Effect of Eccentricity Fluctuations on Elliptic Flow


Color by Roberta Weir

Exploring the secrets of the universe

Art Poskanzer

The Berkeley School 2010

GSI-LBL Collaboration

July 1974 -- 36 years ago

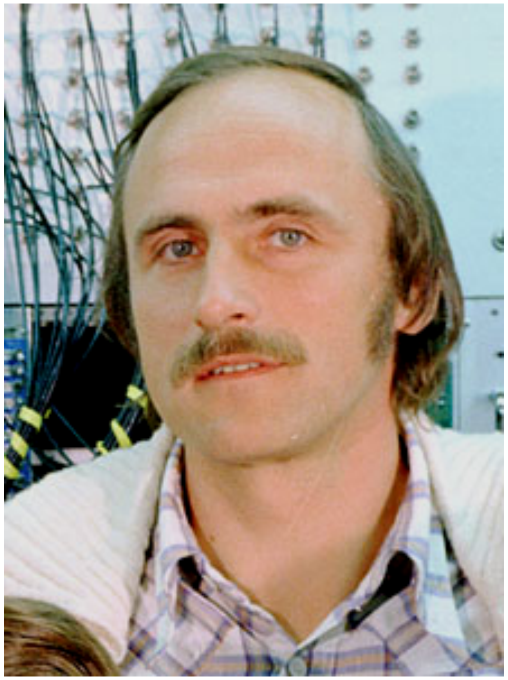
Reinhard and Rudolf Bock walked into my office

Plastic Ball Streamer Chamber

Hermann Grunder

2 Aug 74

Reinhard

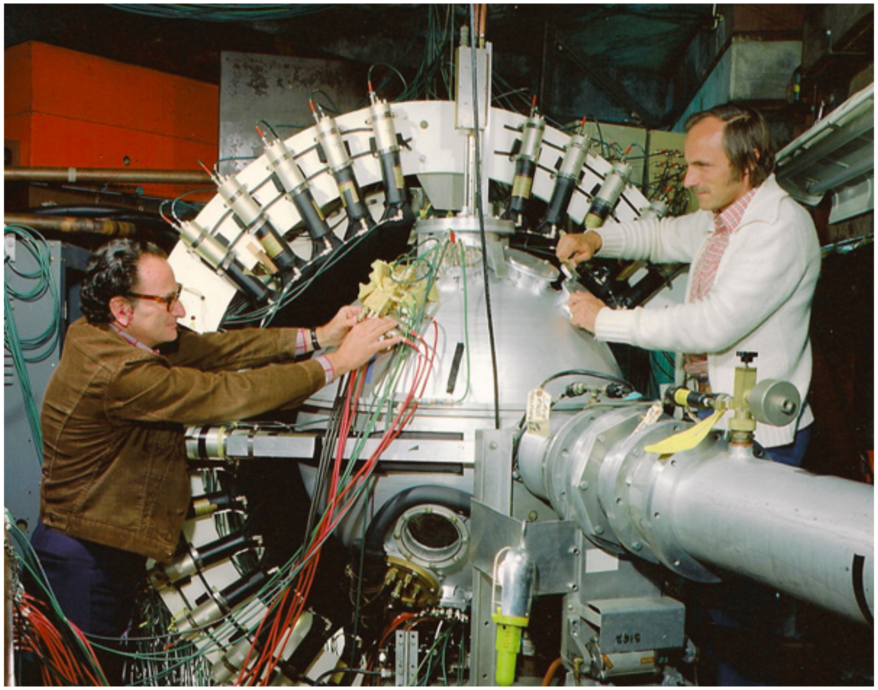


1976

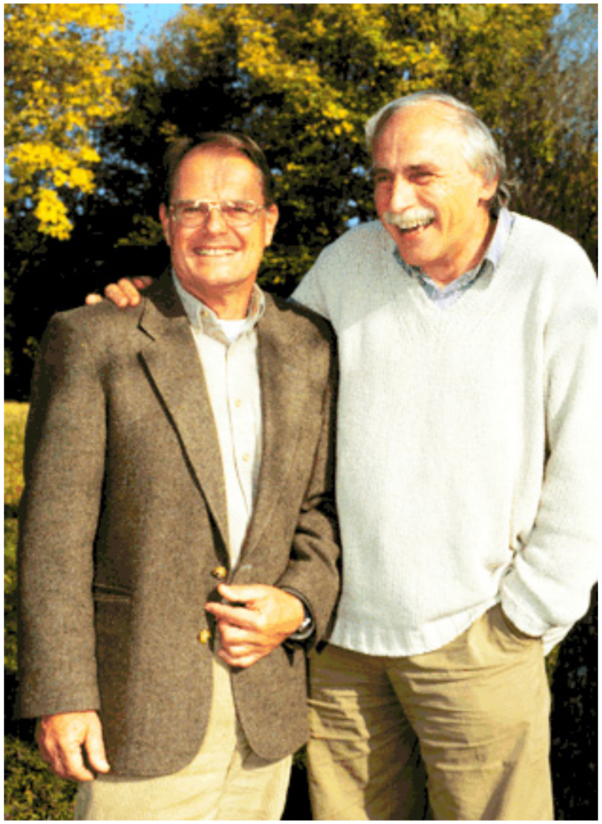
1995

2001

Rudolf Bock
1987



1977

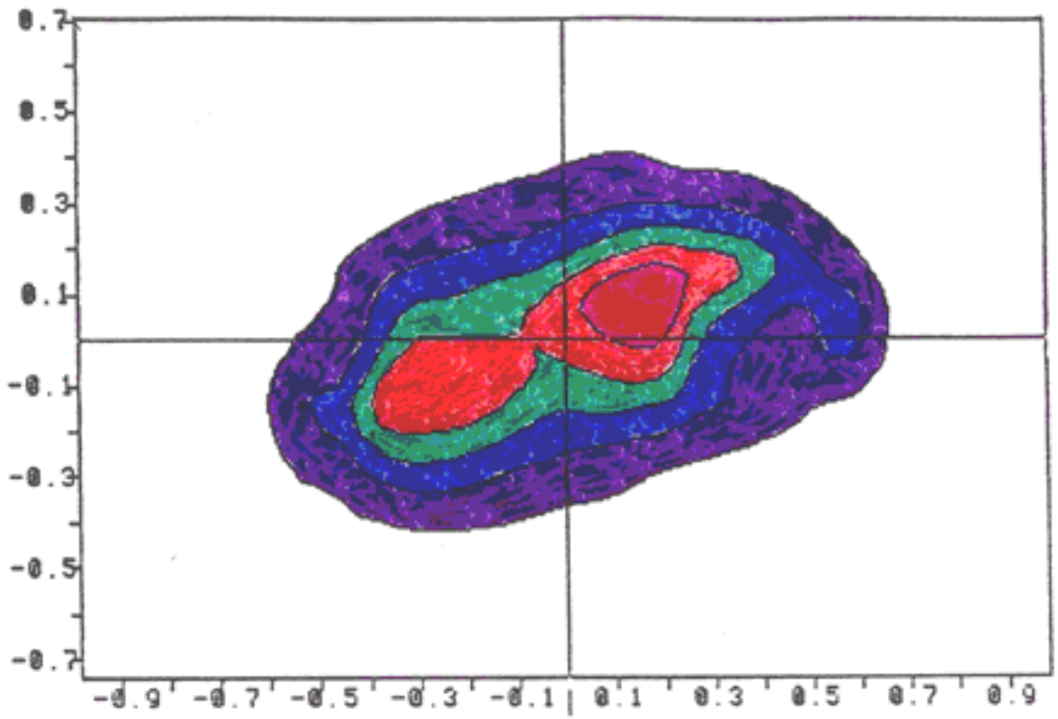


1996

2001

photo by Jef Poskanzer

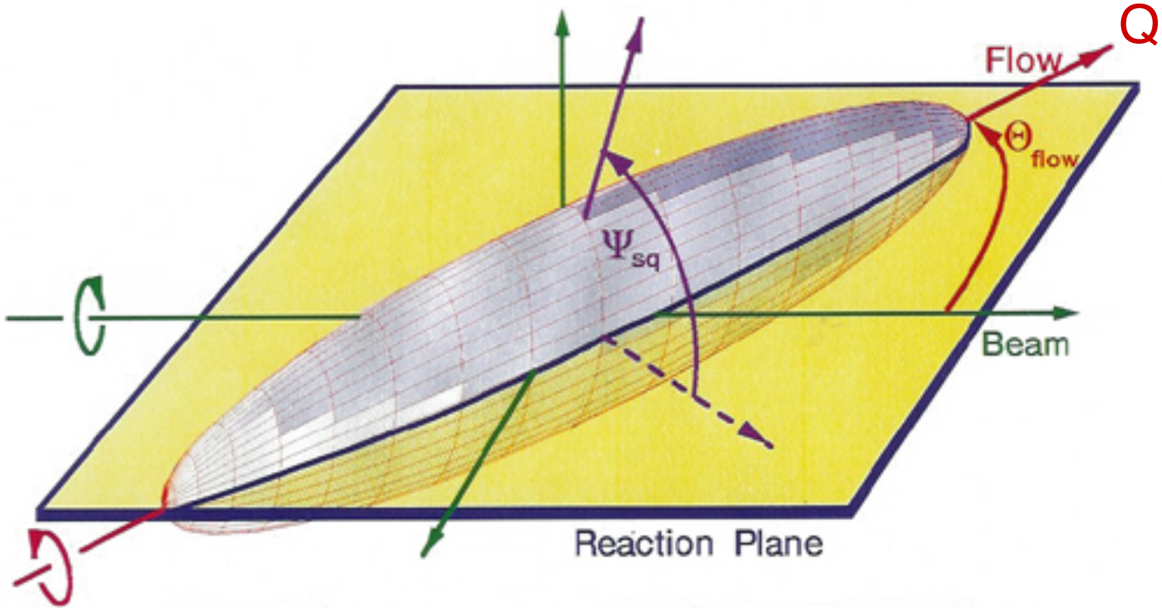
Filter theory to compare with data

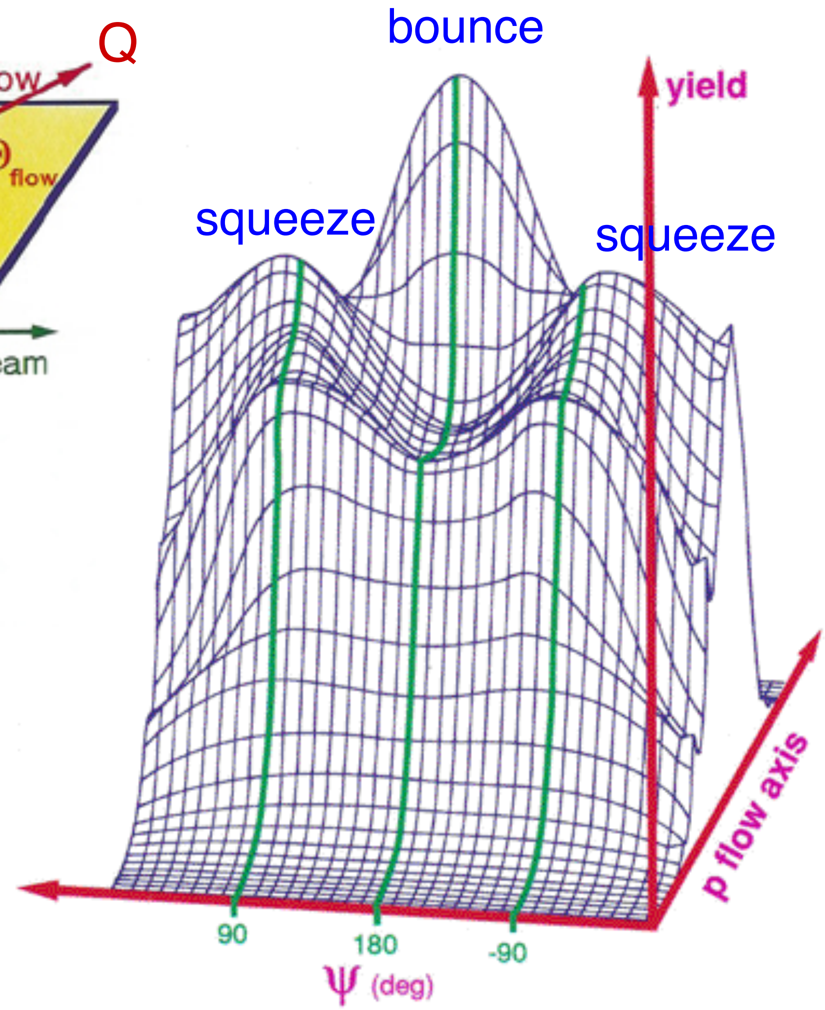


Directed Flow

Plastic Ball, K.G.R. Doss et al., PRL **57**, 302 (1986)

