Azimuthal Anisotropy Distributions

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Publications

- **Elliptic Power Distribution (Simulations)**
  - **PRC 90, 024903 (2014)**
- **QM2014**
  - **arXiv:1408.0709**
- **ATLAS and CMS Data**
  - **arXiv:1408.0921 (submitted to PRL)**

Main Point

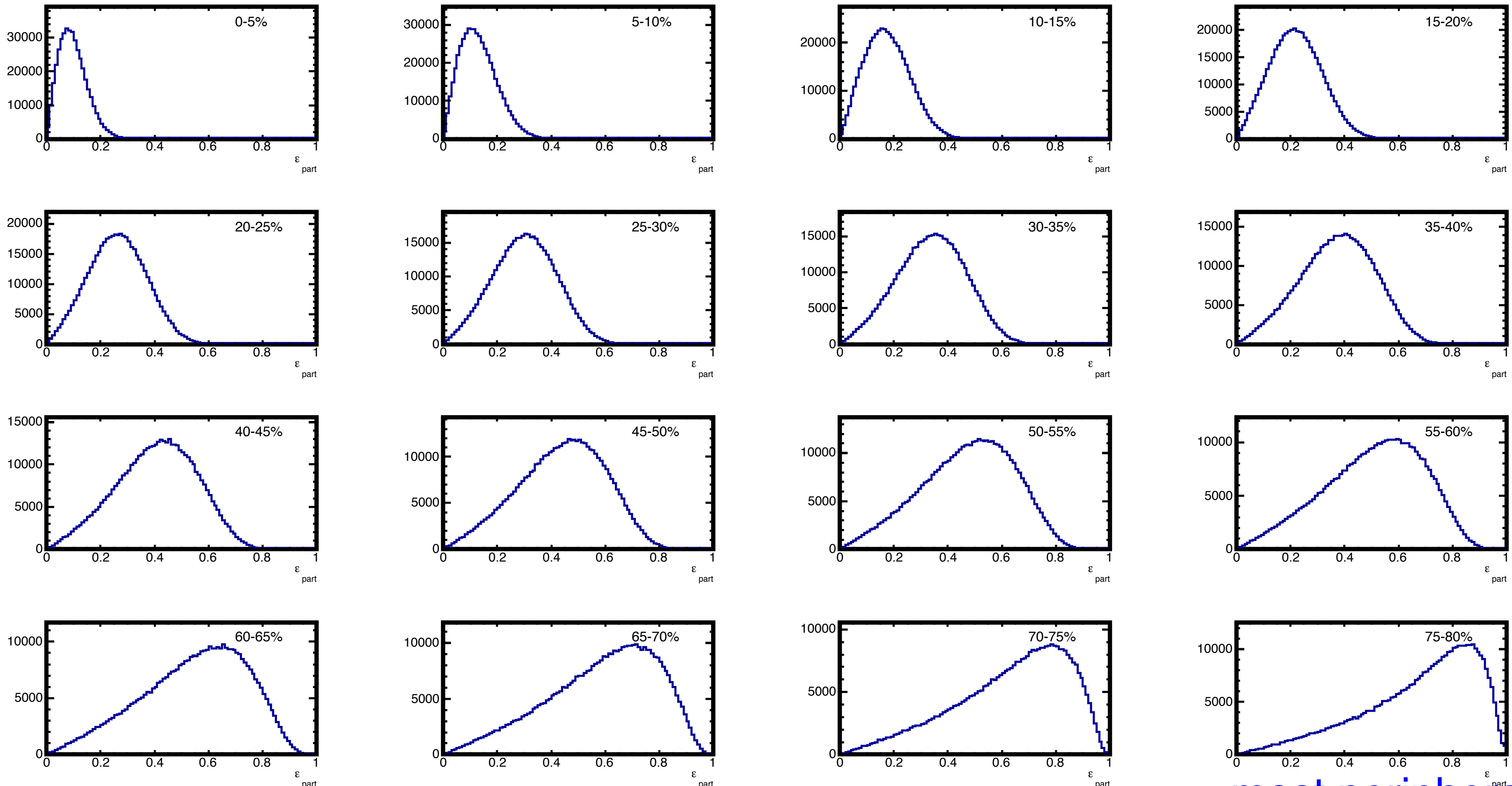
- Initial eccentricity is driving force for flow
- Usual viscous hydro output depends on assumed initial conditions
- We separate hydro response from initial anisotropy based on its non-Gaussian shape
- We obtain hydro response without assuming any particular model of the initial state
- The hydro response can give η/s without any assumptions about the initial state

Participant Eccentricity Distributions

Monte Carlo Glauber for Au + Au $\sqrt{s_{NN}}$ = 200 GeV

Hiroshi Masui

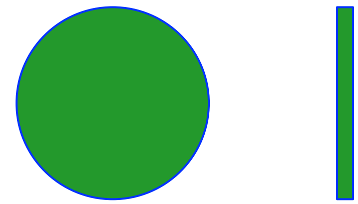
most central



most peripheral

Eccentricity

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



ε=0 ε=1

- **Must be between 0 and 1**
  - **Positive because it is the length of a vector**
  - **Going from Gaussian to Bessel-Gaussian eliminated the negative values**
  - **Going from Bessel-Gaussian to the Elliptic Power distribution eliminates values greater than 1**
- **Participant Eccentricity Ellipse is rotated:**

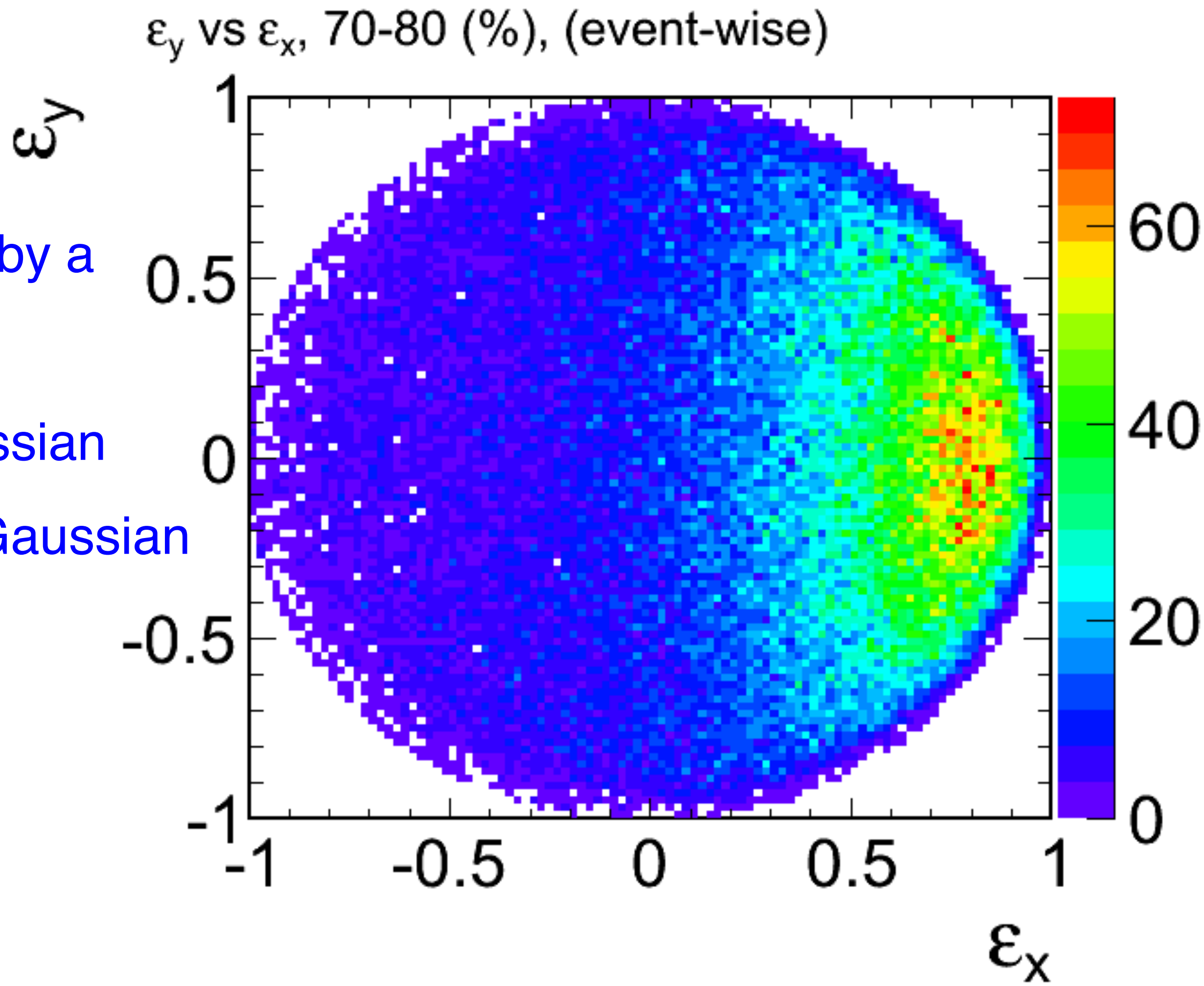
$$(\varepsilon_x, \varepsilon_y) = \left(\frac{\langle \sigma_y^2 - \sigma_x^2 \rangle}{\langle \sigma_y^2 + \sigma_x^2 \rangle}, \frac{\langle 2\sigma_{xy} \rangle}{\langle \sigma_y^2 + \sigma_x^2 \rangle} \right)$$

