Azimuthal Anisotropy Distributions: The Elliptic Power Distribution

Li Yan and Jean-Yves Ollitrault
CNRS, Institut de Physique Théorique de Saclay

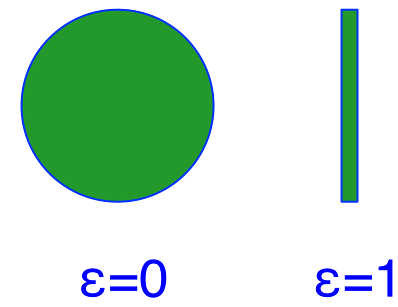
and Art Poskanzer
LBNL

Main Point

- **Initial eccentricity is driving force for flow**
- **Usual viscous hydro output depends on assumed initial conditions**
- **We separate hydro response from initial anisotropy based on its non-Gaussian shape**
- **We obtain hydro response assuming only the initial anisotropy goes to zero at 0 and 1**
- **The decrease of hydro response with decreasing centrality gives η/s**

Eccentricity

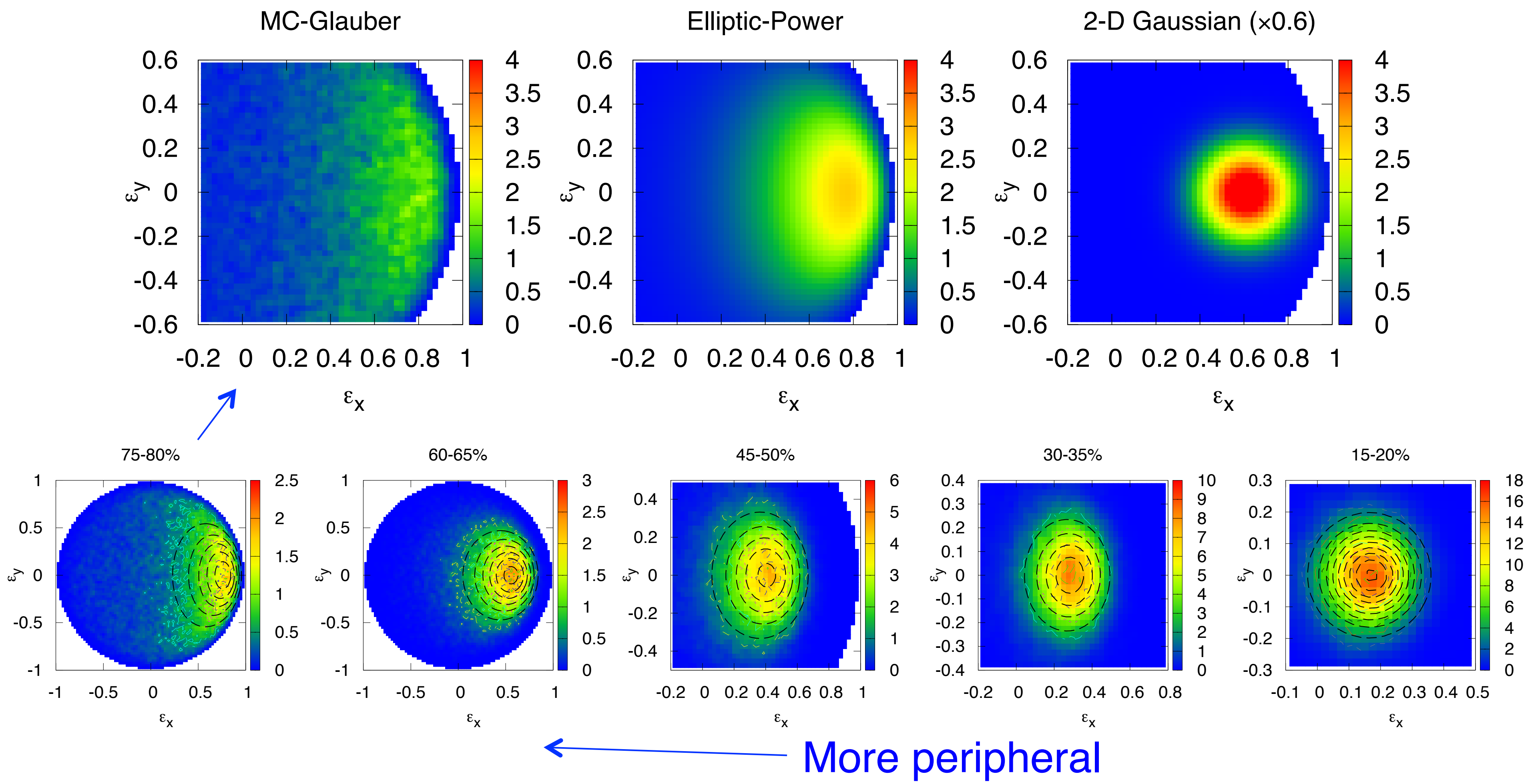
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



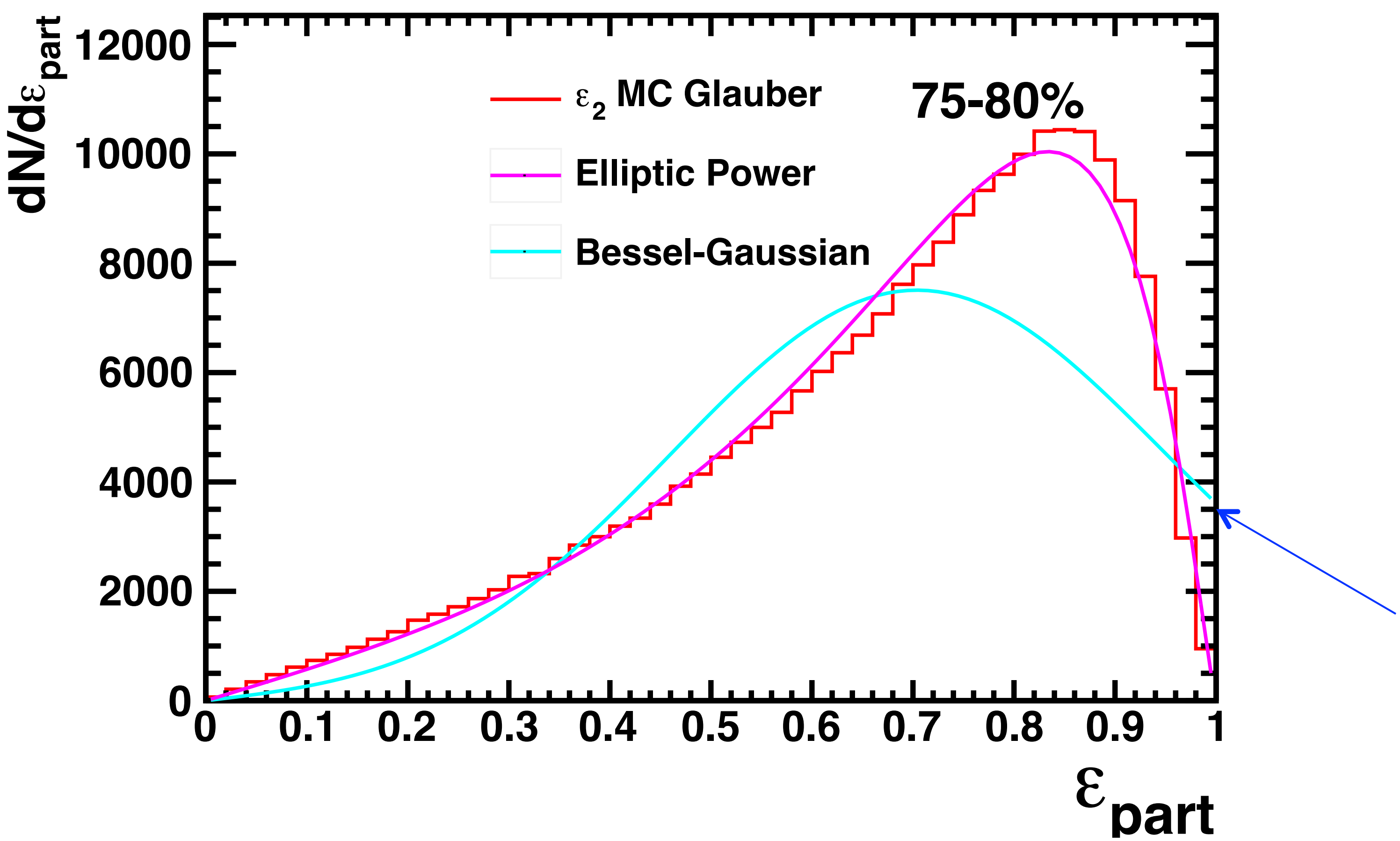
- **Must be between 0 and 1**
  - **Positive because it is the length of a vector**
  - **Going from Gaussian to Bessel-Gaussian eliminated the negative values**
  - **Going from Bessel-Gaussian to the Elliptic Power distribution eliminates values greater than 1**
- **Participant Eccentricity Ellipse is rotated:**

$$(\varepsilon_x, \varepsilon_y) = \left(\frac{\langle \sigma_y^2 - \sigma_x^2 \rangle}{\langle \sigma_y^2 + \sigma_x^2 \rangle}, \frac{\langle 2\sigma_{xy} \rangle}{\langle \sigma_y^2 + \sigma_x^2 \rangle} \right)$$