Best Ellipsoid





Squeeze-out

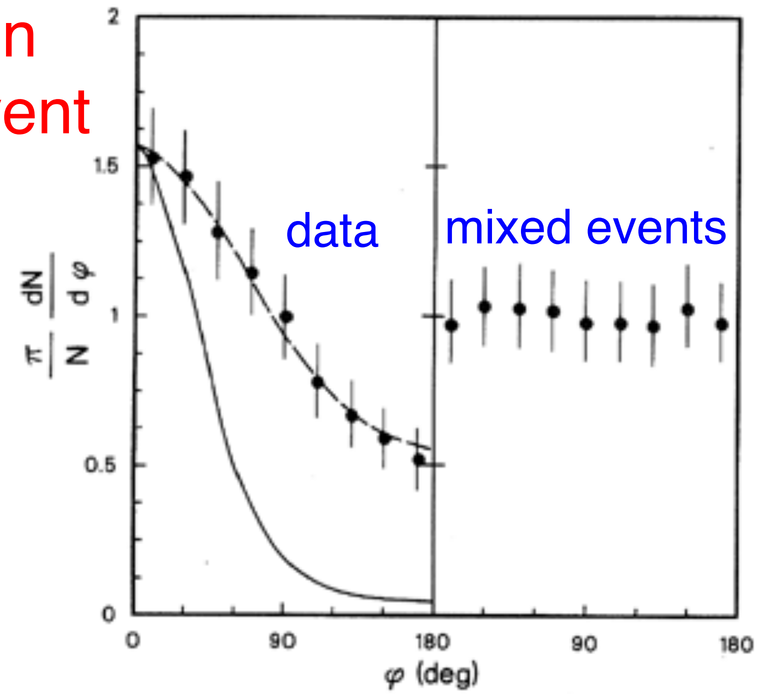
Negative elliptic flow

Plastic Ball, H.H. Gutbrod et al., PRC **42**, 640 (1991)
Diogene, M. Demoulins et al., Phys. Lett. **B241**, 476 (1990)
Plastic Ball, H.H. Gutbrod et al., Phys. Lett. **B216**, 267 (1989)

Analyze in the Transverse Plane

Transverse Momentum Analysis

correlation of sub-event planes



P. Danielewicz and G. Odyniec, Phys. Lett. **157B**, 146 (1985)

Fourier Harmonics

Fourier harmonics:

$$1 + 2v_1 \cos(\phi - \Psi_{RP}) + 2v_2 \cos[2(\phi - \Psi_{RP})] + \cdots$$

$$v_n = \langle \cos[n(\phi_i - \Psi_{RP})] \rangle$$

To remove acceptance correlations

Flatten event plane azimuthal distributions in lab

To measure event plane resolution

Correlate two independent sub-groups of particles

Event plane resolution correction made for each harmonic

Unfiltered theory can be compared to experiment!

Tremendous stimulus to theoreticians!

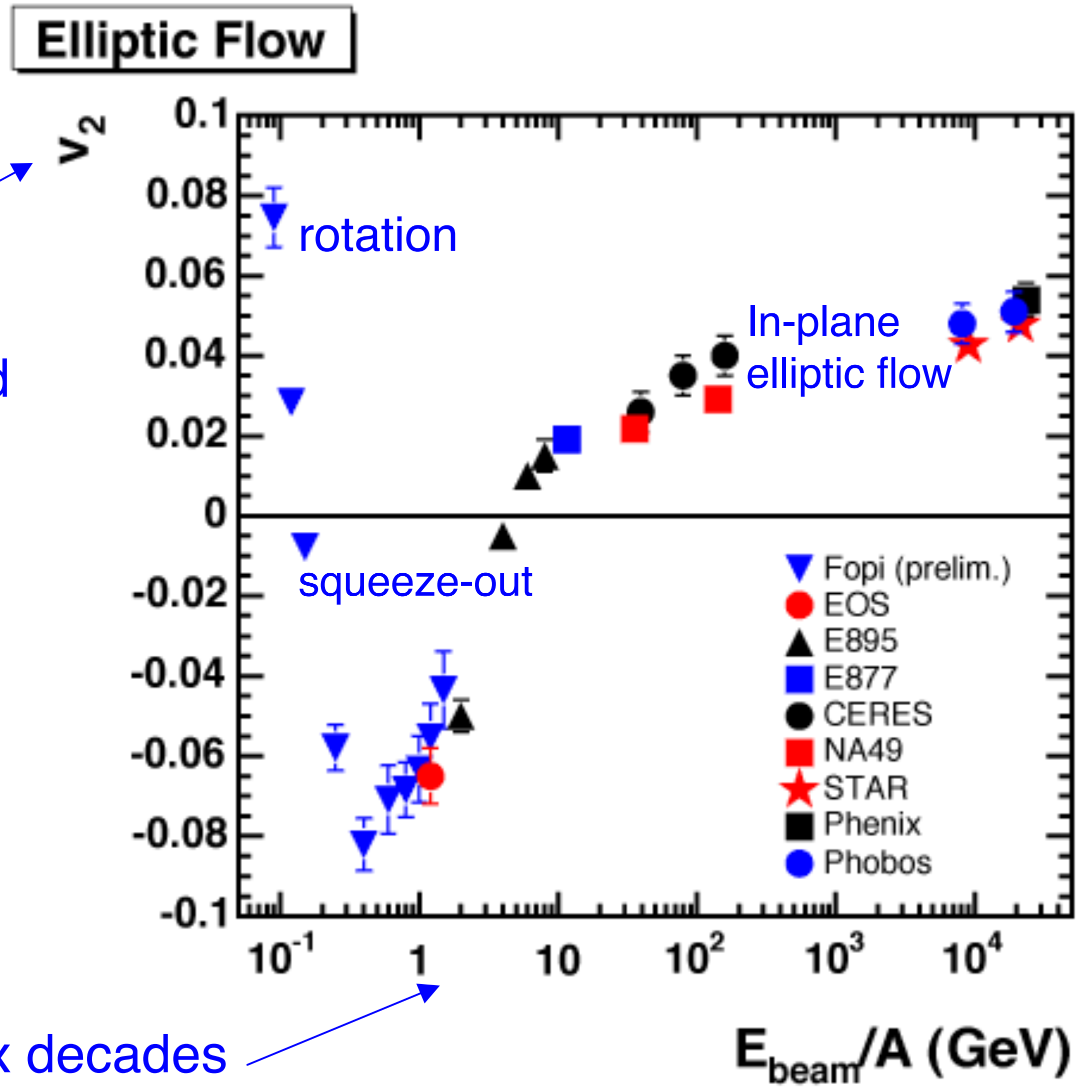
S. Voloshin and Y. Zhang, hep-ph/940782; Z. Phys. C **70**, 665 (1996)
See also, J.-Y. Ollitrault, arXiv nucl-ex/9711003 (1997)
and J.-Y. Ollitrault, Nucl. Phys. **A590**, 561c (1995)
A.M. Poskanzer and S.A. Voloshin, PRC **58**, 1671 (1998)

Elliptic Flow vs. Beam Energy

25% most central
mid-rapidity

all v_2

standard method



six decades

A. Wetzler (2005)

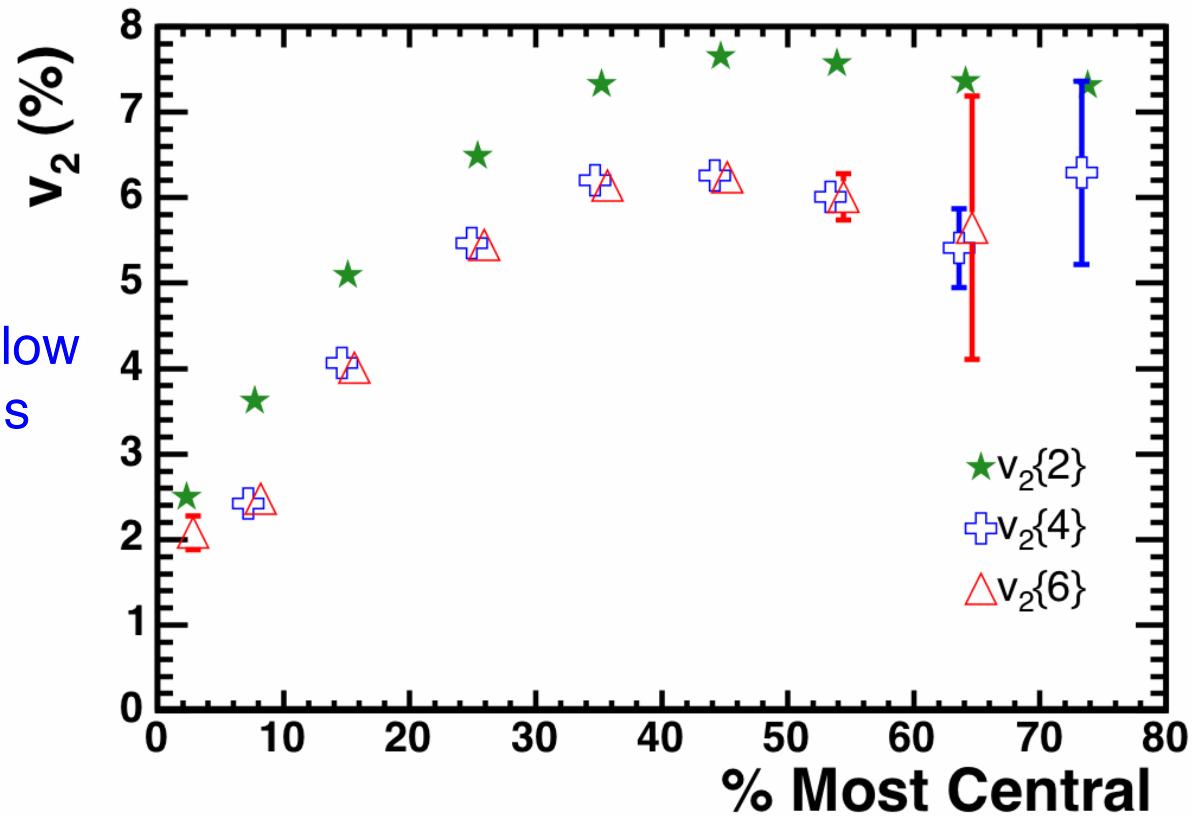
Generating Function Methods

$$\langle\langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle\rangle \equiv \langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle - 2\langle u_{n,1}u^*_{n,2}\rangle^2 = -v_n^4\{4\}$$