Monte-Carlo Glauber in x,y space

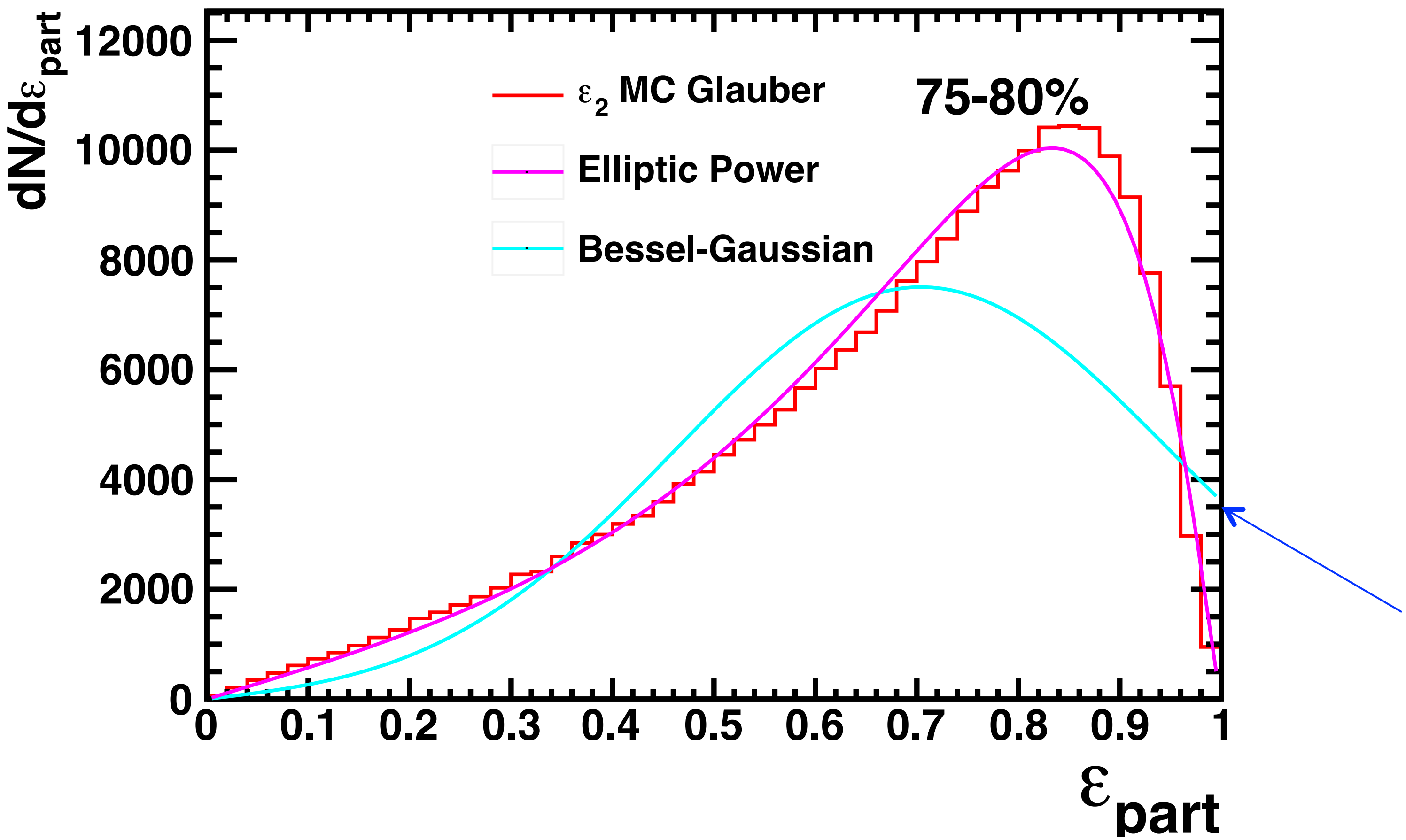
If (ε_x, ε_y) is described by a 2D Gaussian,
then PP distribution would be Bessel-Gaussian

But definitely not 2D Gaussian

$\varepsilon_{part} < 1$



Eccentricity Magnitude ε_2



Bessel-Gaussian goes above 1 but Elliptic Power does not.
Elliptic Power fits much better.

Bessel-Gaussian Distribution

$$\frac{dn}{d\varepsilon} = \frac{\varepsilon}{\sigma_0^2} \exp\left(-\frac{\varepsilon^2 + \varepsilon_0^2}{2\sigma_0^2}\right) I_0\left(\frac{\varepsilon\ \varepsilon_0}{\sigma_0^2}\right)$$

Assumes a 2D Gaussian of width σ_0
in the reaction plane displaced to one side by ε_0.
2 parameters:

ε_0 mean eccentricity in RP
σ_0 eccentricity fluctuations around mean

S.A. Voloshin et al, PLB 659, 537 (2008)

New Elliptic Power Distribution

$$P(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_0^2)^{\alpha+\frac{1}{2}} \frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha-1}}$$

$$\frac{dn}{d\varepsilon} = \frac{2}{\pi}\varepsilon\alpha(1-\varepsilon^2)^{(\alpha-1)}(1-\varepsilon_0^2)^{(\alpha+1/2)}\int_0^{\pi}(1-\varepsilon_0\varepsilon\cos\phi)^{-(1+2\alpha)}d\phi$$

Could be expressed as a hypergeometic function,
but the ROOT version is not defined everywhere needed.
Better to do numerical integration.

Point-like independent sources distributed in a 2D elliptic Gaussian with an eccentricity = ε_0 and a cut off at $\varepsilon = 1$.

2 parameters:

- ε_0: ellipticity parameter is approx. eccentricity in RP
- α : power parameter describes fluctuations

When $\varepsilon_0 \ll 1$ and $\alpha \gg 1$ becomes Bessel-Gaussian
With $\sigma_0 \cong 1/(2\alpha)^{1/2}$

Power Distribution

For ε_0=0 (only fluctuations) the Elliptic Power distribution reduces to the Power distribution:

$$P(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1 - \varepsilon_x^2 - \varepsilon_y^2)^{\alpha-1}$$

$$\frac{dn}{d\varepsilon} = 2\varepsilon\alpha\left(1 - \varepsilon^2\right)^{\alpha-1}$$

Yan and Ollitrault, PRL 112, 082301 (2014)

2D isotropic distribution with a cut off at $\varepsilon = 1$

One parameter:

α : power parameter describes fluctuations

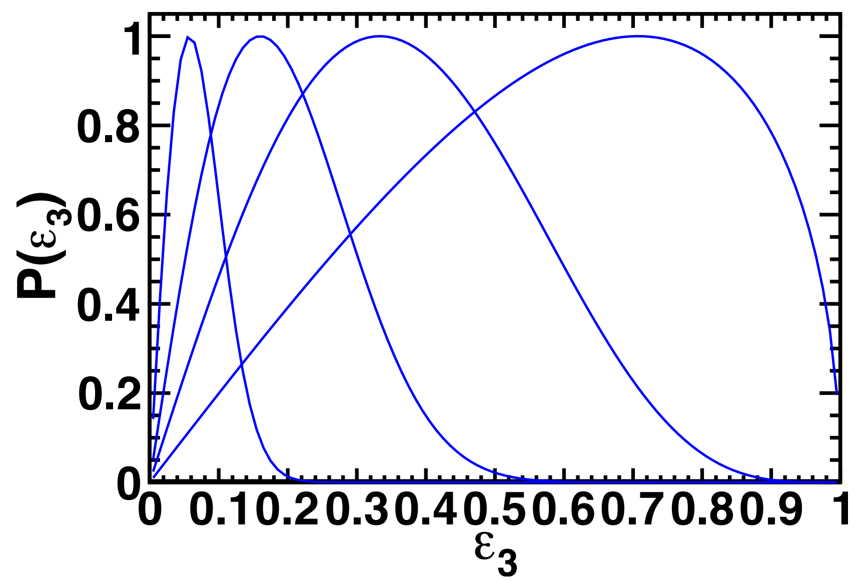
For α >> 1 it becomes a Gaussian*ε with $\sigma^2=1/(2\alpha)$

For Elliptic Power ε_3 Glauber

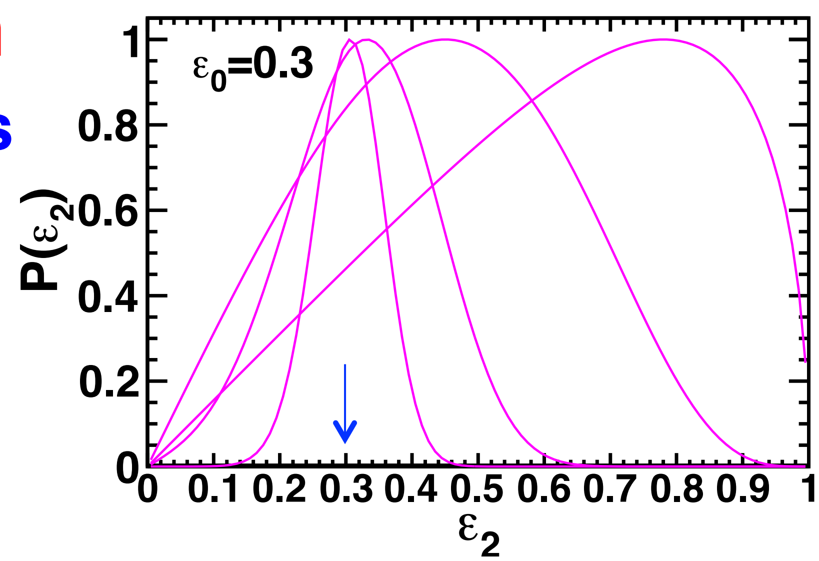
Found $\varepsilon_0 = 0$, thus Power is OK for ε_3