Pb + Pb MC Glauber



Eccentricity Magnitude ε_2



Bessel-Gaussian goes above 1 but Elliptic Power does not.
Elliptic Power fits much better.

Bessel-Gaussian Distribution

$$\frac{dn}{d\varepsilon} = \frac{\varepsilon}{{\sigma_0}^2} \exp\left(-\frac{\varepsilon^2 + {\varepsilon_0}^2}{2{\sigma_0}^2}\right) I_0\left(\frac{\varepsilon\ \varepsilon_0}{{\sigma_0}^2}\right)$$

Assumes a 2D Gaussian of width σ_0
in the reaction plane displaced to one side by ε_0
Two parameters:

ε_0 mean eccentricity in RP
σ_0 eccentricity fluctuations around mean

S.A. Voloshin et al, PLB 659, 537 (2008)

New Elliptic Power Distribution

$$P(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1 - {\varepsilon_0}^2)^{\alpha+\frac{1}{2}} \frac{(1 - \varepsilon_x^2 - \varepsilon_y^2)^{\alpha-1}}{(1 - \varepsilon_0 \varepsilon_x)^{2\alpha-1}}$$

$$\frac{dn}{d\varepsilon} = \frac{2}{\pi}\varepsilon\alpha(1 - \varepsilon^2)^{(\alpha-1)}(1 - {\varepsilon_0}^2)^{(\alpha+1/2)} \int_0^\pi (1 - \varepsilon_0 \varepsilon \cos\phi)^{-(1+2\alpha)} d\phi$$

Could be expressed as a hypergeometic function,
but the ROOT version is not defined everywhere needed.
Better to do numerical integration.

Point-like independent sources distributed in a 2D elliptic Gaussian with an eccentricity = ε_0 and a cut off at $\varepsilon = 1$.
Also two parameters:

ε_0: ellipticity parameter is approx. eccentricity in RP
α : power parameter describes fluctuations

When $\varepsilon_0 << 1$ and $\alpha >> 1$ becomes Bessel-Gaussian
With $\sigma_0 \cong 1/(2\alpha)^{1/2}$

Power Distribution

For ε_0=0 (only fluctuations) the Elliptic Power distribution reduces to the Power distribution:

$$P(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1 - \varepsilon_x^2 - \varepsilon_y^2)^{\alpha - 1}$$

$$\frac{dn}{d\varepsilon} = 2\varepsilon\alpha\left(1 - \varepsilon^2\right)^{\alpha - 1}$$

Yan and Ollitrault, PRL 112, 082301 (2014)

2D isotropic distribution with a cut off at ε = 1

One parameter:

α : power parameter describes fluctuations

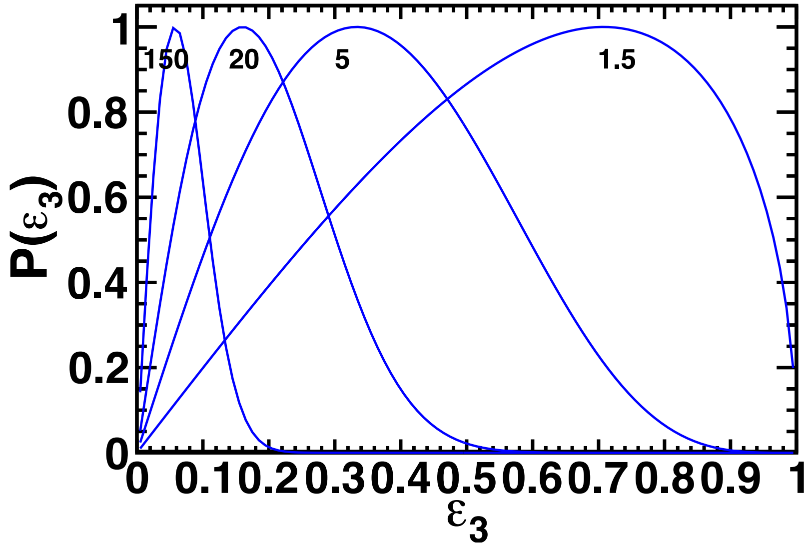
For Elliptic Power ε_3 from Glauber

Found $\varepsilon_0 = 0$, thus Power is OK for ε_3

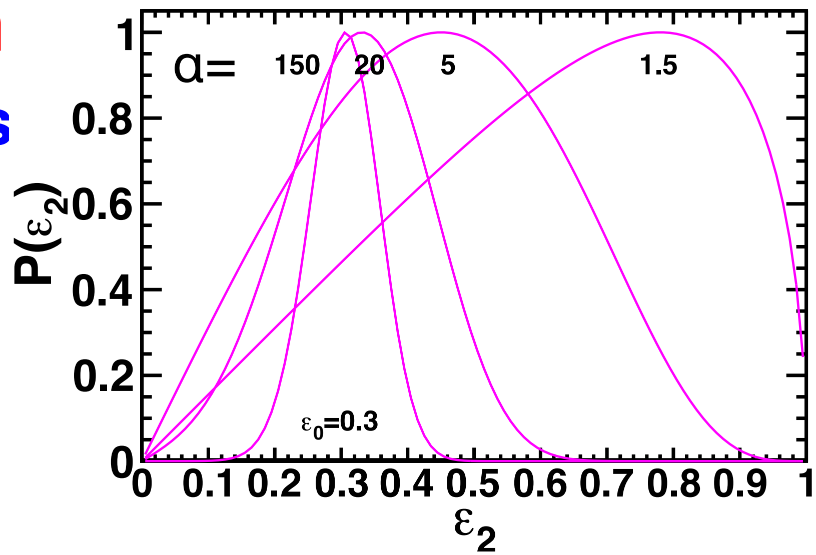
For α >> 1 it becomes a Gaussian*ε with $\sigma^2 = 1/(2\alpha)$