Minimize complex generating function to evaluate four-particle correlation

Cumulants
Lee-Yang Zeros

Multi-particle methods eliminate 2-particle non-flow and Gaussian fluctuations



STAR, J. Adams et al., PRC **72**, 014904 (2005)

Direct Four-Particle Correlation

$$\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle = \frac{(\sum e^{i2(\phi_i-\phi_j)})(\sum e^{i2(\phi_k-\phi_l)}) - degeneracies}{N(N-1)(N-2)(N-3)}$$

$$\begin{aligned} degeneracies = & \sum \left(e^{i2(\phi_i-\phi_j)}\right)^2 \\ + & \; 2\left(\sum e^{i2(\phi_i-\phi_j)} e^{i2(\phi_i-\phi_k)}\right) \\ + & \; 2\left(\sum e^{i2(\phi_i-\phi_j)} e^{i2(\phi_k-\phi_i)}\right) \end{aligned}$$

4-particle correlations without a generating function

Sergei Voloshin (2006)
Dhevan Gangadharan, thesis, UCLA (2010)

$v_2\{4\}$ Direct Cumulant

$$\left\langle\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = \left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle$$
$$- 2\left\langle e^{i2(\phi_1-\phi_3)}\right\rangle\left\langle e^{i2(\phi_2-\phi_4)}\right\rangle$$
$$- \left\langle e^{i2\phi_1}\right\rangle\left\langle e^{i2(\phi_2-\phi_3-\phi_4)}\right\rangle$$
$$- \left\langle e^{i2\phi_2}\right\rangle\left\langle e^{i2(\phi_1-\phi_3-\phi_4)}\right\rangle$$
$$- 2\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{i2(\phi_1+\phi_2-\phi_4)}\right\rangle$$
$$- \left\langle e^{i2(\phi_1+\phi_2)}\right\rangle\left\langle e^{-i2(\phi_3+\phi_4)}\right\rangle$$
$$+ 4\left\langle e^{i2\phi_1}\right\rangle\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{i2(\phi_2-\phi_4)}\right\rangle$$
$$+ 4\left\langle e^{i2\phi_2}\right\rangle\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{i2(\phi_1-\phi_4)}\right\rangle$$
$$+ 2\left\langle e^{i2\phi_1}\right\rangle\left\langle e^{i2\phi_2}\right\rangle\left\langle e^{-i2(\phi_3+\phi_4)}\right\rangle$$
$$+ 2\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{-i2\phi_4}\right\rangle\left\langle e^{i2(\phi_1+\phi_2)}\right\rangle$$
$$- 6\left\langle e^{i2\phi_1}\right\rangle\left\langle e^{i2\phi_2}\right\rangle\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{-i2\phi_4}\right\rangle$$

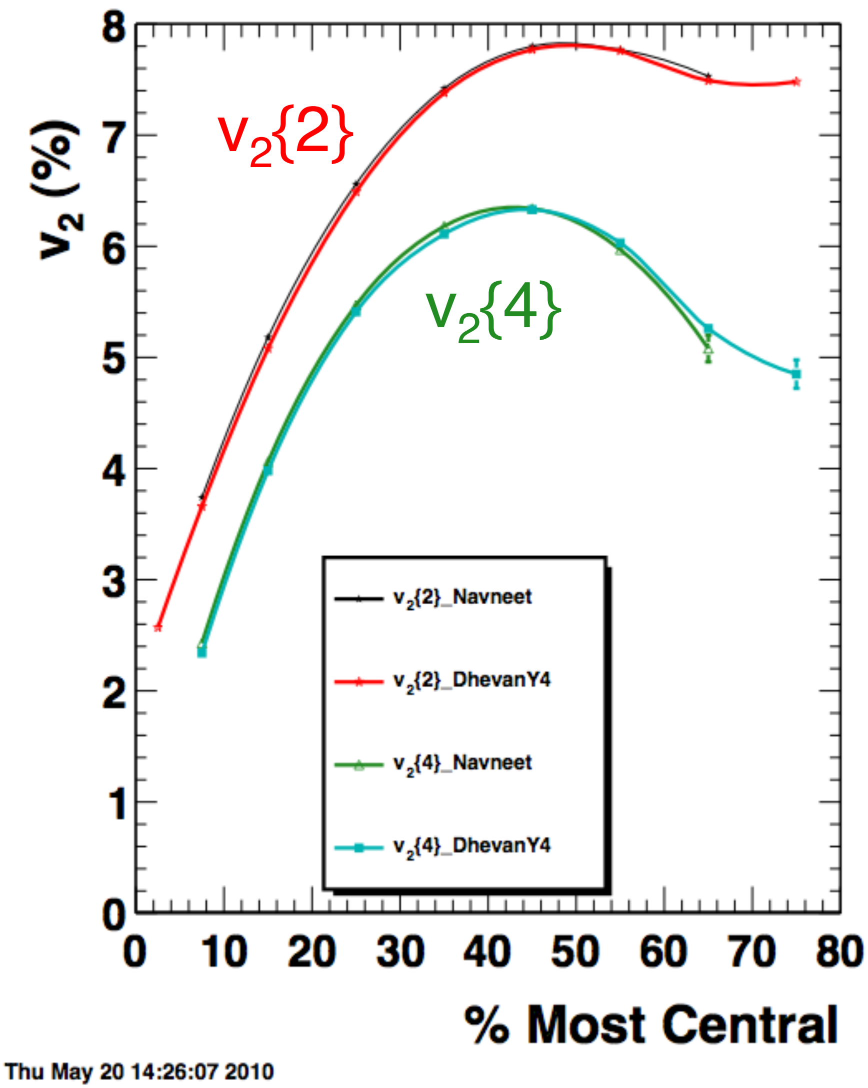
acceptance corrections

$$\left\langle\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = -v_2^4\{4\}$$

Dhevan Gangadharan, thesis, UCLA (2010)

Direct $v_2\{4\}$

Two independent implementations agree



STAR preliminary

Sergei Voloshin, STAR
Dhevan Gangadharan, STAR
Navnett Pruthi, STAR
Ante Bilandzic, ALICE

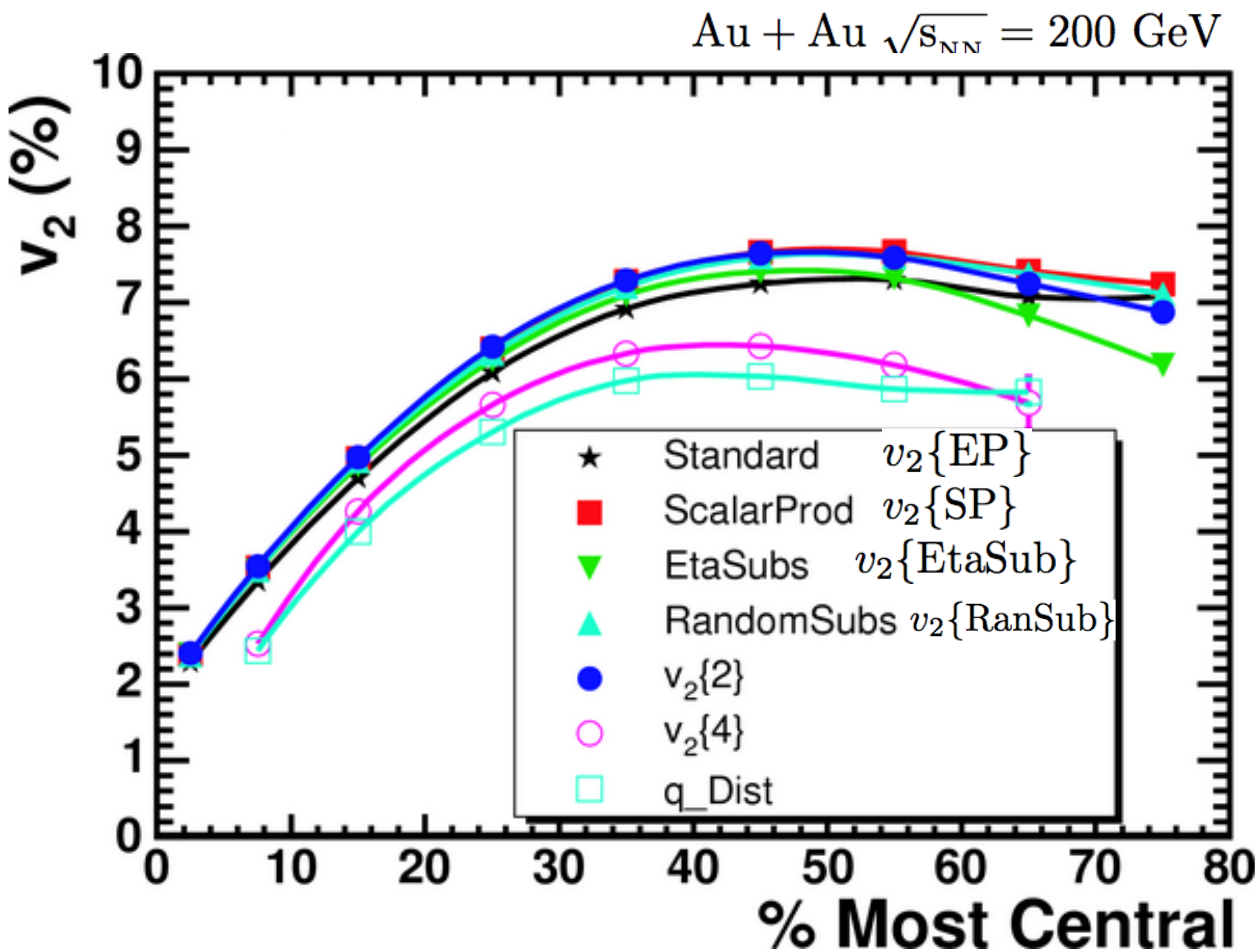
Methods

- “Two-particle”:
  - $v_2\{2\}$: each particle with every other particle
  - $v_2\{subEP\}$: each particle with the EP of the other subevent
  - $v_2\{EP\}$ “standard”: each particle with the EP of all the others
  - $v_2\{SP\}$: same, weighted with the length of the Q vector

- Many-particle:
  - $v_2\{4\}$: 4-particle - 2 * (2-particle)^2
    - Generating function or Direct Cumulant
  - $v_2\{q\}$: distribution of the length of the Q vector
  - $v_2\{LYZ\}$: Lee-Yang Zeros multi-particle correlation

review of azimuthal anisotropy:
arXiv: 0809.2949

Integrated v_2



STAR, J. Adams et al., PRC **72**, 014904 (2005)