Eccentricity Parameters

- **Power Distribution**
  - Eccentricity fluctuations
  - alpha

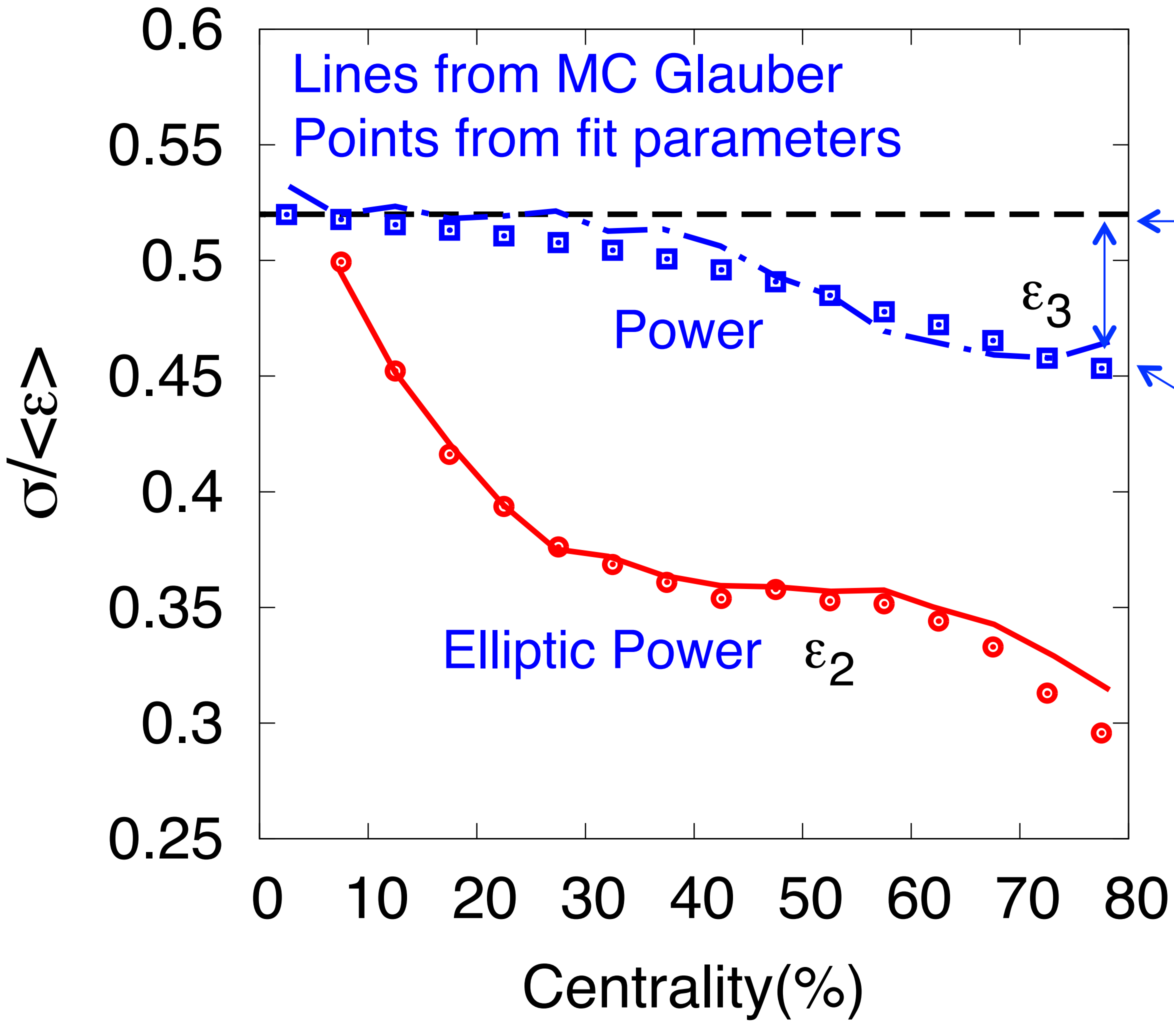


- **Elliptic Power Distribution**
  - Correlated with a plane plus fluctuations
  - ε_0 and alpha



MC Glauber Fluctuations

$$\frac{\sigma_\varepsilon}{\langle\varepsilon\rangle} = \frac{\sqrt{\langle\varepsilon^2\rangle - \langle\varepsilon\rangle^2}}{\langle\varepsilon\rangle}$$