Eccentricity Parameters

- **Power Distribution**
  - **Eccentricity fluctuations**
  - **alpha**

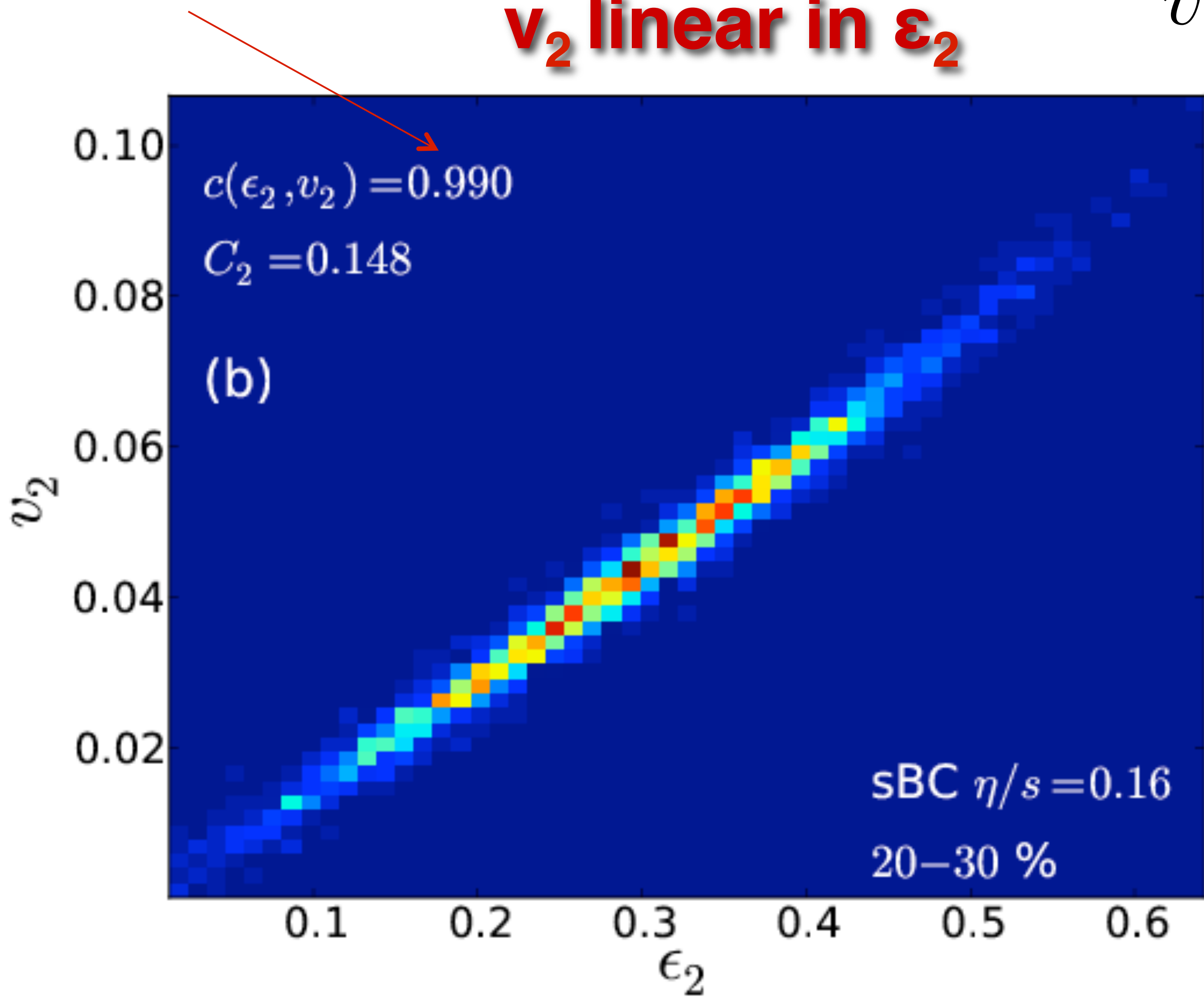


- **Elliptic Power Distribution**
  - **Correlated with a plane plus eccentricity fluctuations**
  - **ε_0 and alpha**



v_2 linear in ε_2

$$v_n = \kappa_n \varepsilon_n$$



Non-linear effect discussed later

Event-by-event viscous hydro

Niemi, Denicol, Holopainen, and Huovinen, PRC **87**, 054901 (2013)

κ Hydro Response

$$v_n = \kappa_n \varepsilon_n$$

New parameter:

κ is the response of the media to the initial configuration

assume $v_n = \kappa_n \varepsilon_n$ n= 2,3

$$\frac{dn}{dv_n} = 1/\kappa_n \frac{dn}{d\varepsilon}$$

The v_n distribution is the ε_n distribution rescaled by κ_n

Parameters of v_n Distributions

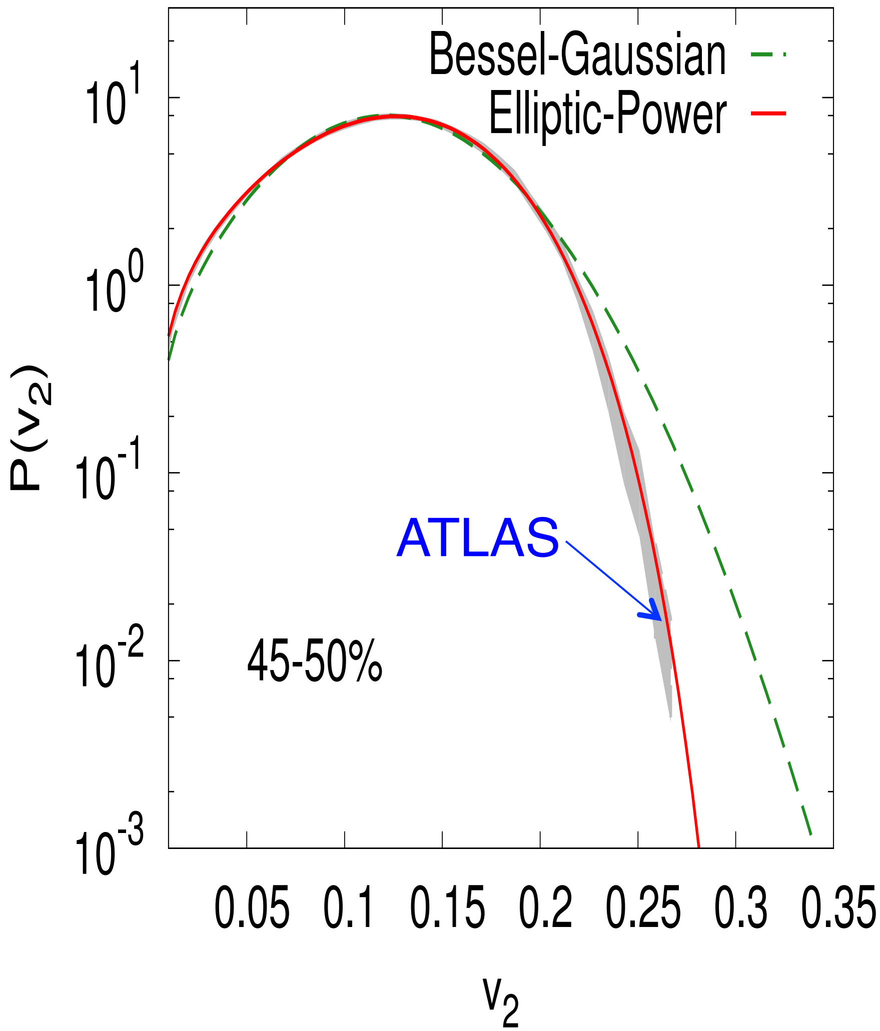
- Power Distribution
  - Flow fluctuations
  - 2: alpha and kappa
  - Good for A+A v_3
  - Good for p+A v_2
- Elliptic Power Distribution
  - Flow correlated with the reaction plane plus flow fluctuations
  - 3: ε_0, alpha, and kappa
  - Good for A+A v_2
- Bessel-Gaussian
  - 3: $\kappa\varepsilon_0$ and $\kappa\sigma$
  - Can not determine kappa