Measurements

- **Two-particle methods**
  - **contain nonflow** $\langle \cos\phi_1 - \cos\phi_2 \rangle = \langle v^2 \rangle + \delta$ ← nonflow
  - **mean of some power of the distribution in the Participant Plane** $v_2\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$

- **Multi-particle methods**
  - **suppress nonflow**
  - **mean in the Reaction Plane in Gaussian approx.**

Bhalerao and Ollitrault, Phys Lett. **B 641**, 260 (2006)
Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

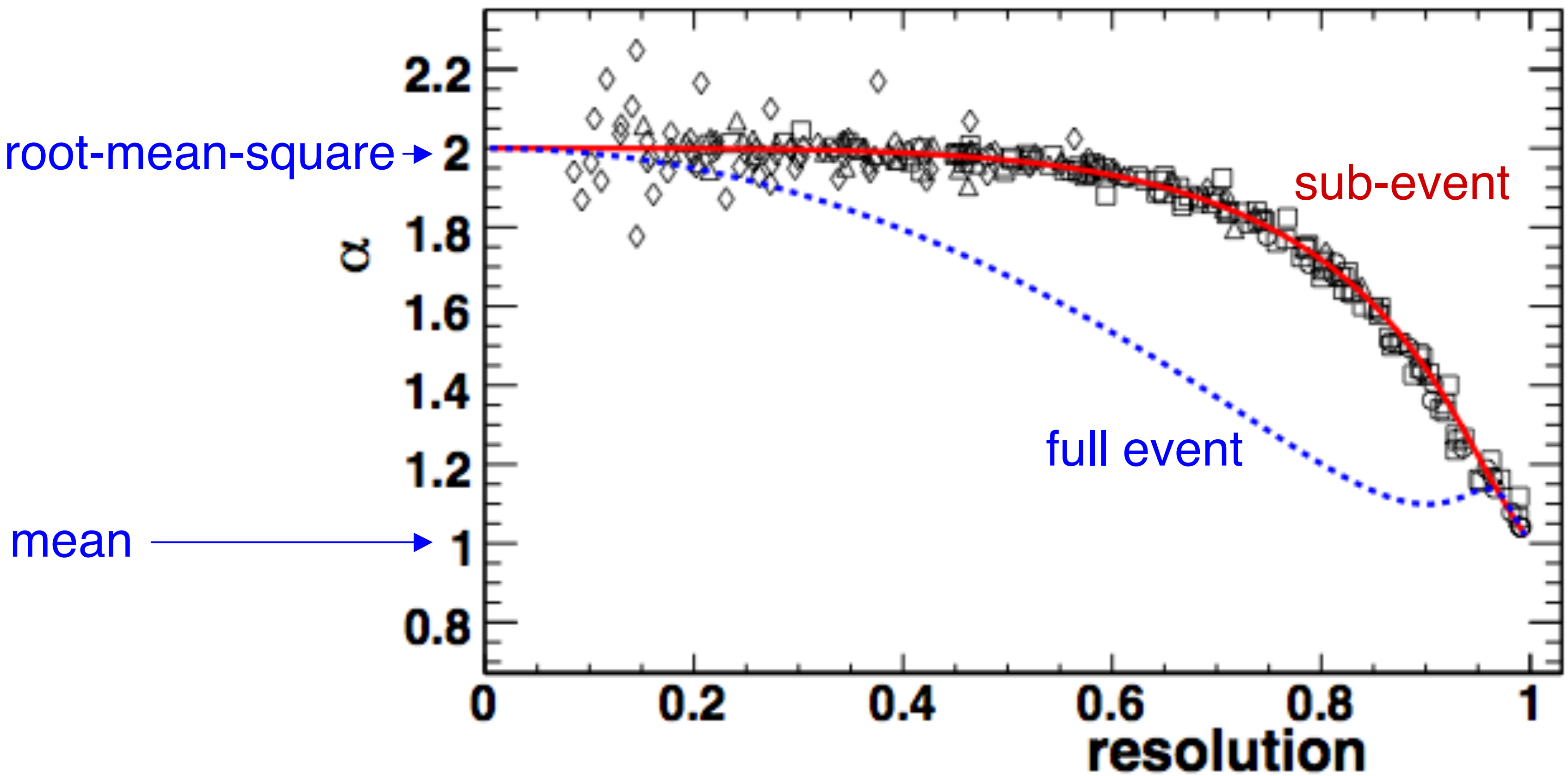
Effect of Eccentricity Fluctuations on Elliptic Flow

- Mean of some power of the distribution

- Participant plane fluctuations

Effect of Fluctuations on the Mean

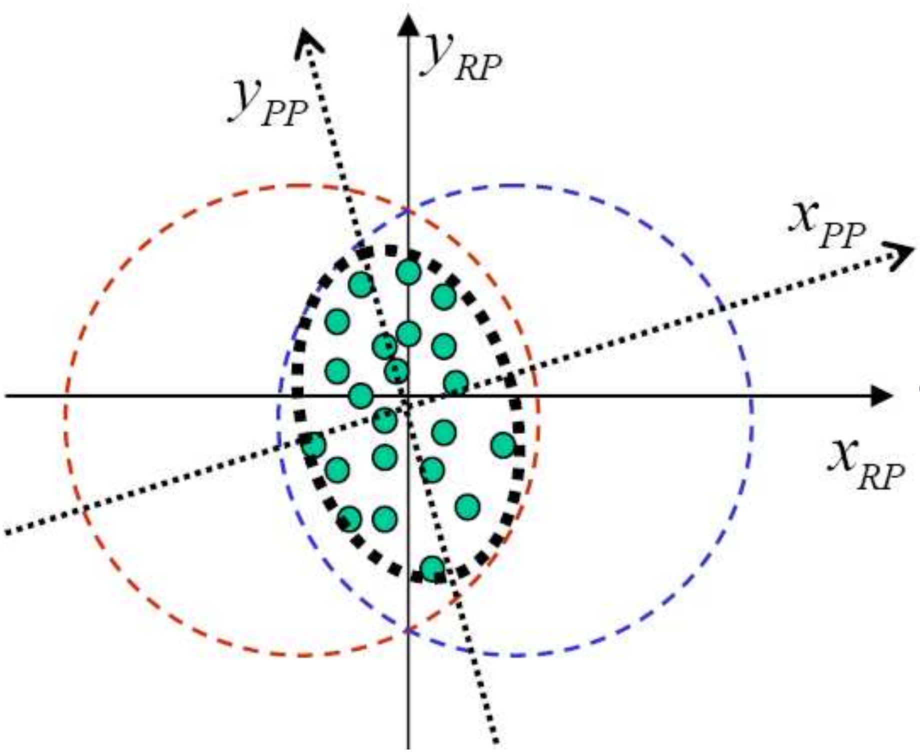
$$v_2\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$$



$$v_2\{\ \}^2 = \langle v \rangle^2 + (\alpha - 1)\sigma_v^2$$

Points: simulations by PHOBOS+
Ollitrault, Poskanzer, and Voloshin, PRC **80**, 014904 (2009)

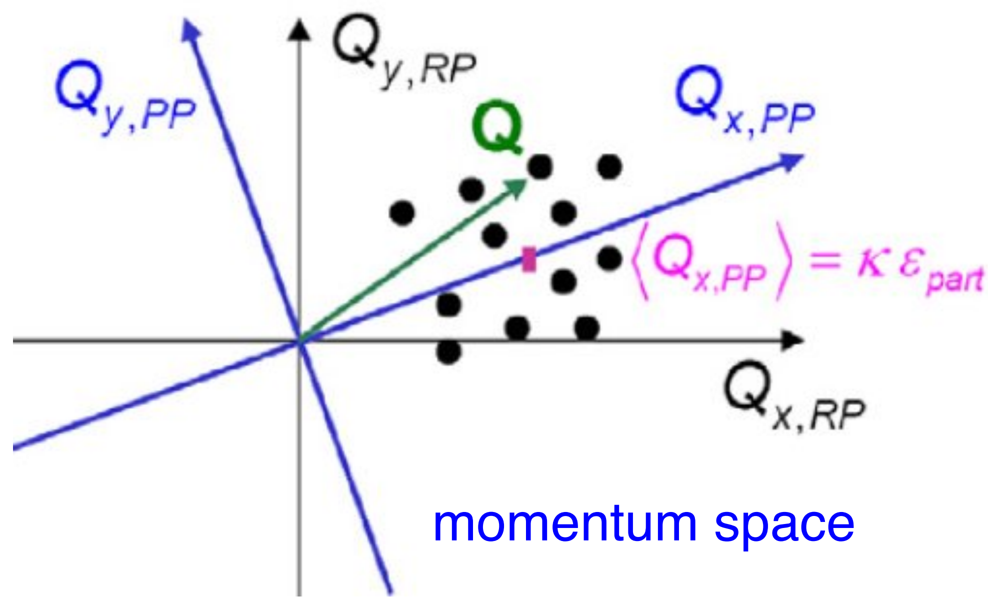
Reaction, Participant, and Event Planes



determined by particles struck

determined by impact parameter

coordinate space



participant plane

momentum space

Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

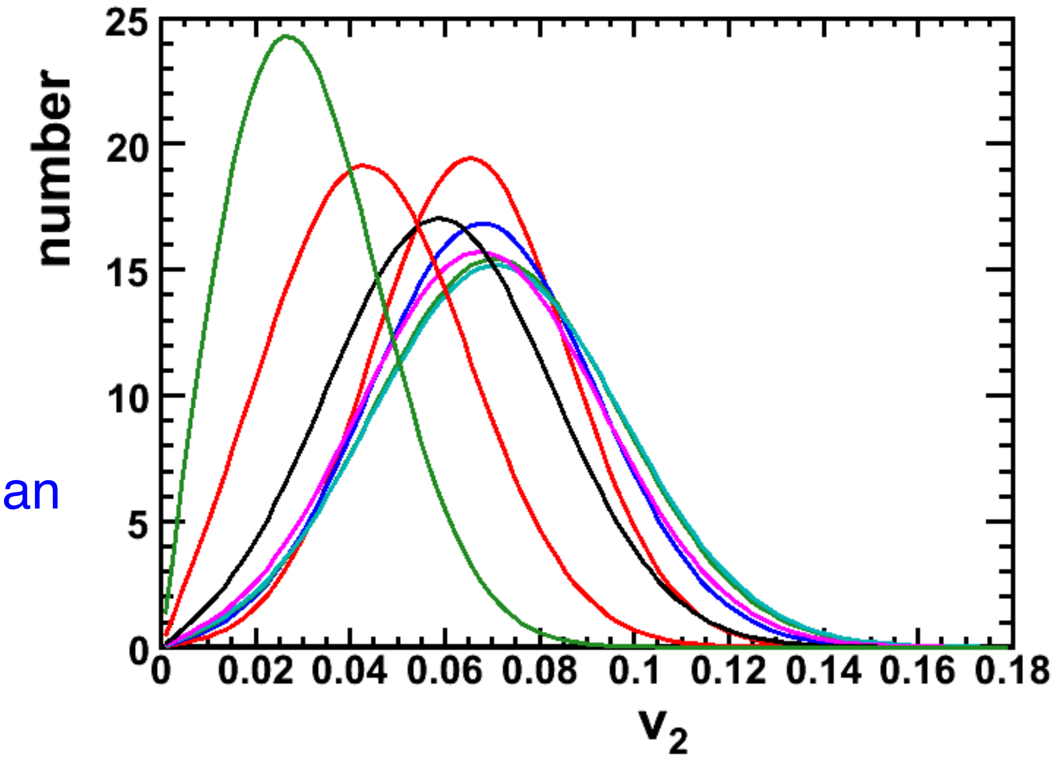
v_2 Fluctuations from ε_{part} Fluctuations

Assume width with same percent width as ε_{part}: $\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$

σ_ε is from standard deviation of nucleon MC Glauber of ε_{part}

Bessel-Gaussian: $$\frac{dn}{dv} = \frac{v}{\sigma_0^2} I_0\left(\frac{v\ v_0}{\sigma_0^2}\right) \exp\left(-\frac{v^2 + v_0^2}{2\sigma_0^2}\right)$$

2D Gaussian fluctuations in ε_x and ε_y in the reaction plane lead to Bessel-Gaussian fluctuations along the participant plane axis



Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

Theory is in RP, except...