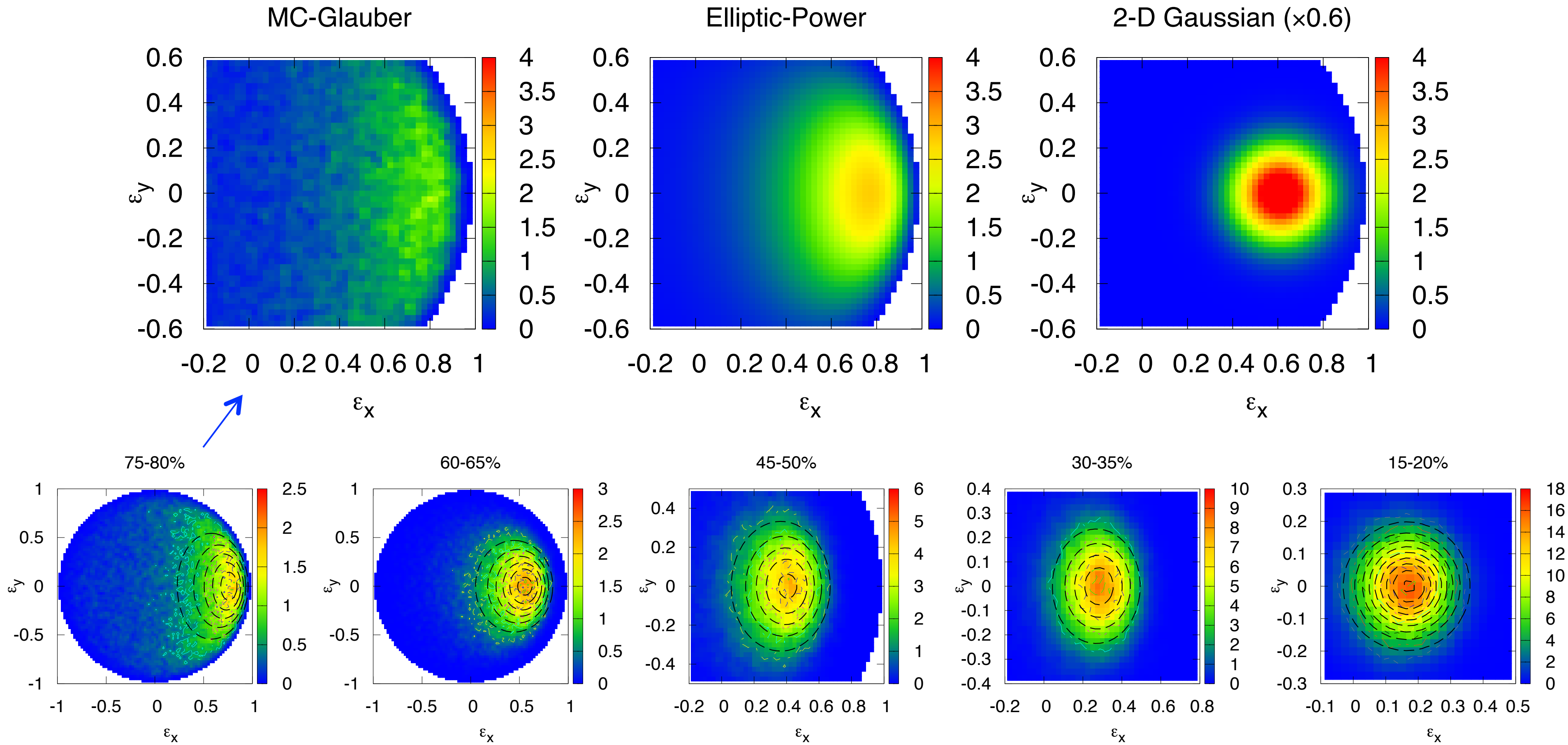


$$\sqrt{4/\pi - 1} = 0.52$$

Pure Gaussian fluctuations

Non-Gaussian

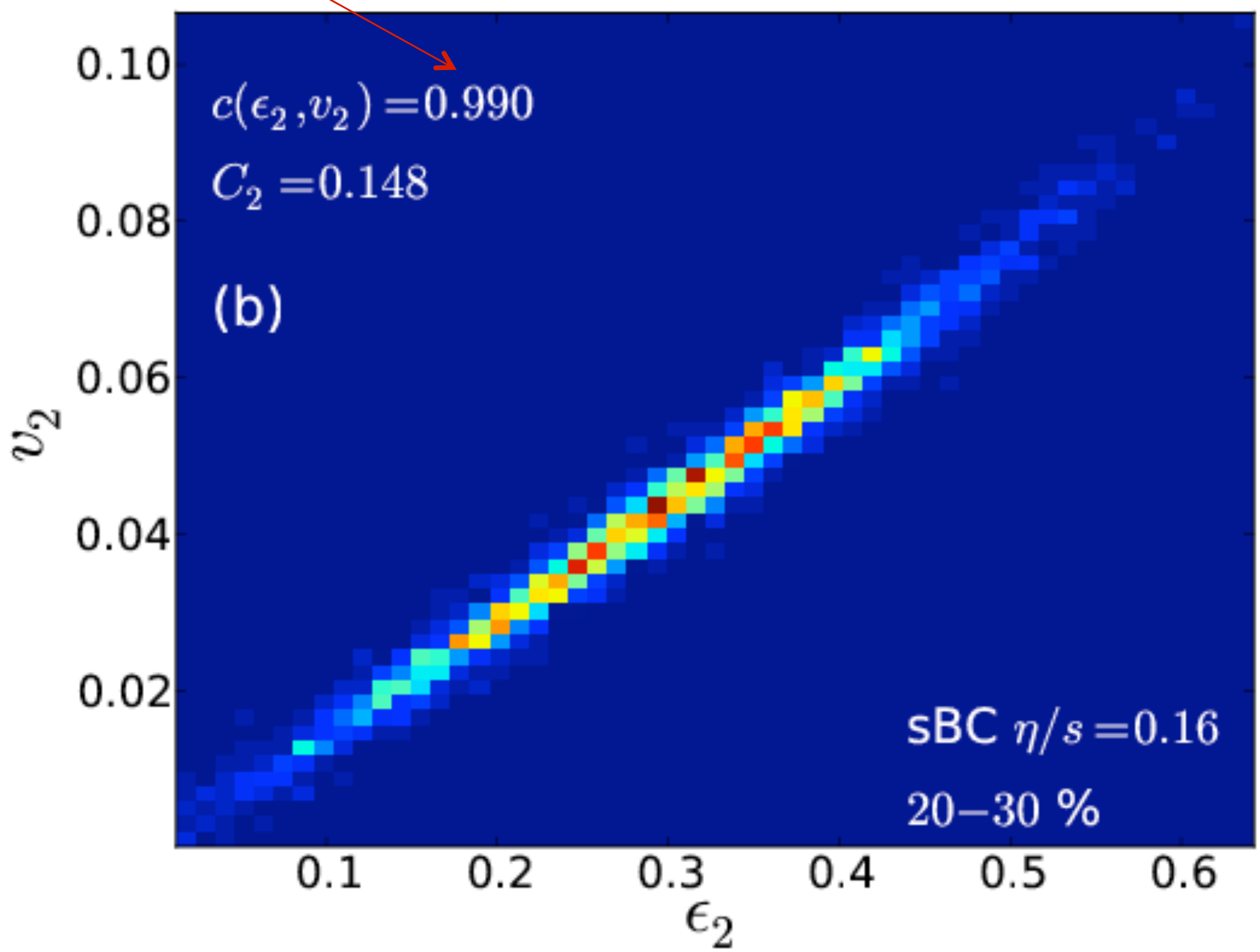
Pb + Pb MC Glauber



More peripheral

v_2 linear in ε_2

$$v_n = \kappa_n \varepsilon_n$$



Event-by-event viscous hydro

Niemi, Denicol, Holopainen, and Huovinen, PRC **87**, 054901 (2013)

κ Hydro Response

$$v_n = \kappa_n \varepsilon_n$$

κ used to be called v/ε
But that implies you need to know ε
We determine κ without knowing ε

New parameter:
κ is the response of the media to the initial configuration

assume $v_n = \kappa_n \varepsilon_n$ n= 2,3

$$\frac{dn}{dv_n} = 1/\kappa_n \frac{dn}{d\varepsilon_n}$$

The v_n distribution is the ε_n distribution rescaled by κ_n

v_n Parameters

- Power Distribution
  - Flow fluctuations
  - alpha and kappa
  - For A+A v_3
  - For p+A v_2

- Elliptic Power Distribution
  - Flow correlated with the reaction plane plus fluctuations
  - ε_0, alpha, and kappa
  - For A+A v_2

A + A Data

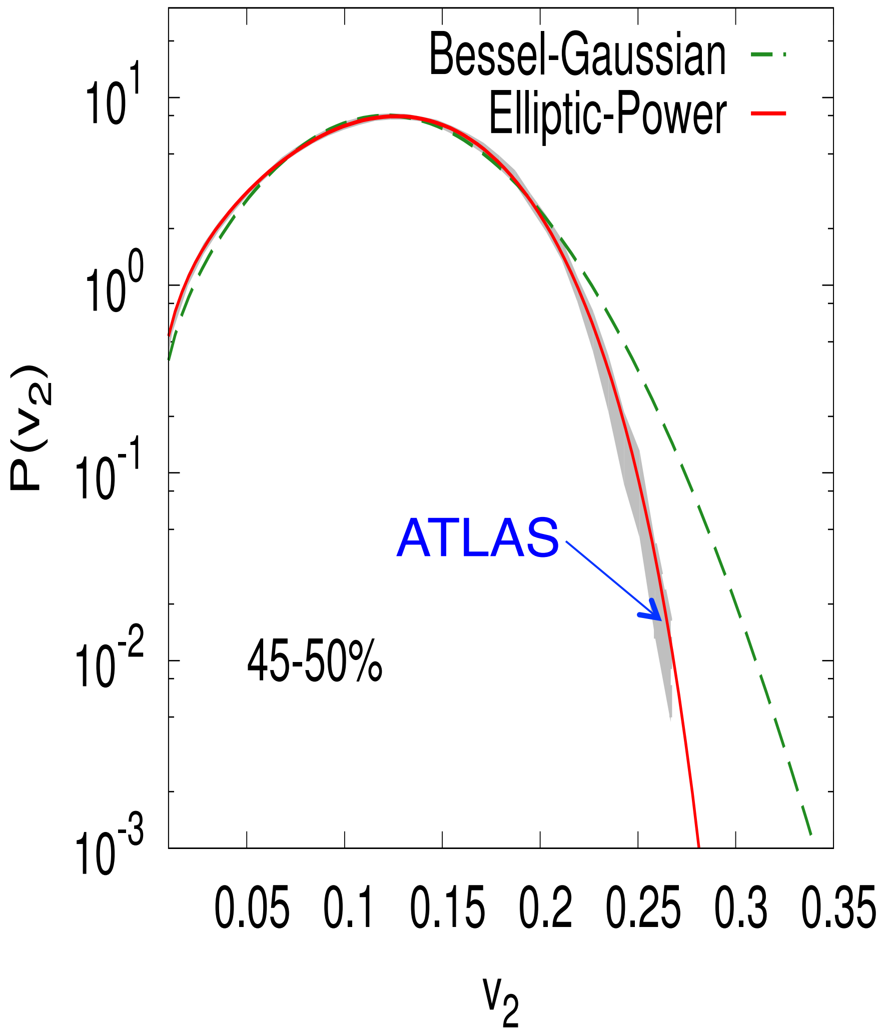
- ATLAS $\sqrt{s}$ 2.76 TeV Pb+Pb
  - JHEP 11, 183 (2013); arXiv:1305.2942
- Very high multiplicity with large statistics
- Also ALICE and STAR
  - But not yet enough statistics