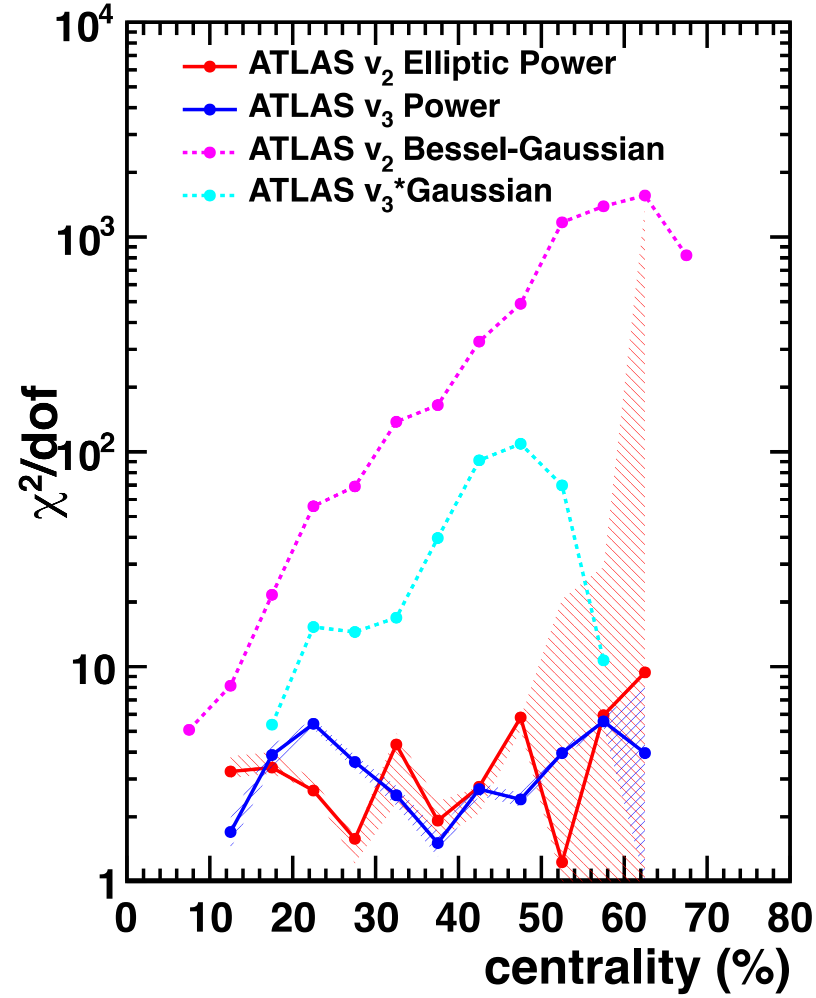
ATLAS Bayesian Unfolding

- ATLAS √s 2.76 TeV Pb+Pb
  - JHEP 11, 183 (2013); arXiv:1305.2942
- v_n distribution with dispersion unfolded
  - To remove “most” of non-flow and fluctuations
- Uses η sub-event method
- Difference of flow vectors
  - Real flow signal cancels
  - Dispersion is twice that of the flow signal
  - Iterative procedure
- Called event-by-event, but it is not
  - Because v_2 for a single event is not known

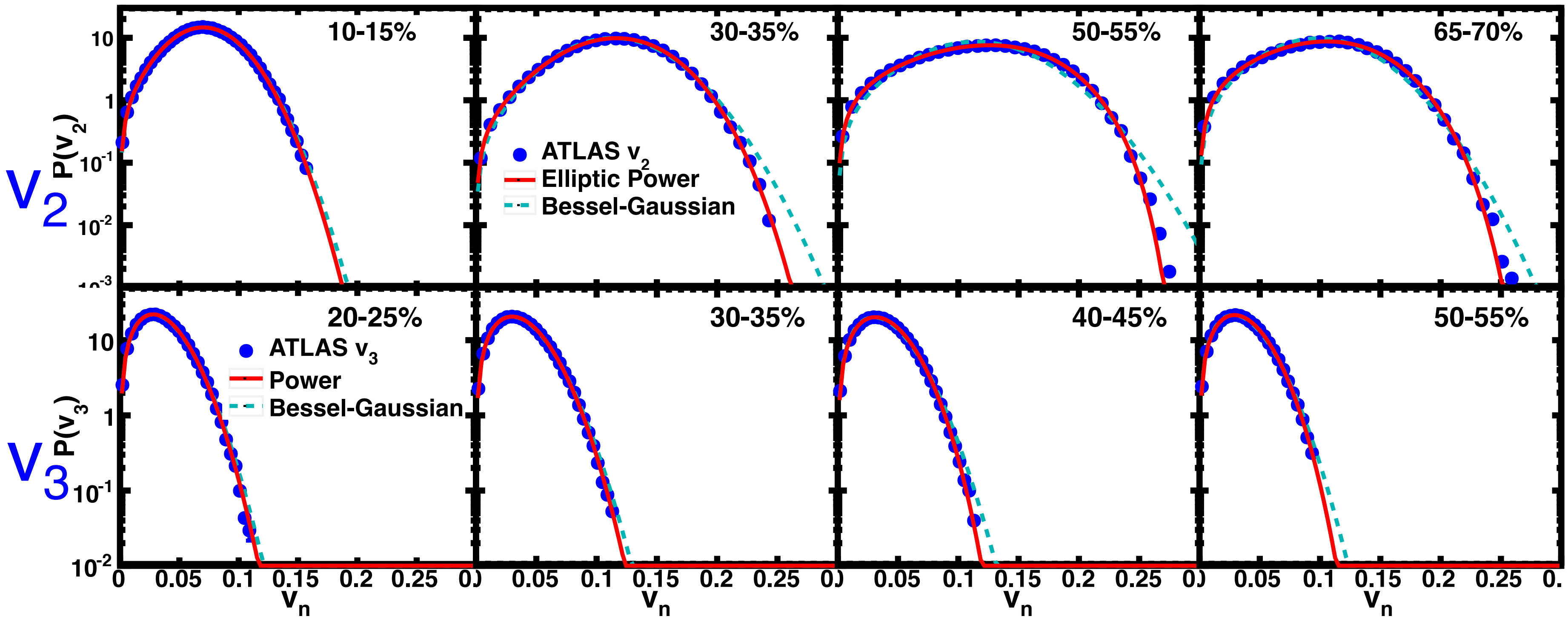
Jiangyong Jia and Mohapatra, PRC 88, 014907 (2013)

Goodness of Fit





ATLAS Distributions



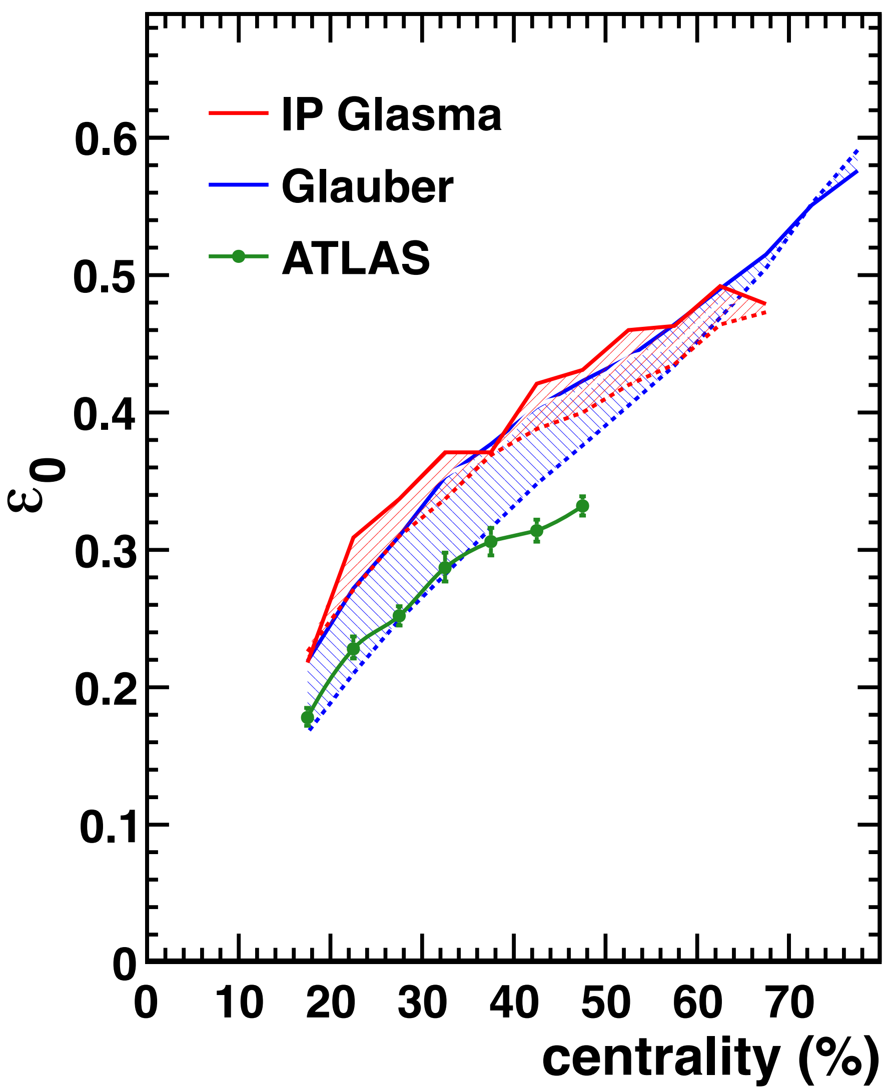
Bessel-Gaussian can not determine κ because width = κ σ

