quenching patterns
  - Rotates Participant sQGP Density in η,x plane
  - Induces Dynamic Twist Effect θ_3 in $R_{AA}(\varphi,\eta)$
  - Can measure via non-flow directed v1 and octupole! $v_3(\eta,p_T)$ Fourier moments in addition to usual v2,v4
- Will help to test details and sort out
  - CGC vs Shadowing
  - Cronin