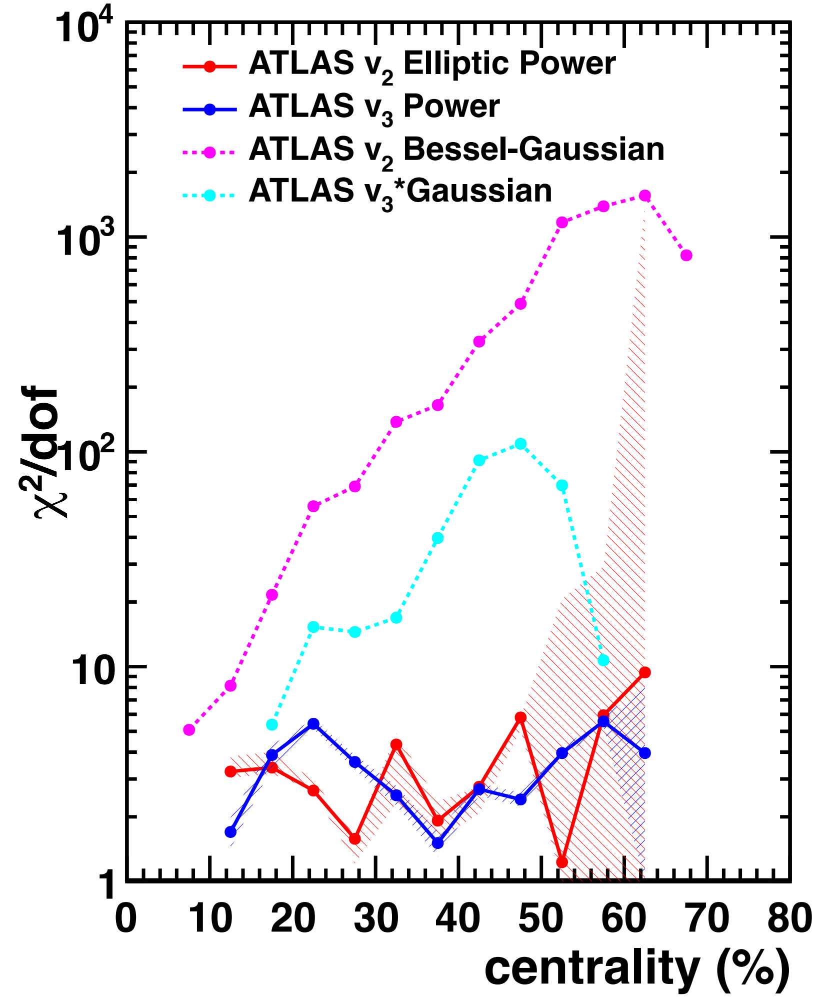
ATLAS Bayesian Unfolding

- v_n distribution with dispersion unfolded
  - To remove "most" of non-flow and fluctuations
- Uses η sub-event method
- Difference of flow vectors
  - Real flow signal cancels
  - Dispersion is twice that of the flow signal
  - Iterative procedure
- Called event-by-event, but it is not
  - Because v_2 for a single event is not known

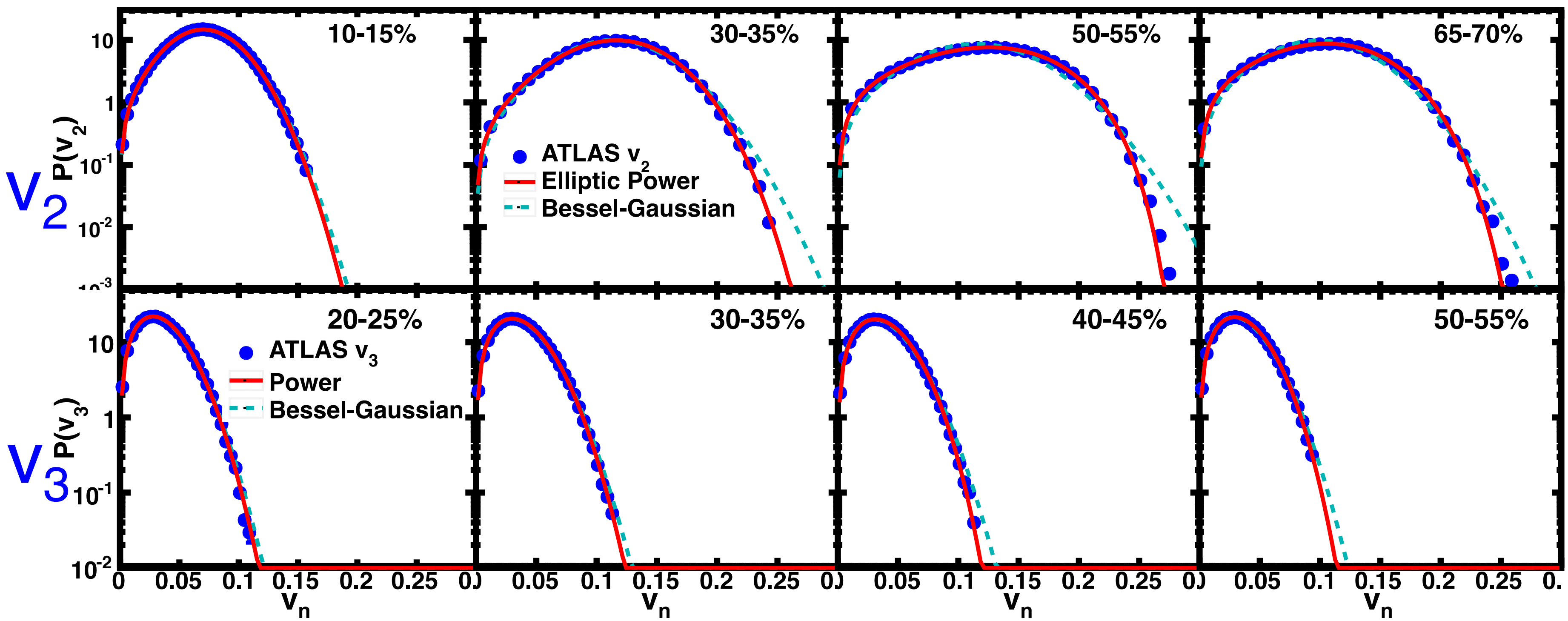
Jiangyong Jia and Mohapatra, PRC 88, 014907 (2013)

Goodness of Fit





ATLAS Distributions



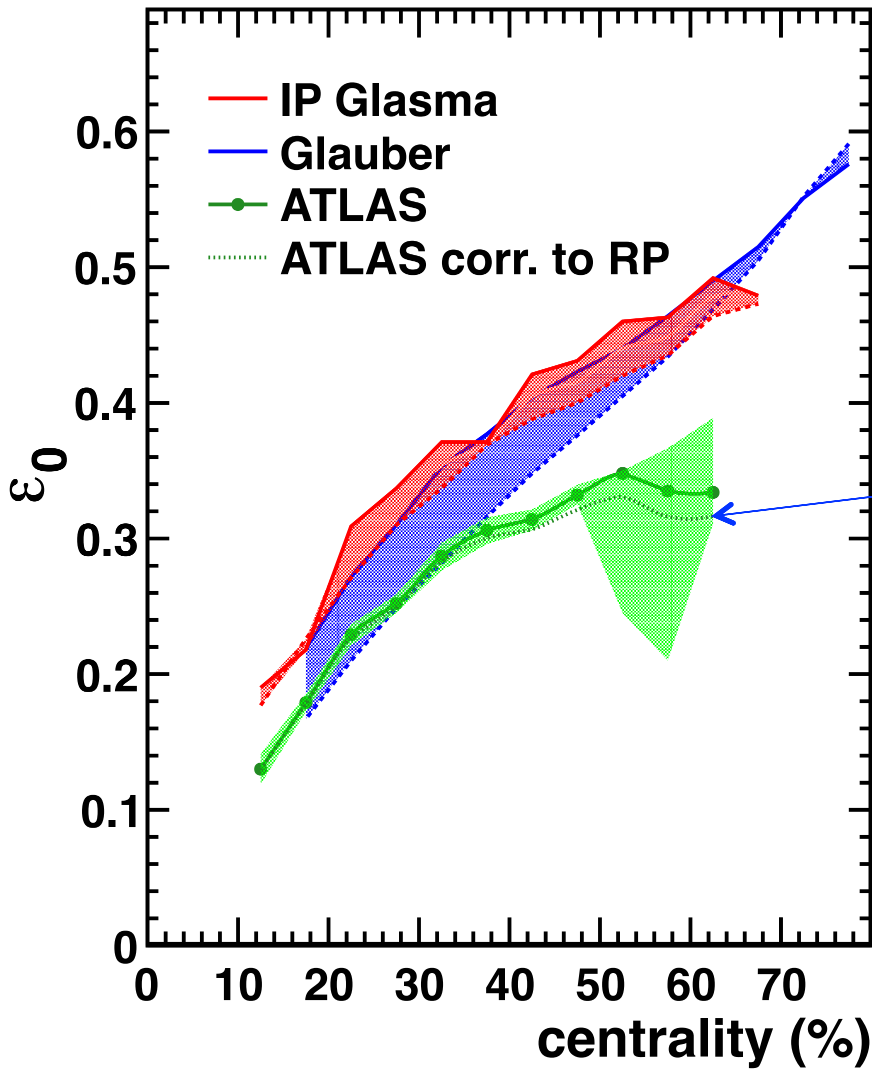
Bessel-Gaussian can not determine κ

Elliptic Power shape becomes non-Gaussian at v_n values close to κ (ε values close to 1)

Ability to measure κ depends on this non-Gaussian shape

(v_3 distributions are more Gaussian)