Elliptic Power shape becomes non-Gaussian for ε values close to 1

Ability to measure κ depends on this non-Gaussian shape

(v_3 distributions are more Gaussian)

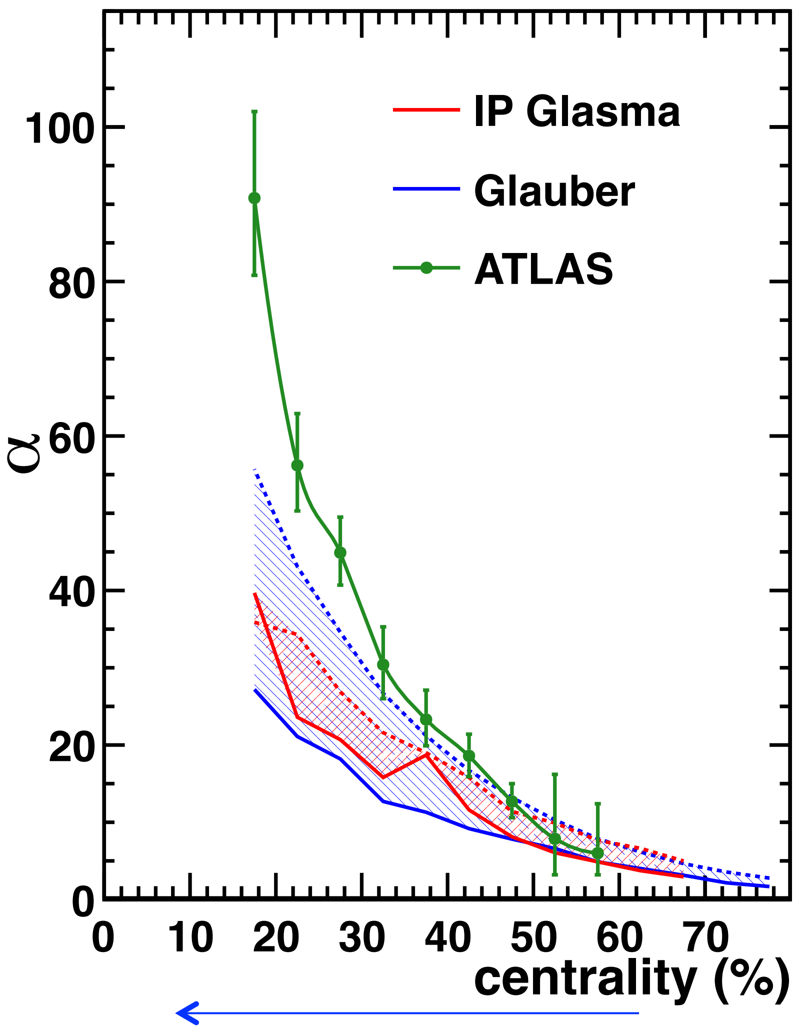
ε_0 Approx. RP Eccentricity



Simulations are about the same

Real data with the linear assumption have ε_0 values smaller than simulations for peripheral collisions

α Fluctuation Parameter



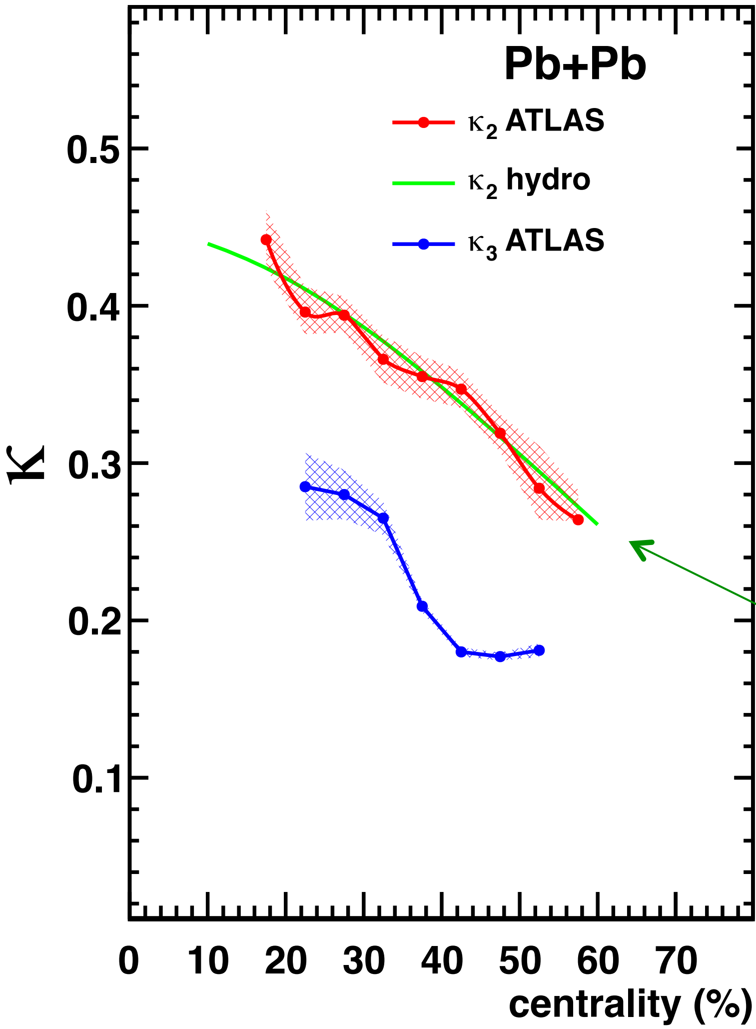
more particles, less fluctuations, more Gaussian

α accounts for both initial state fluctuations and Gaussian fluctuations during the expansion

Real data with the linear assumption are more Gaussian than simulations

α prop. to number of sources in Glauber simulation

κ Hydro Response Parameter



ATLAS p_T> 0.5 GeV/c

K_3 has only statistical errors

κ_3 is lower than κ_2 because the finer details of higher harmonics are damped more by viscous effects.

Both decrease for peripheral collisions because the smaller size leads to larger viscous damping (~1/R)