- Event-by-Event without impact parameter

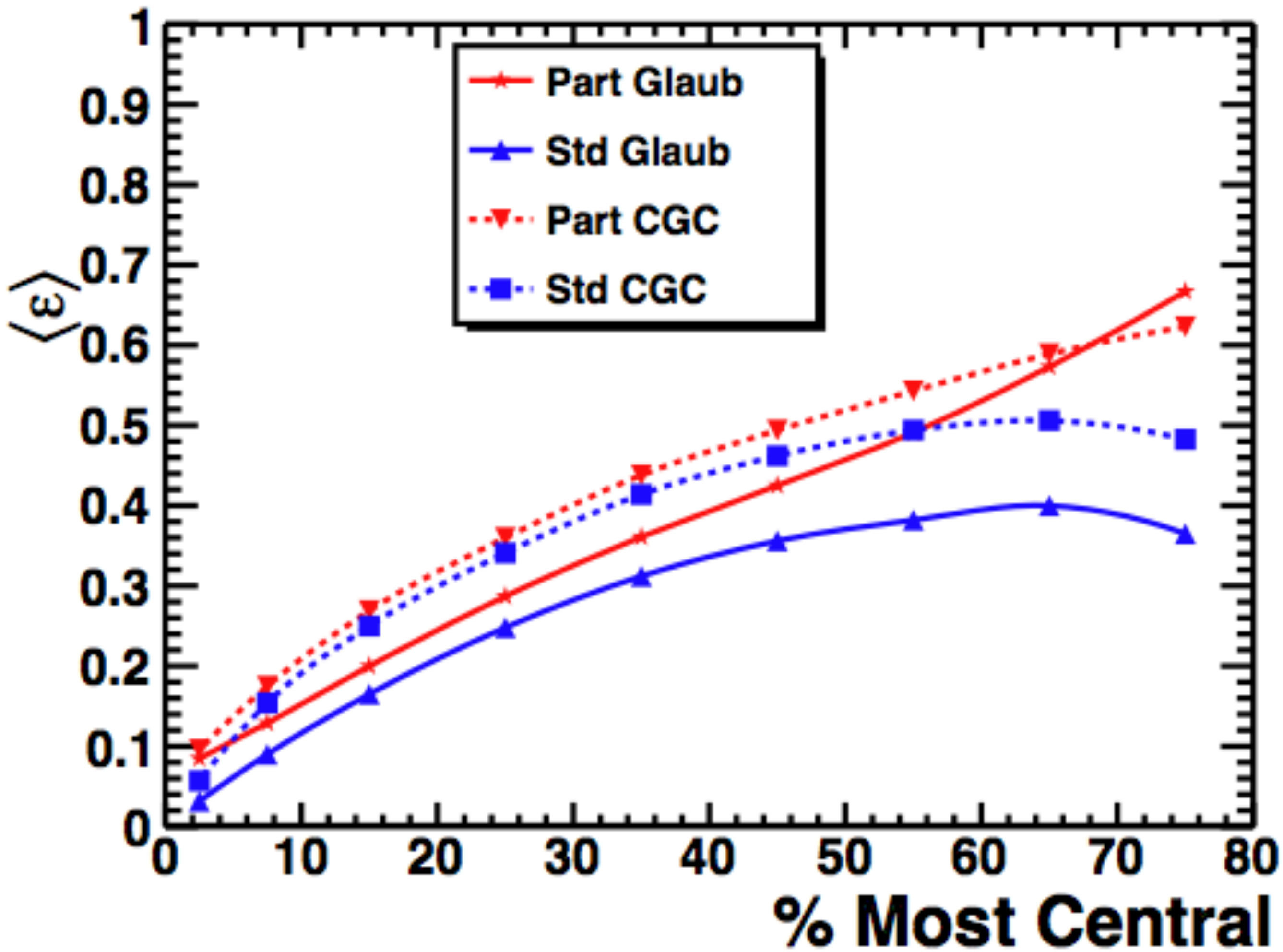
- Kodama
  - NeXSPheRIO
  - Hydro for event-by-event participants

- Hirano
  - Determine PP for each event
  - Rotate event to RP
  - Thus include PP fluctuations in initial conditions

R. Andrade et al., Phys. Rev. Lett. **97**, 202302 (2006)

T. Hirano and Y. Nara, PRC **79**, 064904 (2009)

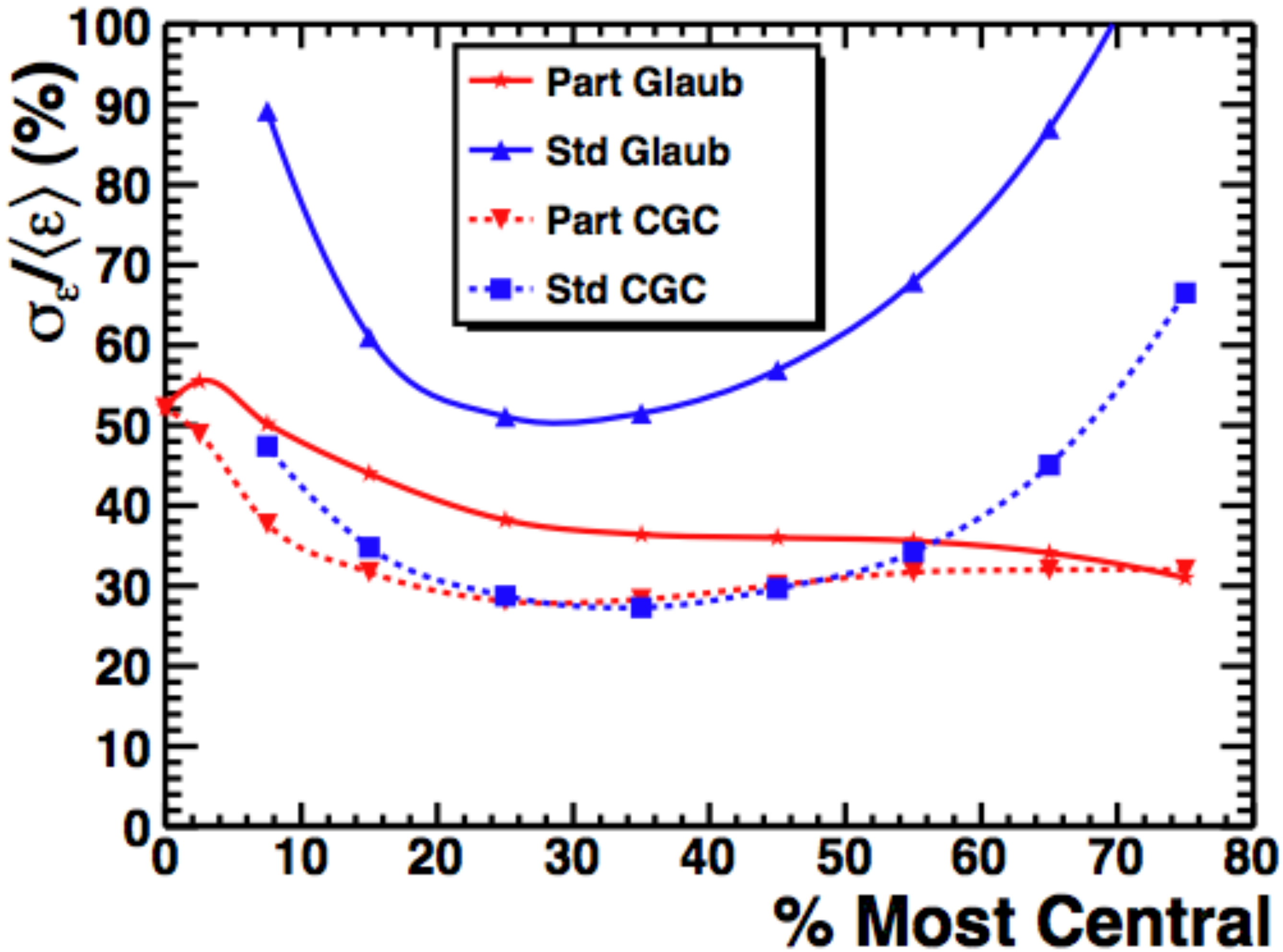
Eccentricities



Hiroshi Masui

Drescher and Nara, PRC **76**, 041903(R) (2007)

Eccentricity Fluctuations



Hiroshi Masui
Drescher and Nara, PRC **76**, 041903(R) (2007)

An Application to Data

- **Correct for nonflow**

$$\langle \cos\phi_1 - \cos\phi_2 \rangle = \langle v^2 \rangle + \delta$$ ← nonflow

- **Correct to mean v_2 in PP**
- **Correct to RP**
- **Assumptions**

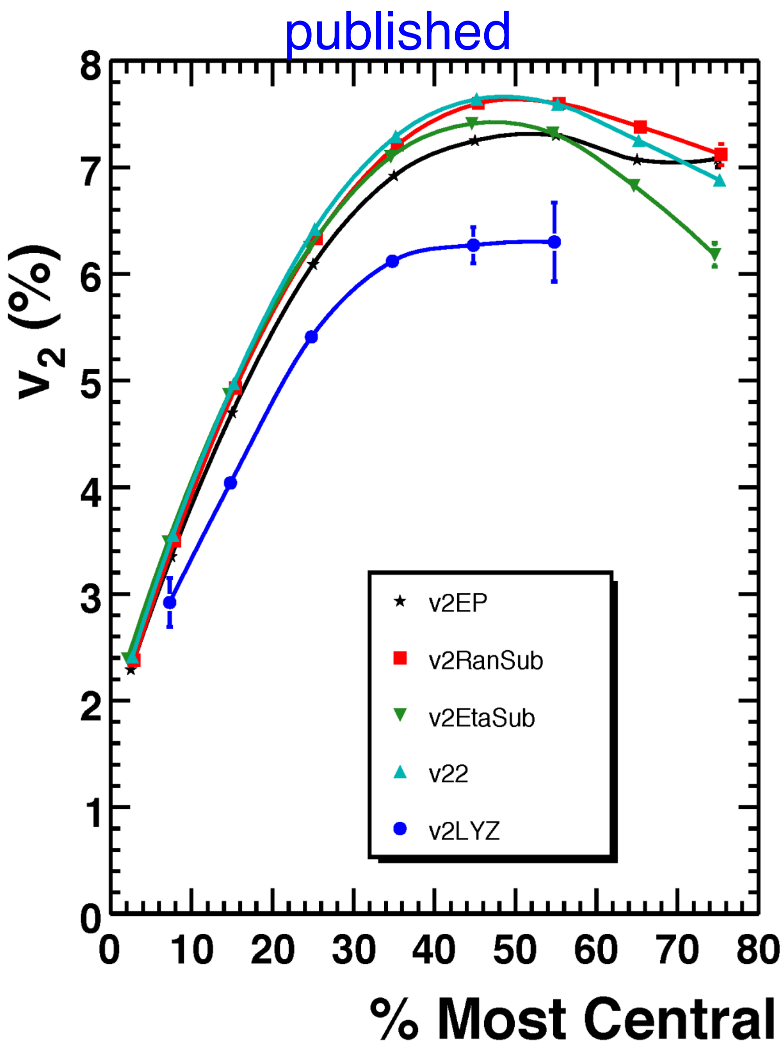
$$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$$ MC ε participant