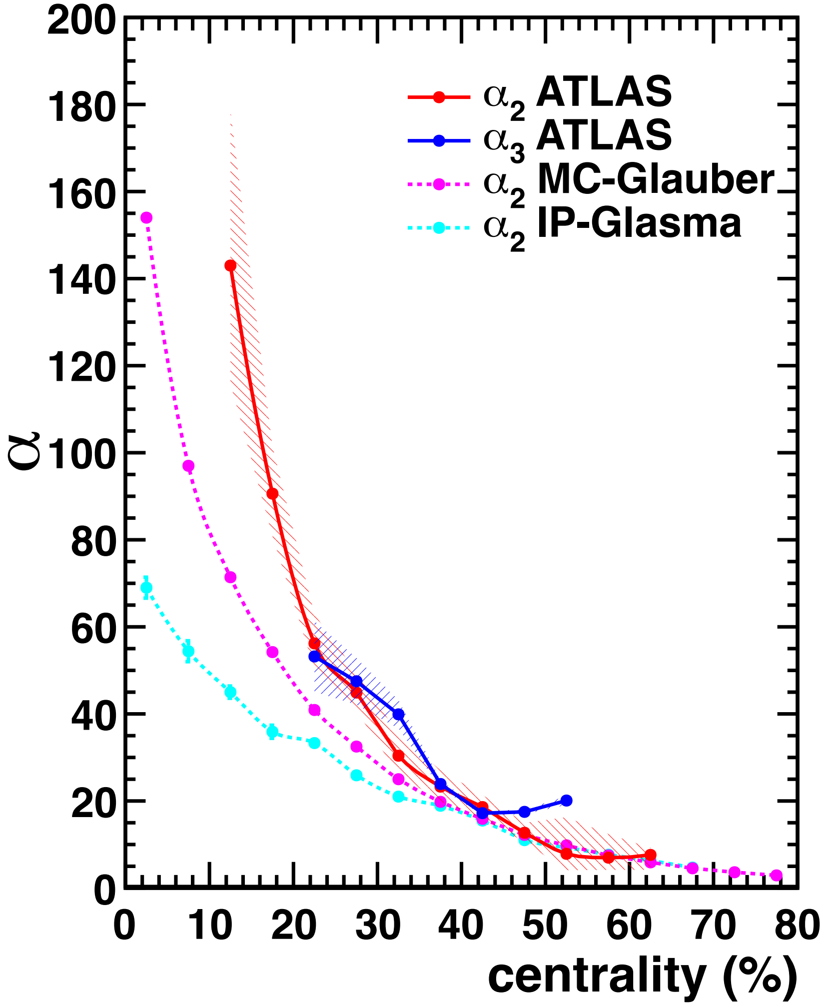
ε_0 Approx. RP Eccentricity



Simulations are about the same
Real data has ε_0 values smaller than simulations

Correction to reaction plane small

α Fluctuation Parameter



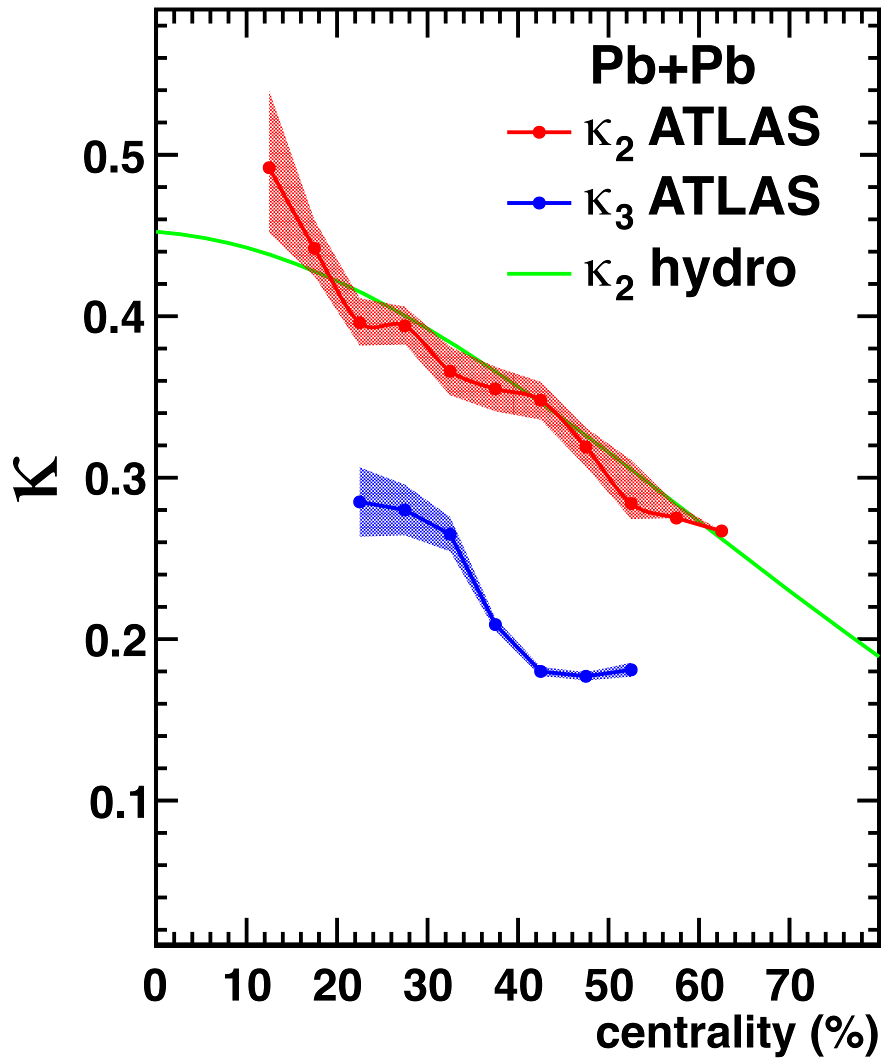
α accounts for both initial state fluctuations and those during the expansion

Real data is more Gaussian than Simulations

α prop. to number of sources in Glauber simulation

← more particles, less fluctuations, more Gaussian

κ Hydro Response Parameter



ATLAS p_T> 0.5 GeV/c

K_3 has only statistical errors

κ extracted independent of initial eccentricity

κ_3 is lower than κ_2 because the finer details of higher harmonics are damped more by viscous effects.

Both decrease for peripheral collisions because the smaller size leads to larger viscous damping (~1/R)

viscous hydro calc. of v/ε

←η/s=0.18

(normalized vertically)

(no systematic errors)

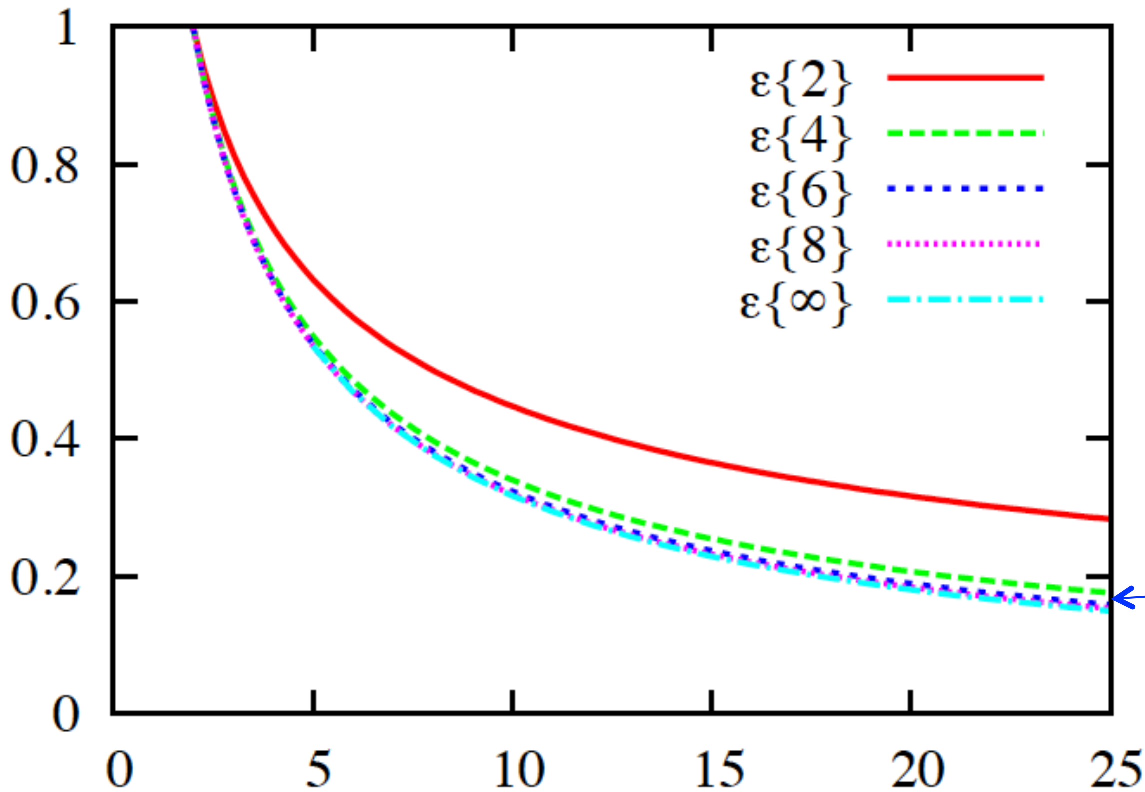
(τ varies little with centrality)

viscous hydro from Teaney and Yan

p + A

- Probably no almond shaped overlap region
- But lots of fluctuations
  - Especially for large impact parameters
- Claims of “Flow” in p + A
- Large $v_2\{4\}$ is a non-Gaussian effect
- No v_2 distributions
  - Must use cumulants

Power Distribution Cumulants



For Gaussian all = 0
For Bessel-Gaussian all equal to ε_0

← small N Number of particles

No in-plane flow, just large flow fluctuations

The non-Gaussian shape causes:

p+A $v_2\{4\}$ to be finite

A+A $v_3\{4\}$ to be finite

The non-Gaussian shape also allows the determination of the parameters!