viscous hydro calc. of v/ε
for η/s=0.19

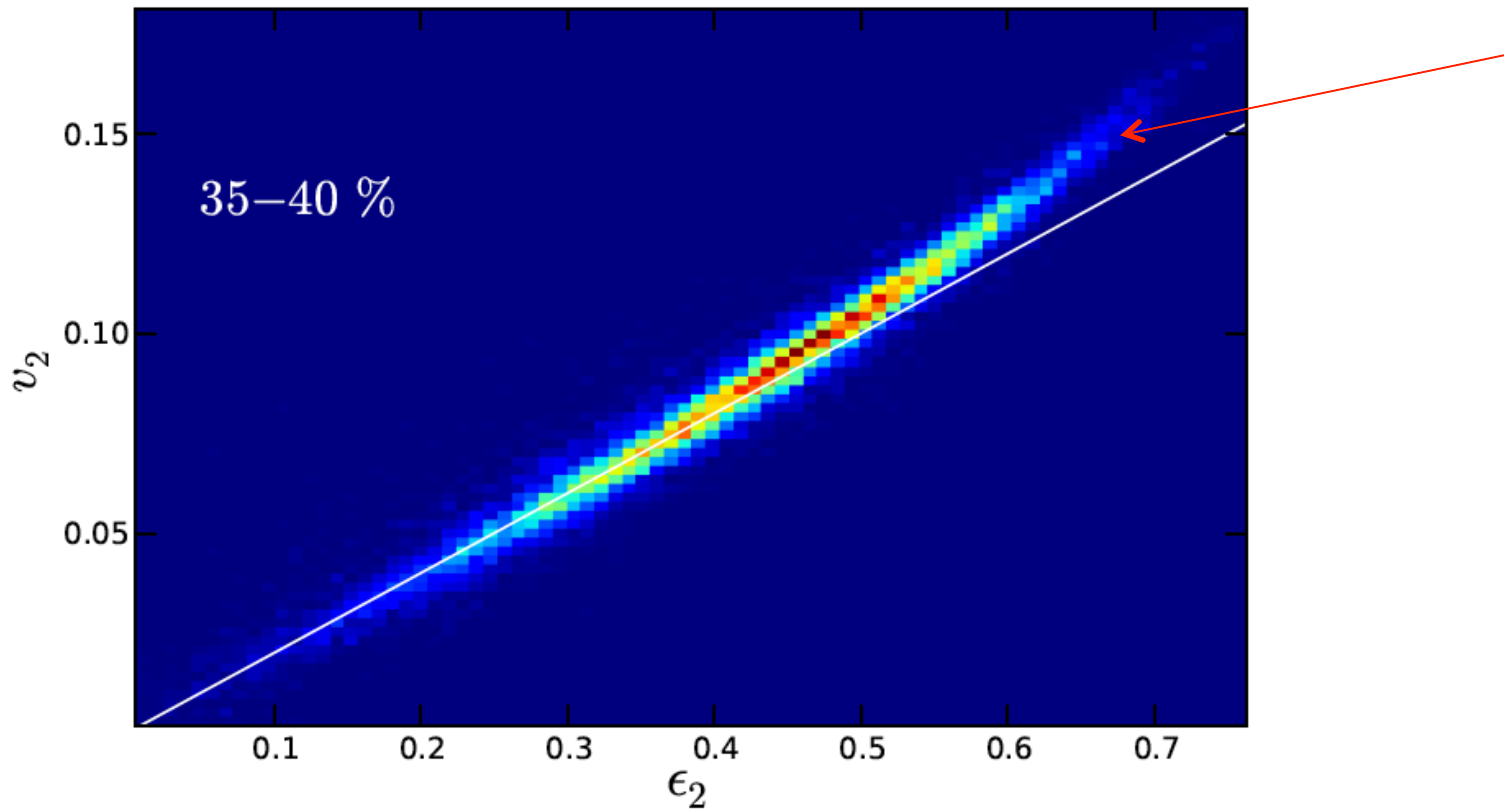
(normalized vertically by 1.7)
(no systematic errors)
(temperature varies little with centrality)

viscous hydro from Teaney and Yan

Non-Linear

$$v_2 = \kappa_2 \varepsilon_2 + \kappa' \kappa_2 \varepsilon_2^3 \qquad \frac{dN}{dv_2} = \frac{d\varepsilon_2}{dv_2}\frac{dN}{d\varepsilon_2}$$

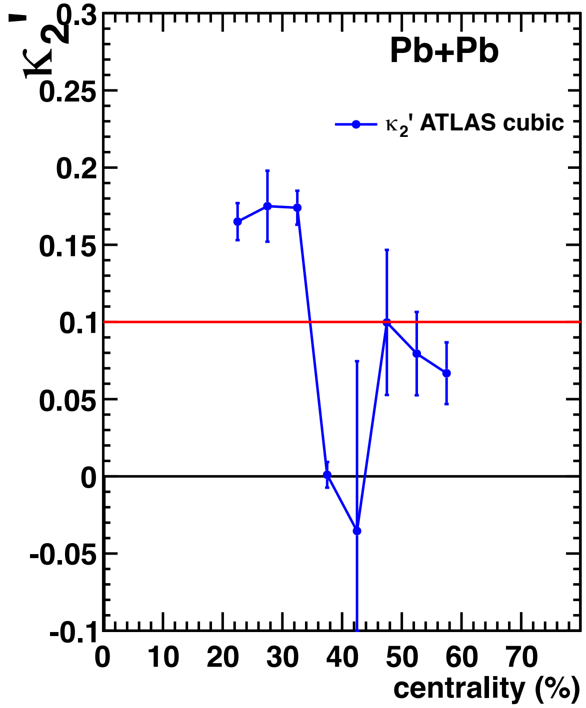
First significant non-linear term is cubic



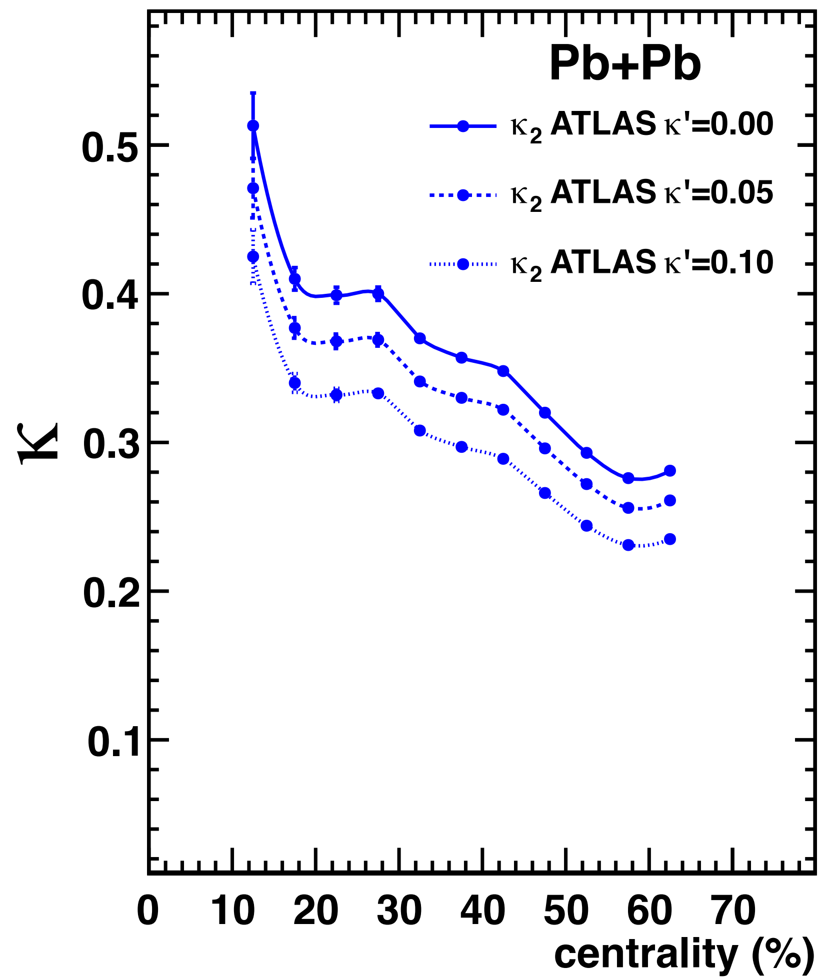
Paatelainen, Niemi, Eskola, and Tuominen, QM2014 poster