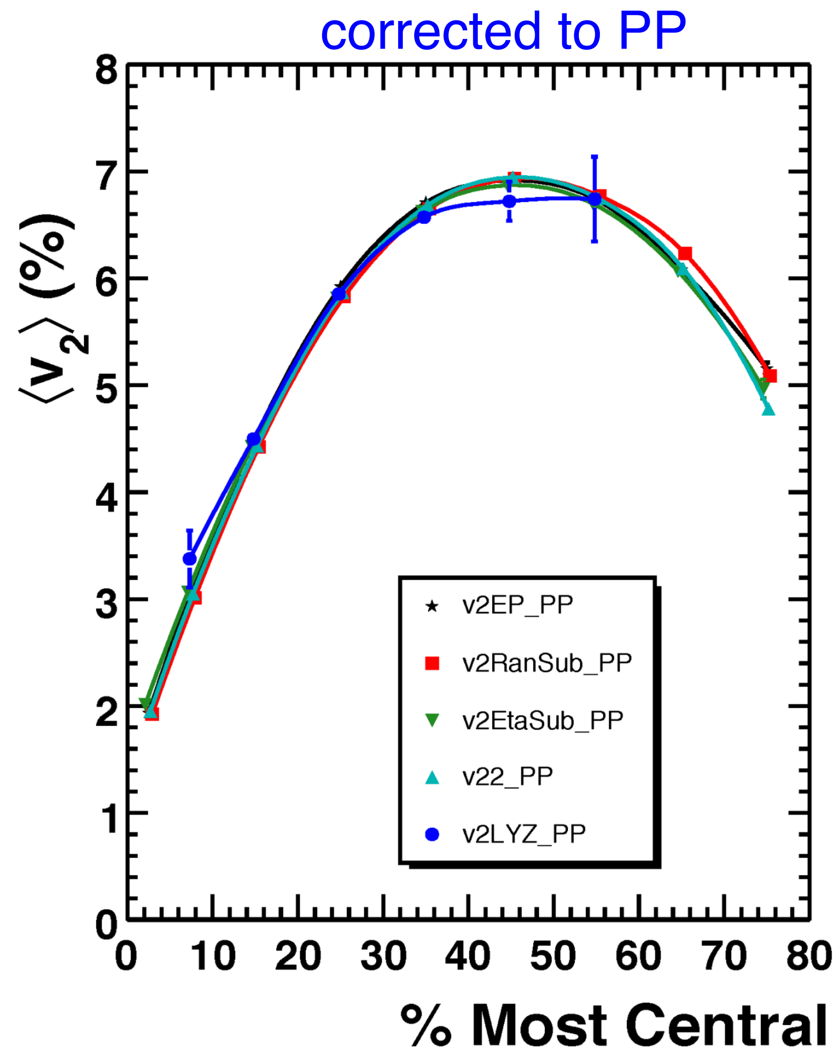
$$\delta_2 = 2\ \delta_{pp}/N_{\rm part} \qquad \delta_{pp} = 0.0145$$

$$\delta_{\rm etaSub} = 0.5\ \delta_2$$ less nonflow

Ollitrault, Poskanzer, and Voloshin, PRC **80**, 014904 (2009)

Data Corrected to <v_2>





agreement for mean v_2
in participant plane

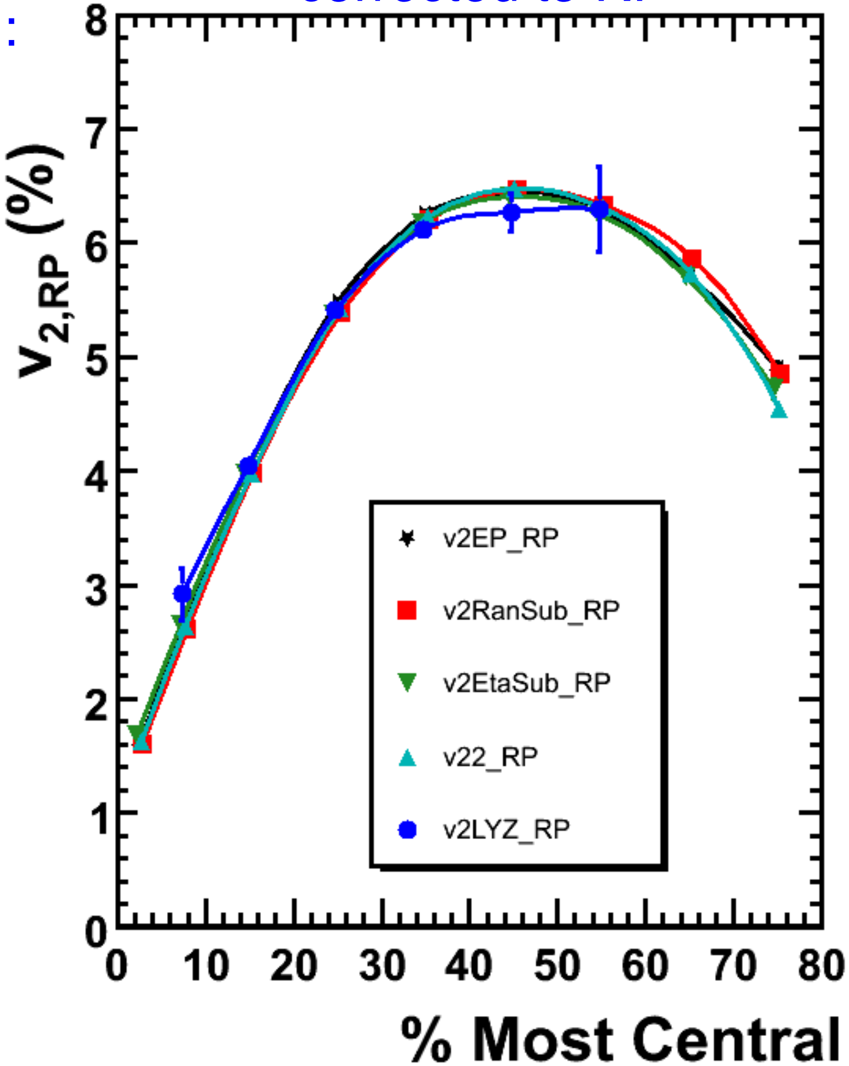
Ollitrault, Poskanzer, and Voloshin, PRC **80**, 014904 (2009)

v_2 in the Reaction Plane

in Gaussian fluctuation approximation:

$$\langle v_{2,\mathrm{PP}} \rangle^2 \simeq \langle v_{2,\mathrm{RP}} \rangle^2 + \sigma^2_{v2,\mathrm{part}}$$

corrected to RP



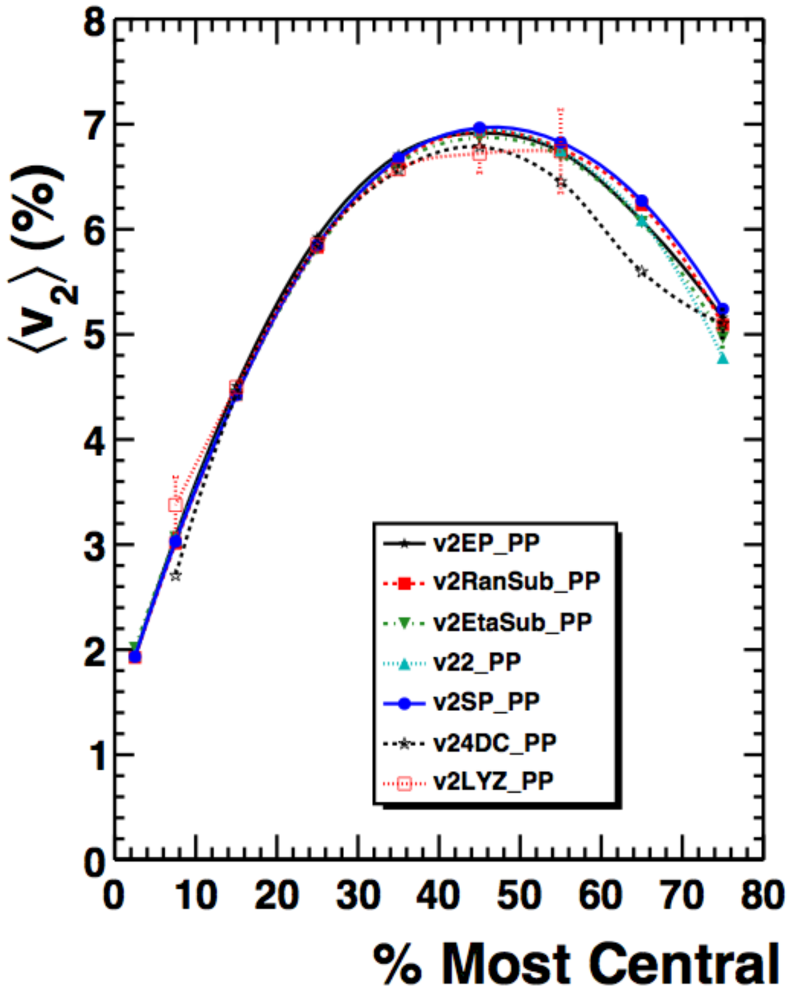
Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