κ from Cumulants of Power Distribution

assume $\varepsilon_0 = 0$ (Power Distribution)

$$\varepsilon_n\{4\}/\varepsilon_n\{2\} = (1+\alpha/2)^{-1/4}$$

$$\varepsilon_n\{2\} = 1/\sqrt{1+\alpha}$$

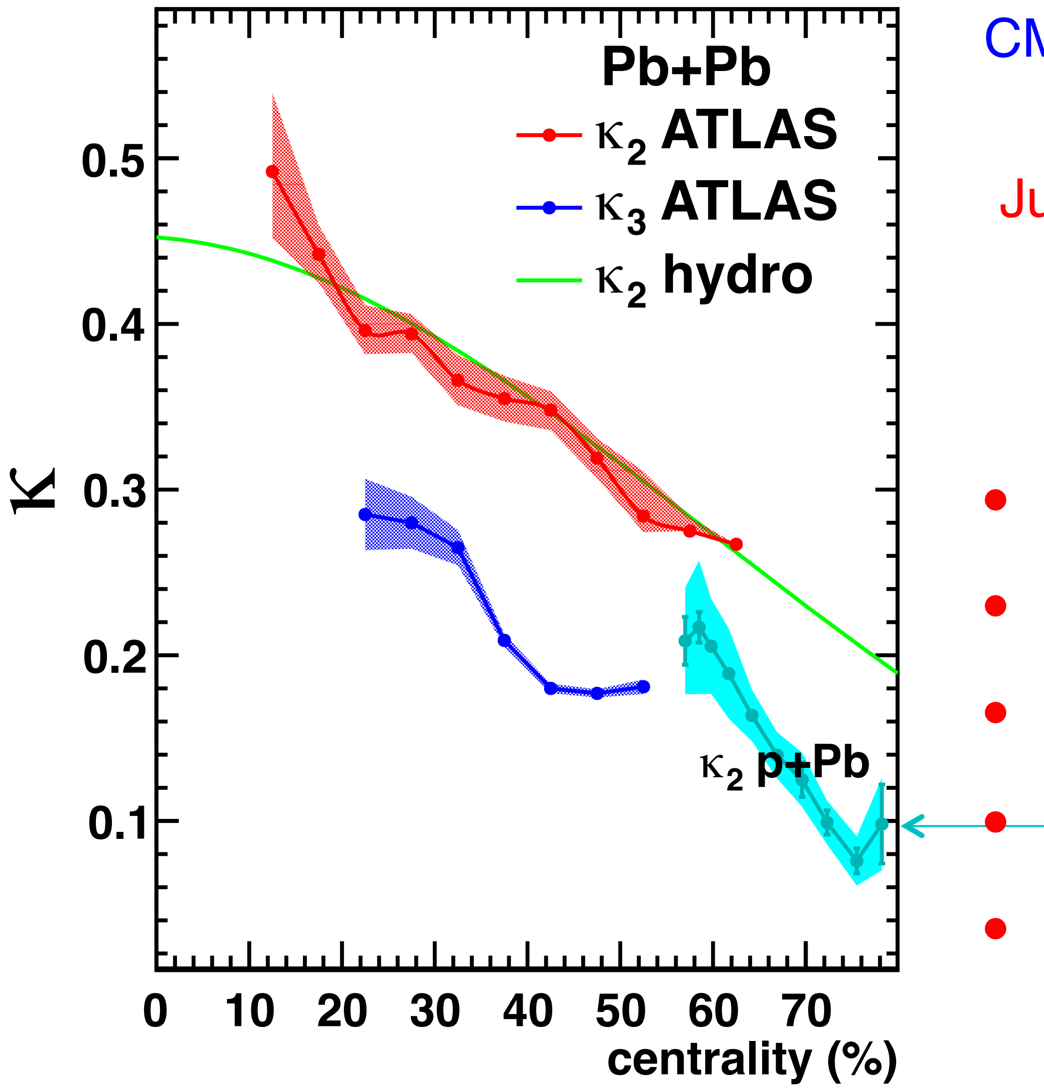
assume $v_n = \kappa_n \varepsilon_n$ (linear)

$$\kappa_n = v_n\{2\}\sqrt{2\left(\frac{v_n\{2\}}{v_n\{4\}}\right)^4 - 1}$$

The cumulant ratio gives the non-Gaussian shape

Independent of the ε distribution!

CMS p + Pb



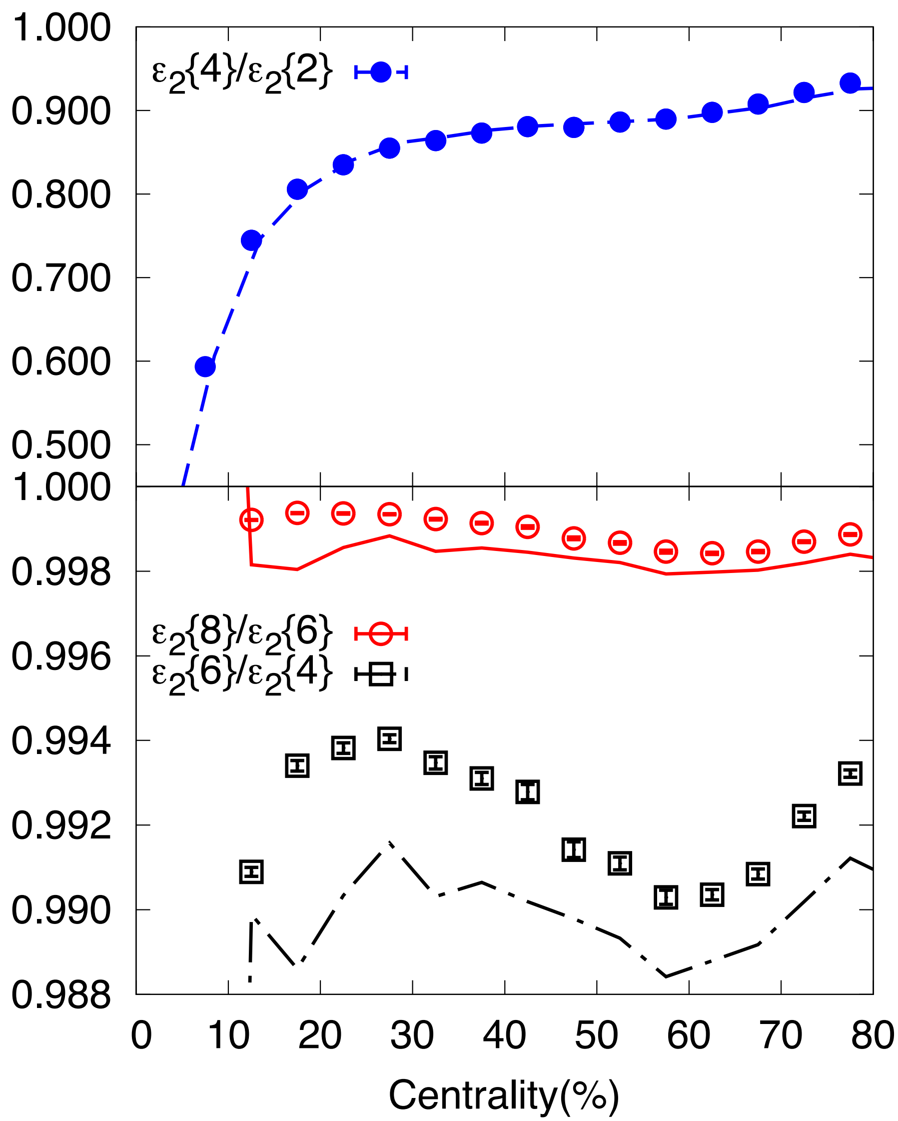
CMS because ATLAS has small η gap

Just non-Gaussian flow fluctuations!

- **CMS √s 5.02 TeV**
- **PLB 724, 213 (2013)**
- **Peripheral subtraction**
- **From cumulants**
- **Equivalent centrality**
  - **Based on N_{tracks}**

So why not cumulants of the Elliptic Power?