Non-Linear Effect

κ’ fit



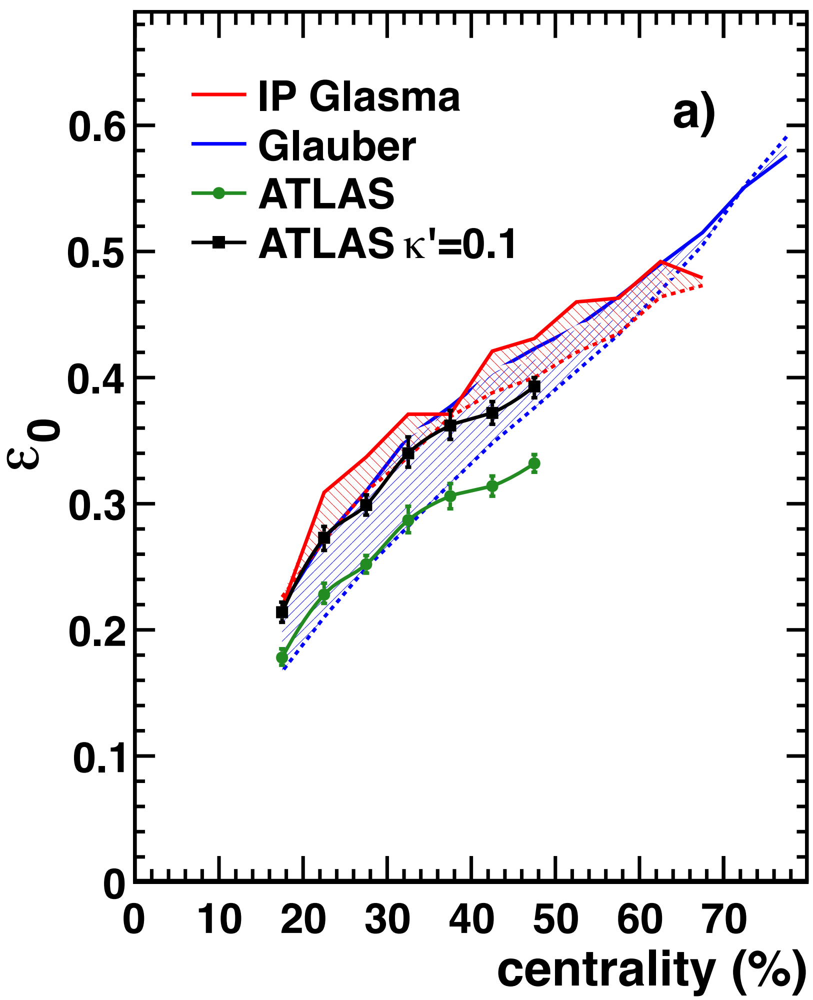
κ’ fixed

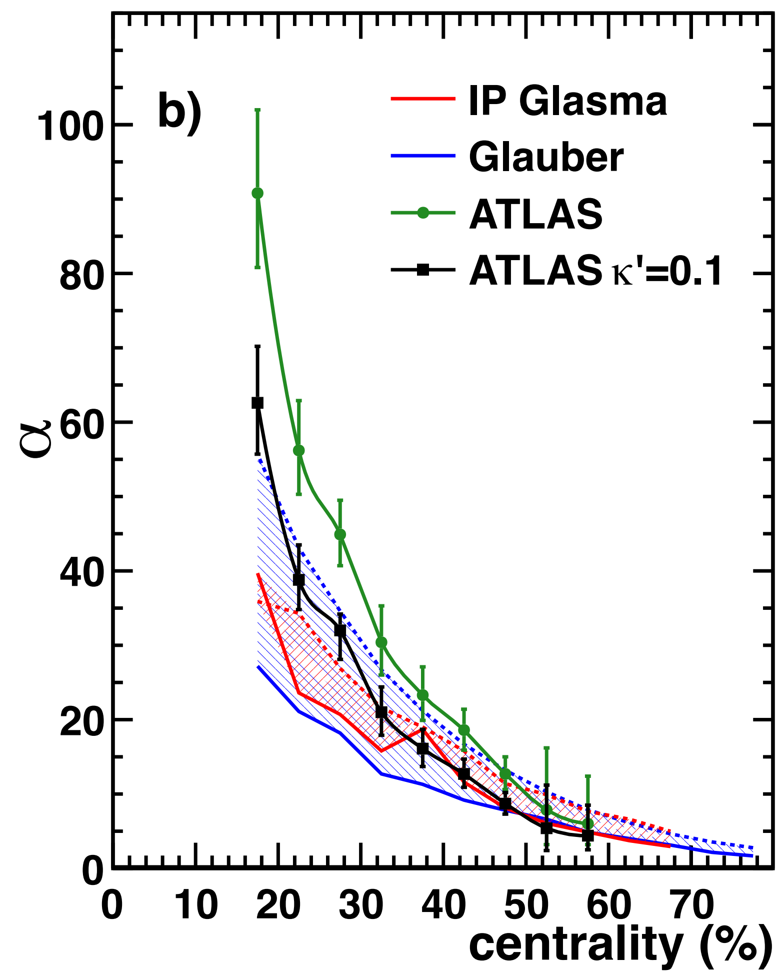


κ’ affects the normalization
but not the slope

$\Delta\kappa/\kappa \cong -1.7\kappa'$

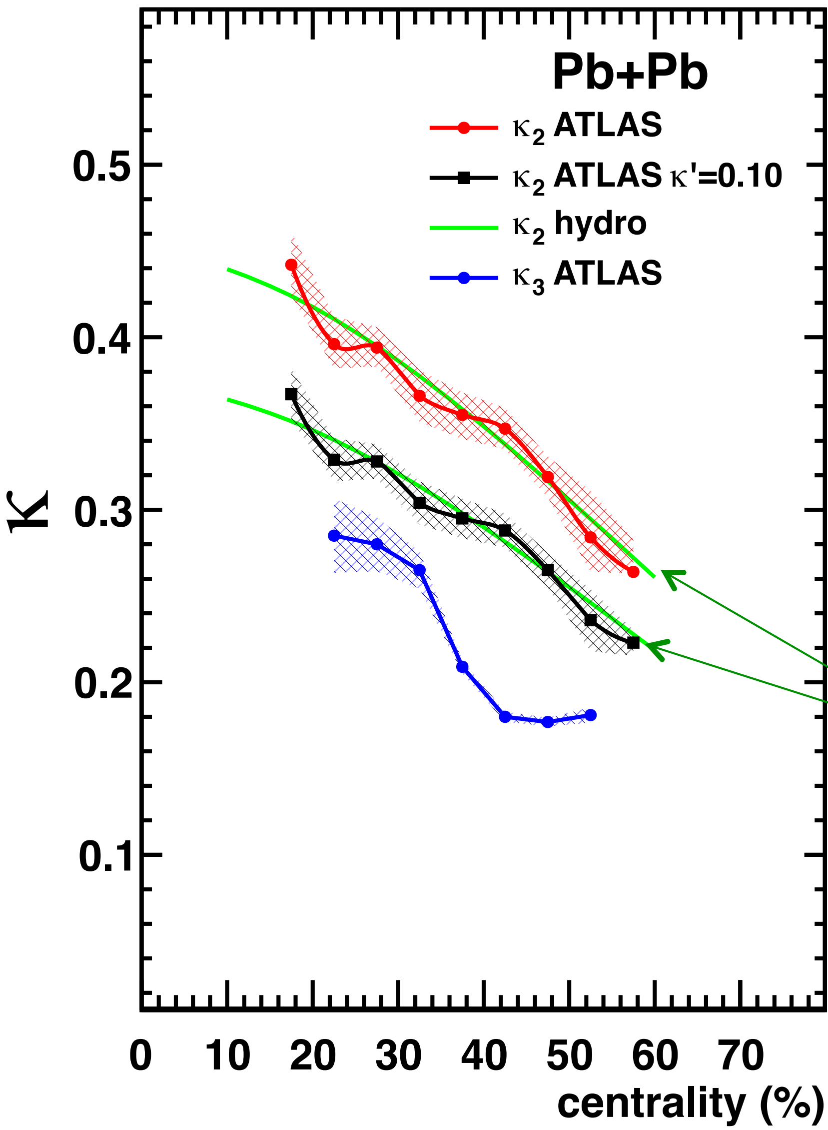
ε_0 and α





Non-linearity gives agreement with initial state models

κ Hydro Response Parameter



Assuming κ' does not vary with centrality

viscous hydro calc. of v/ε
for η/s=0.19

Summary

- Rescaled Power and Elliptic Power give good fits
- Non-Gaussian shape allows determination of parameters
- Bigger effect for peripheral collisions
  - More fluctuations to larger anisotropies
- Bigger effect for smaller systems
  - Should study lighter collision systems
- Fits to viscous hydro indicate low value of $\eta/s = 0.19$

Acknowledgments

- Sergei Voloshin for the name Elliptic Power and valuable comments on both papers
- Matt Luzum for pushing the cumulants

- Elliptic Power Distribution (Simulations)
  - PRC 90, 024903 (2014) (arXiv:1405.6595)
- QM2014
  - Nucl. Phys. A, arXiv:1408.0709
- ATLAS and CMS Data analyzed
  - arXiv:1408.0921, ver 2