New Results

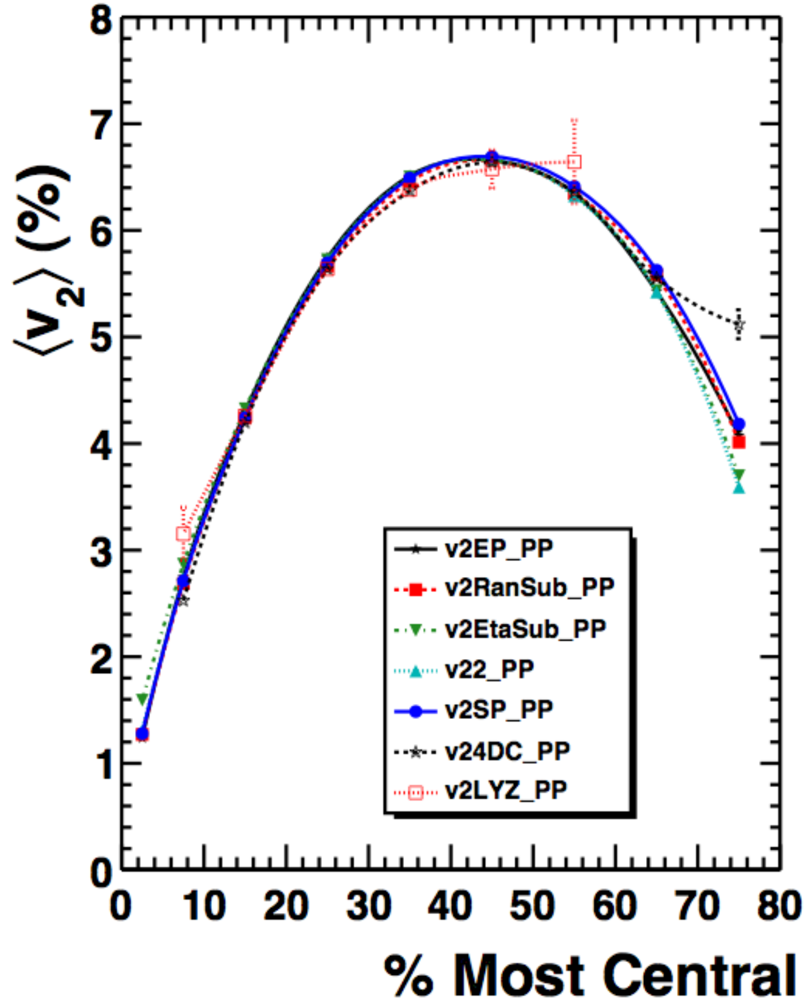
- Direct Cumulants
- Non-Gaussian behavior

Participant Plane

Glauber



CGC



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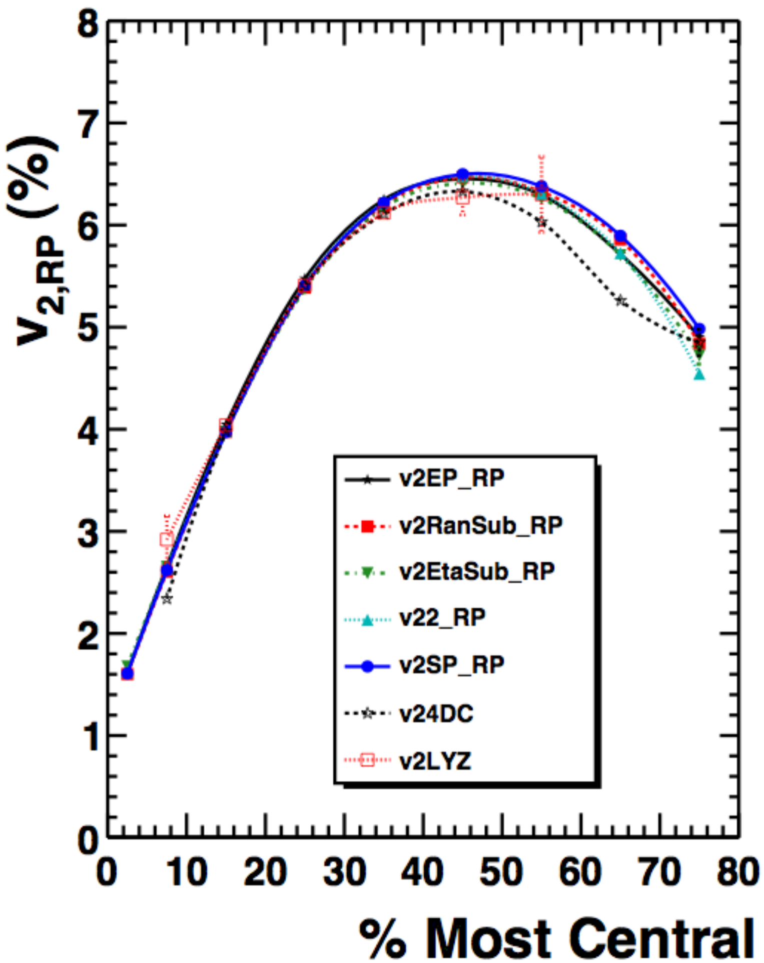
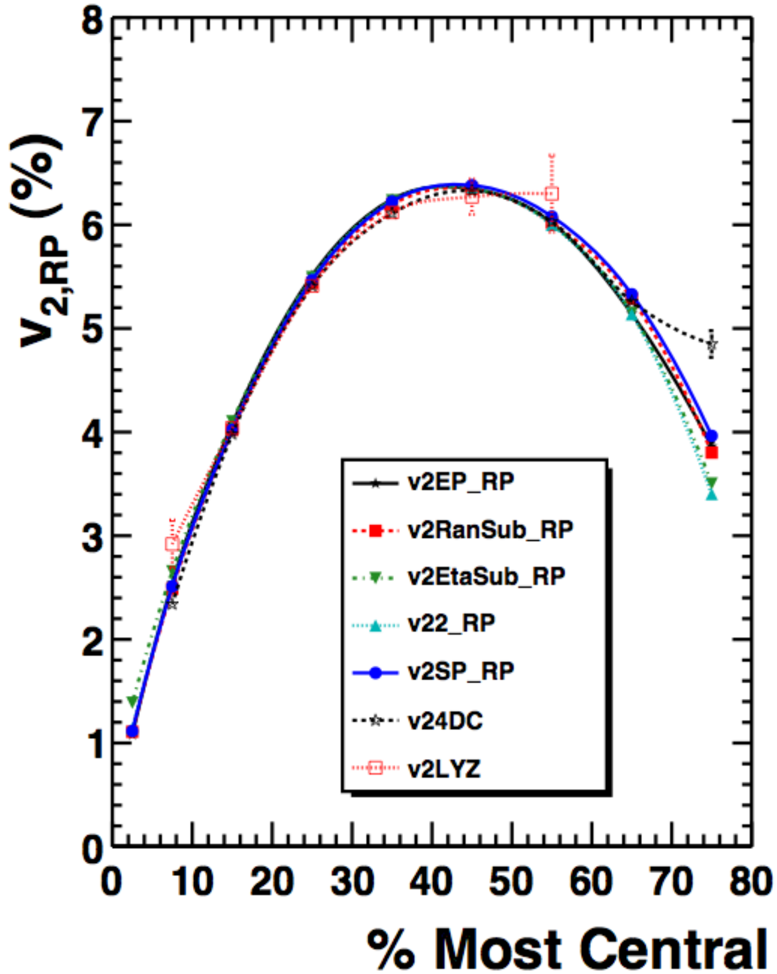
Reaction Plane

$$v_{RP} = v_{PP}\sqrt{1-(\sigma_\varepsilon/\varepsilon)^2}$$

Glauber



CGC



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Can Compare to Theory

Because we now:

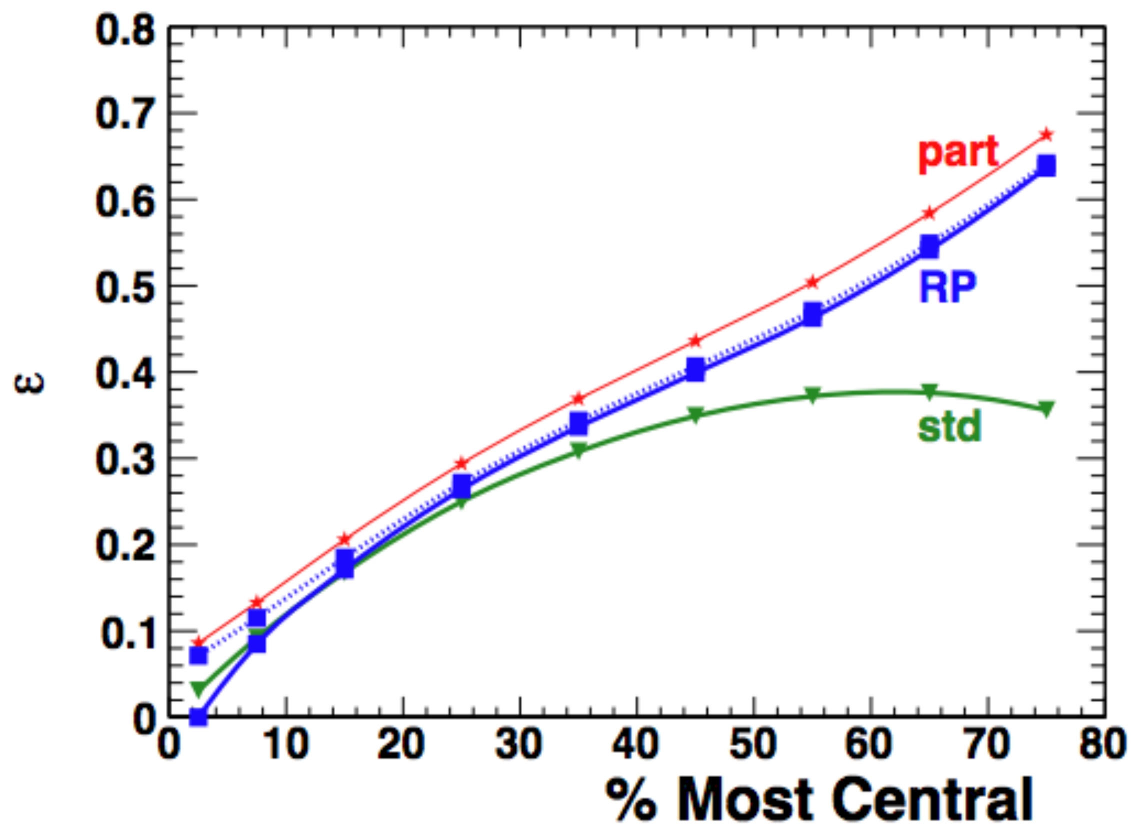
- **Remove acceptance correlations**
- **Correct for Event Plane resolution**
- **Correct for mean of a power of the distribution**
- **Correct for fluctuations of the PP**