Pb + Pb MC Glauber Cumulant Ratios



1%

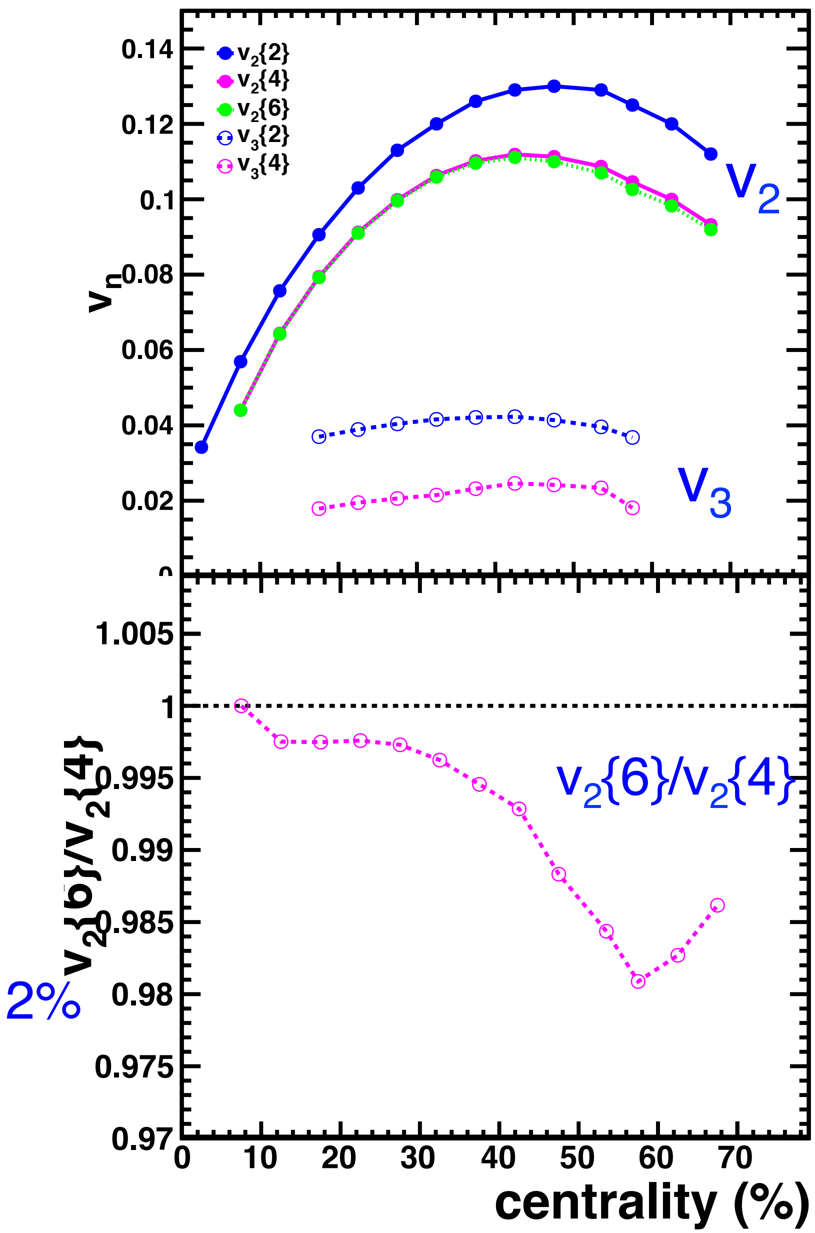
Test of Elliptic Power with cumulants of Glauber

Lines are direct MC Glauber cumulants

Points are Elliptic Power cumulants with parameters from fits to distributions

Good, but not perfect

ATLAS Pb + Pb Cumulants



$v_2\{4\}$ and higher orders:
For Gaussian all = 0
For Bessel-Gaussian
all equal

$v_3\{4\}$ finite because of
non-Gaussian effects

not = 1 because of
non-Gaussian effects

Cumulants of Elliptic Power

$$f_k \equiv \langle (1-\varepsilon_n^2)^k \rangle = \frac{\alpha}{\alpha+k}\left(1-\varepsilon_0^2\right)^k {}_2F_1\left(k+\frac{1}{2}, k; \alpha+k+1; \varepsilon_0^2\right)$$

$$\varepsilon_n\{2\} = (1-f_1)^{1/2}$$

$$\varepsilon_n\{4\} = (1-2f_1+2f_1^2-f_2)^{1/4}$$

$$\varepsilon_n\{6\} = \left(1+\frac{9}{2}f_1^2-3f_1^3+3f_1(\frac{3}{4}f_2-1)-\frac{3}{2}f_2-\frac{1}{4}f_3\right)^{1/6}$$

need

k above is the order

Data does not need unfolding

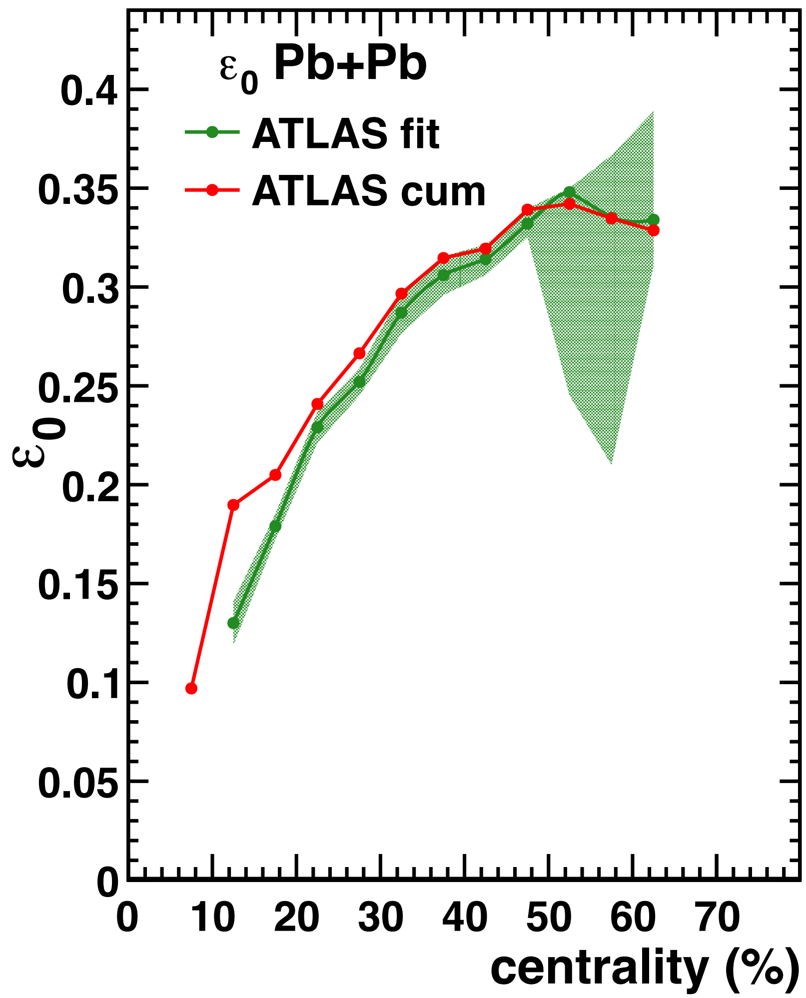
but now 3 parameters

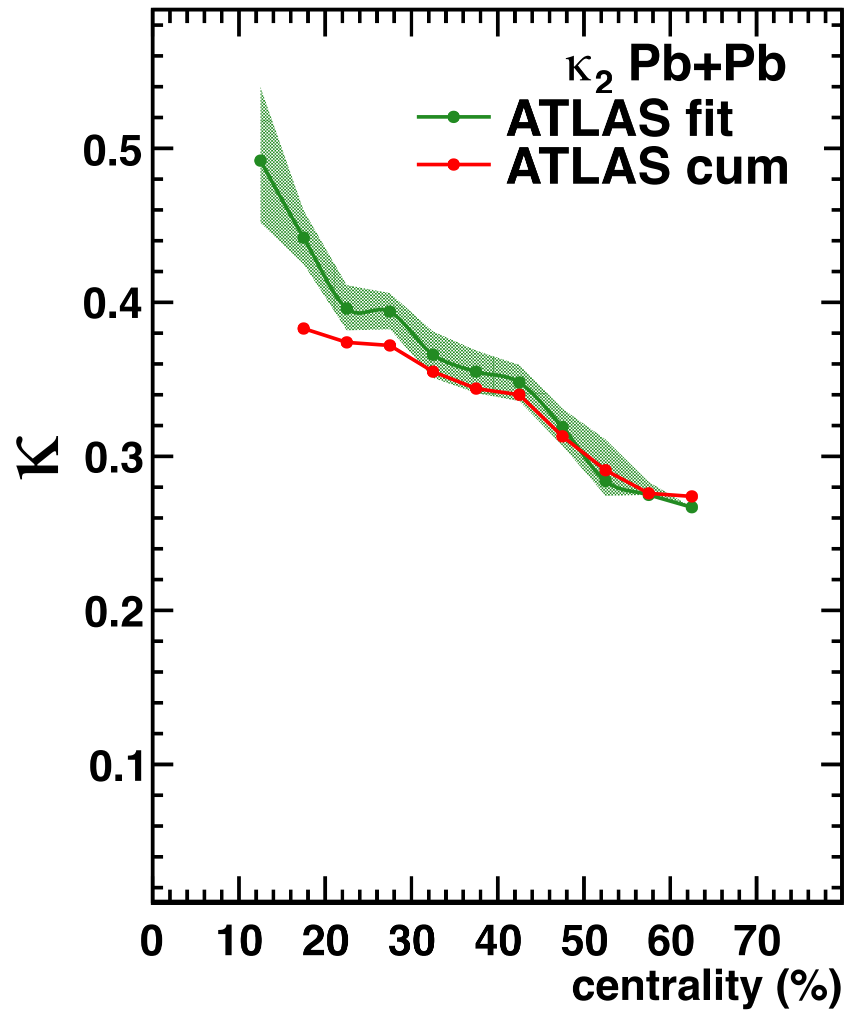
2 ratio equations in 2 unknowns

Then scale by κappa for hydro response

$v_n\{2\}$ may have non-flow (need large η-gap)

Parameters from ATLAS Cumulants





Parameters agree with those from distribution fits

No errors from cumulants

Summary

- **Rescaled Power and Elliptic Power give good fits**
- **Non-Gaussian shape allows determination of parameters**
- **No initial state model assumed**
- **Bigger effect for peripheral collisions**
  - **More fluctuations to larger anisotropies**
- **Bigger effect for smaller systems**
  - **Should study lighter collision systems**
- **Fits to viscous hydro indicate low value of η/s**

Acknowledgments

- Sergei Voloshin for the name Elliptic Power and valuable comments on both papers
- Matt Luzum for pushing the cumulants