Test Method Using ε_{part} to ε_{std}

dashed blue uses $\varepsilon_{RP} = \varepsilon_{PP}\sqrt{1 - (\sigma_{\varepsilon,PP}/\varepsilon_{PP})^2}$

solid blue uses exact equations in Gaussian Model paper

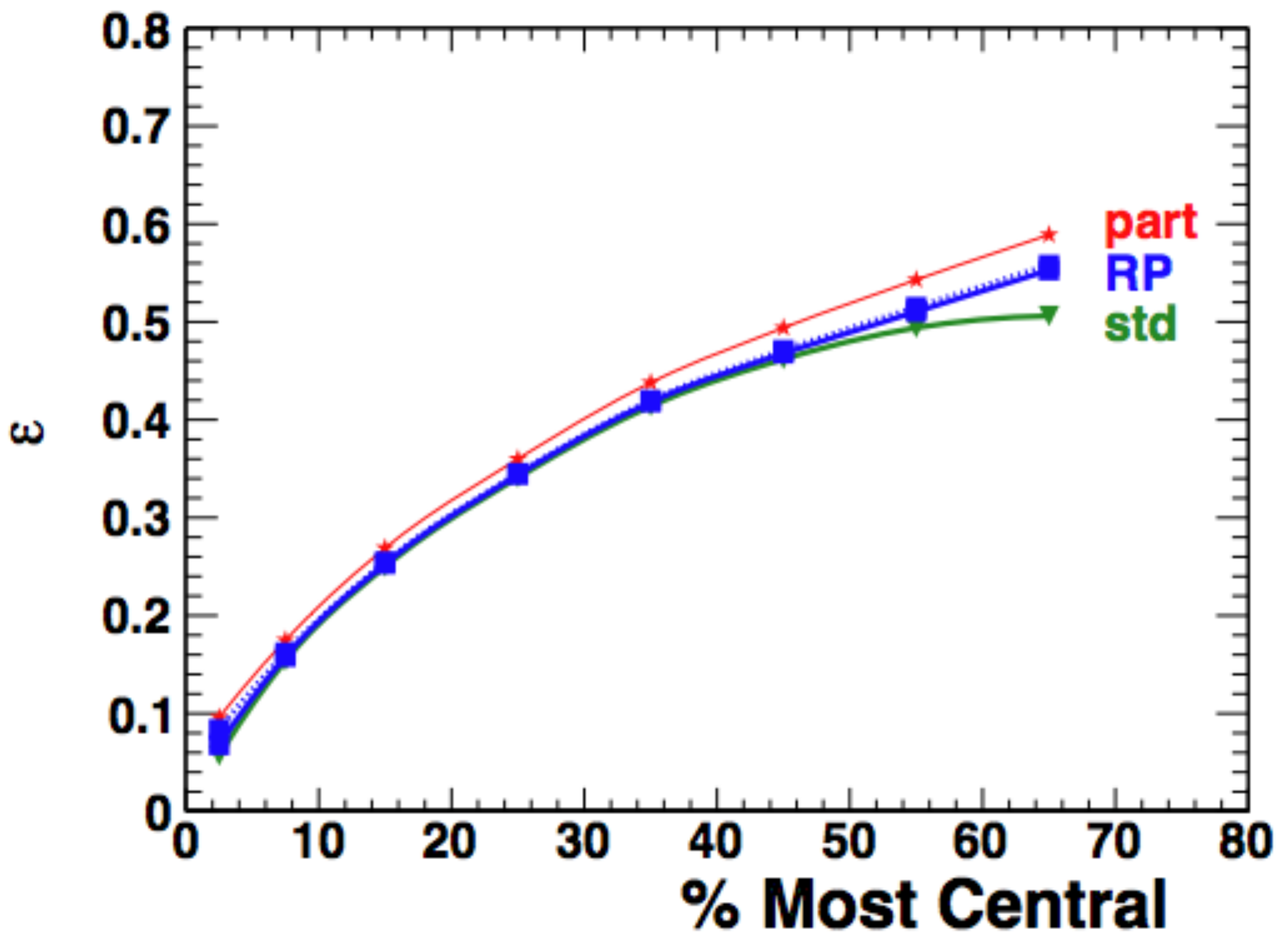
Glauber



Why does blue not go down to green?

Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

CGC

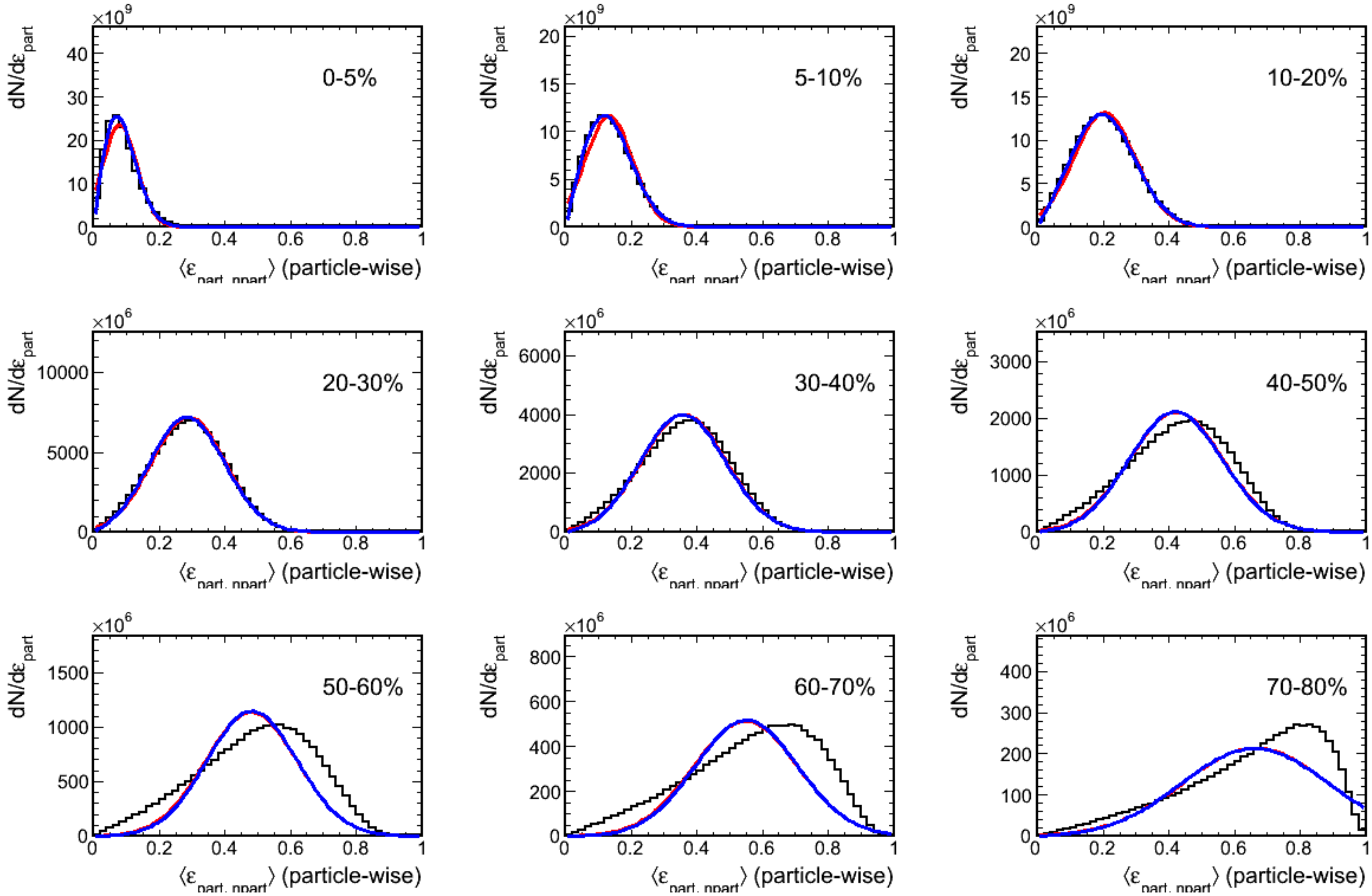


CGC participant fluctuations less
part and std much closer together
so can say RP = std to more peripheral collisions

Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

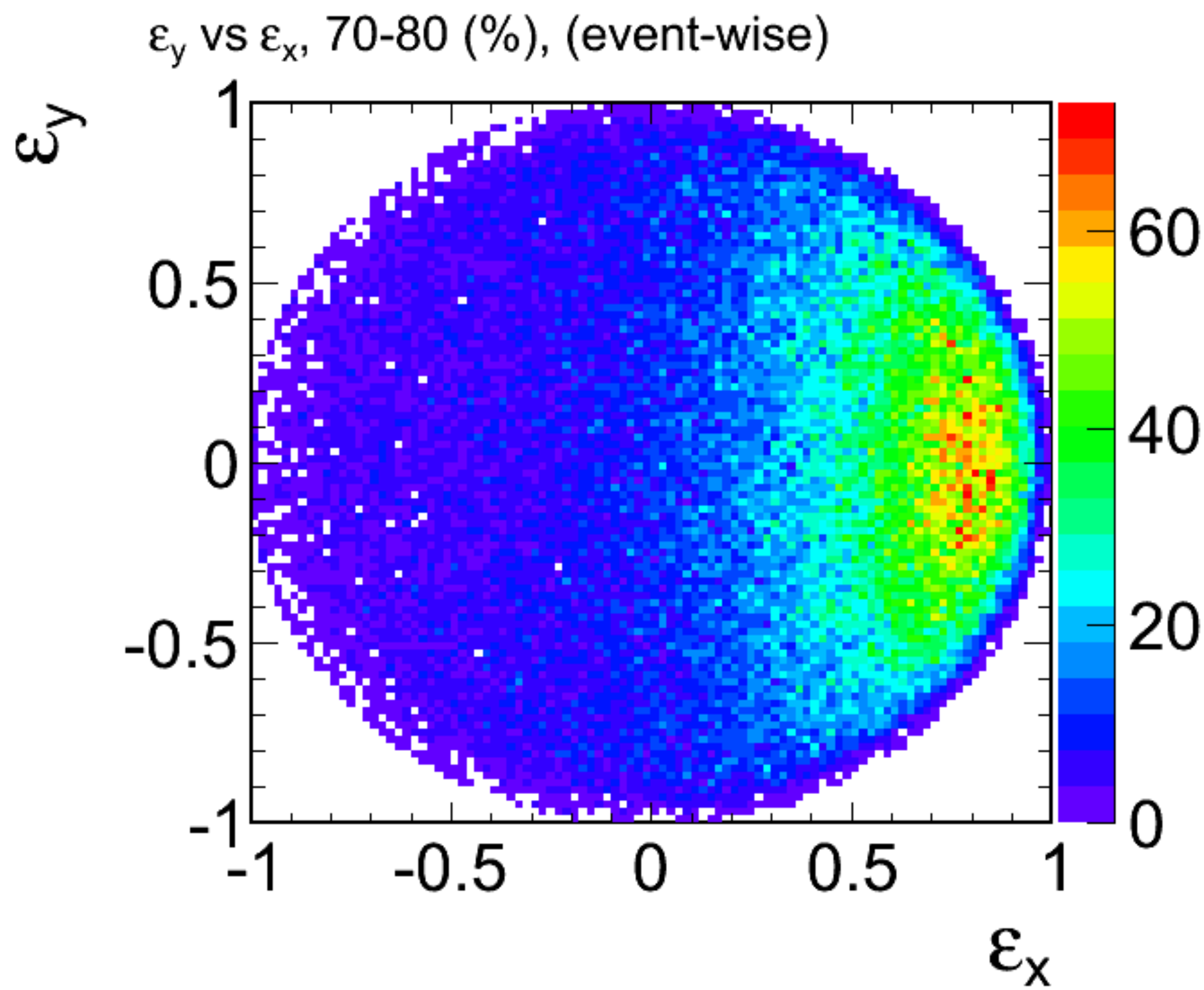
Glauber ε_{part} Distributions

Gaussian and Bessel-Gaussian fits to the black calculations



Non-Gaussian for peripheral collisions

$\varepsilon_{part} < 1$



ε can not be greater than 1

Status

- Even though the eccentricity distribution is not Gaussian, still could be:

$$v_2 \propto \varepsilon_2 \qquad \sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$$

- However,

$$v_2\{4\} \neq v_{2,\mathrm{RP}}$$

- for peripheral collisions
  - as estimated from Glauber calculations

Emphasize Direct $v_2\{4\}$

- No Event Plane
- Corrects for acceptance correlations
- One pass through the data
- Eliminates 2-particle nonflow correlations
- Gives mean of the distribution
- Gives v_2 in the RP
  - except for peripheral collisions