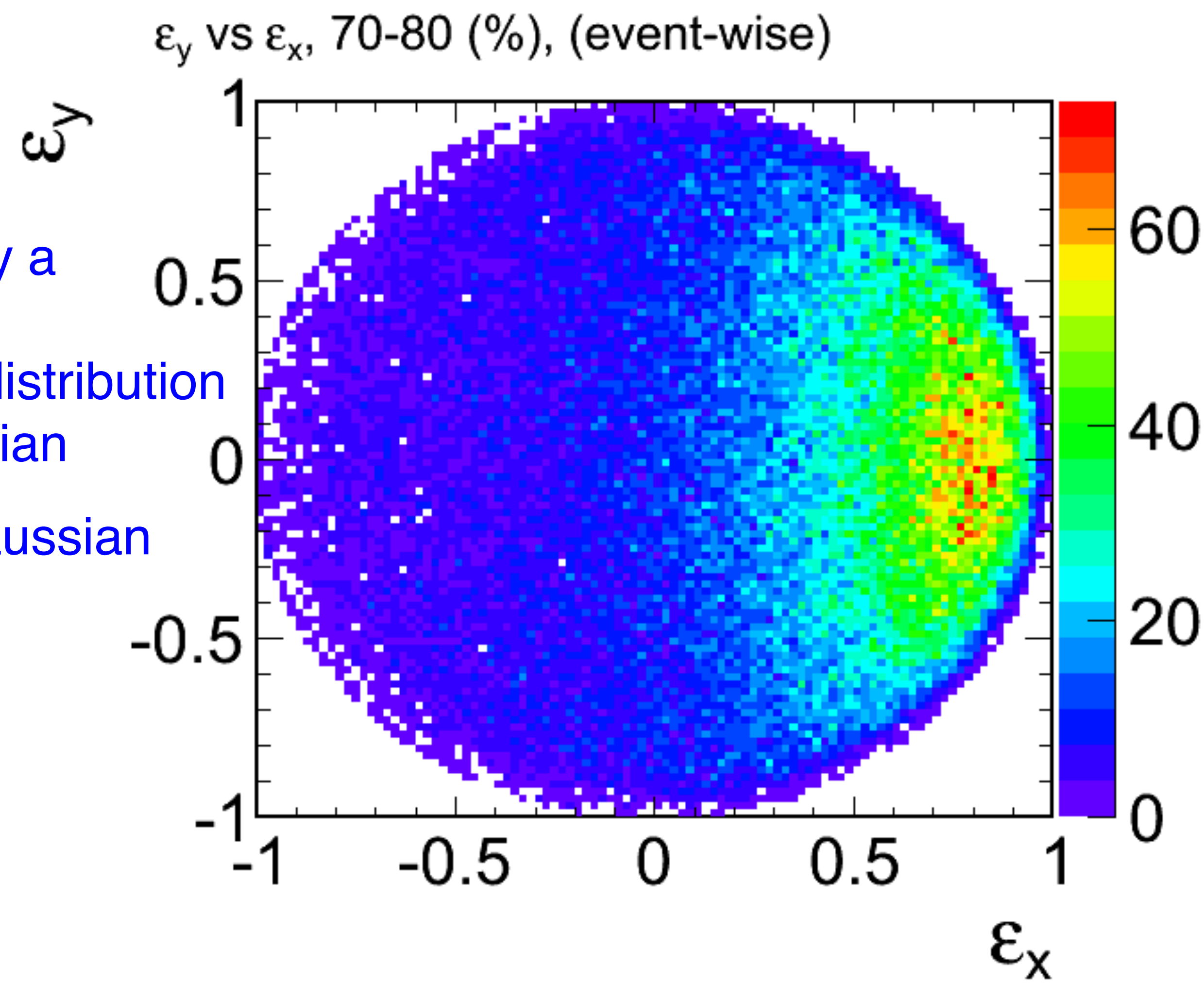
Backup

Monte-Carlo Glauber in x,y space

If (ε_x, ε_y) is described by a
2D Gaussian,
then Participant Plane distribution
would be Bessel-Gaussian

But definitely not 2D Gaussian

$\varepsilon_{part} < 1$



Gaussian Noise

$$v_n = \kappa_n \varepsilon_n + X_n$$

X_n is 2D Gaussian noise uncorrelated with ε_n

$$v_n\{2\}^2 = \kappa_n^2 \varepsilon_n\{2\}^2 + \langle|X_n|^2\rangle$$

$$v_n\{4\} = \kappa_n \varepsilon_n\{4\}$$

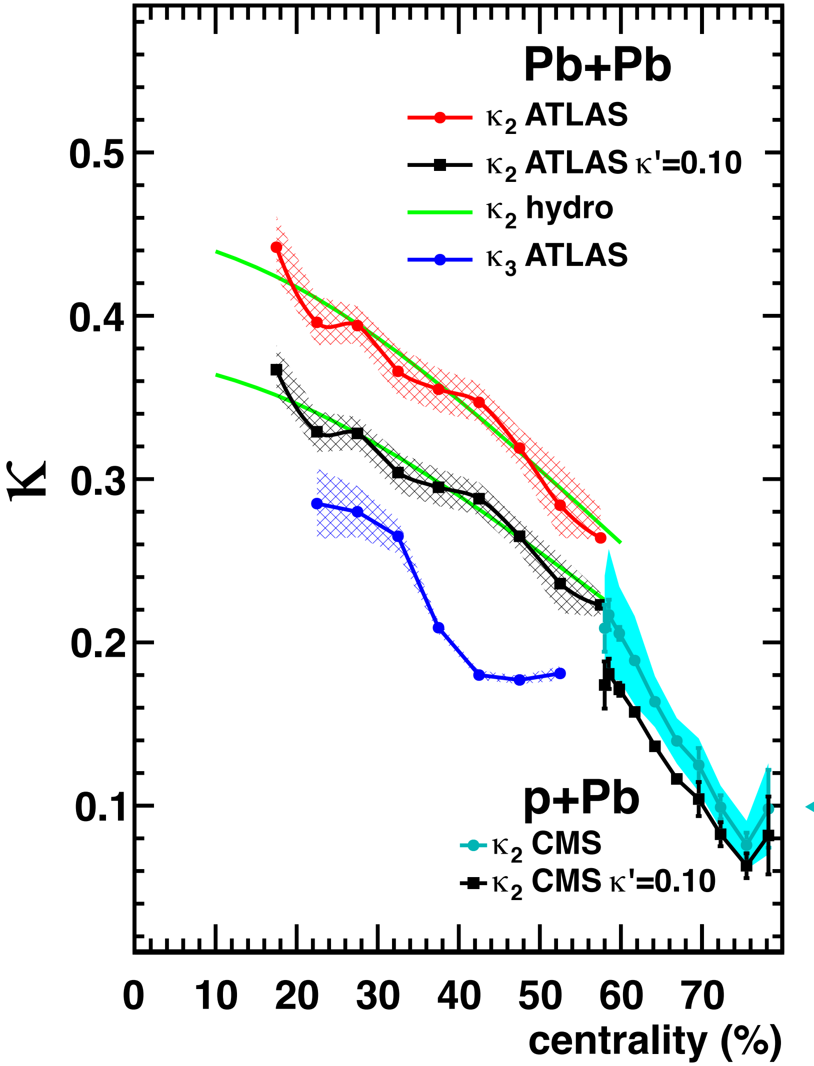
$$v_n\{6\} = \kappa_n \varepsilon_n\{6\}$$

Reduces κ_n by constant factor independent of centrality

p + A

- Probably no almond shaped overlap region
- But lots of fluctuations
  - Especially for large impact parameters
- Claims of “Flow” in p + A
- Large $v_2\{4\}$ is a non-Gaussian effect
- No v_2 distributions
  - Must use cumulants

CMS p + Pb



Just non-Gaussian flow fluctuations!

- CMS $\sqrt{s}$ 5.02 TeV
- PLB 724, 213 (2013)
- Peripheral subtraction
- From cumulants
- Equivalent centrality
  - Based on N_{tracks}

κ from Cumulants of Power Distribution

assume $\varepsilon_0 = 0$ (Power Distribution)

$$\varepsilon_n\{4\}/\varepsilon_n\{2\} = (1 + \alpha/2)^{-1/4}$$

$$\varepsilon_n\{2\} = 1/\sqrt{1+\alpha}$$

assume $v_n = \kappa_n \varepsilon_n$ (linear)

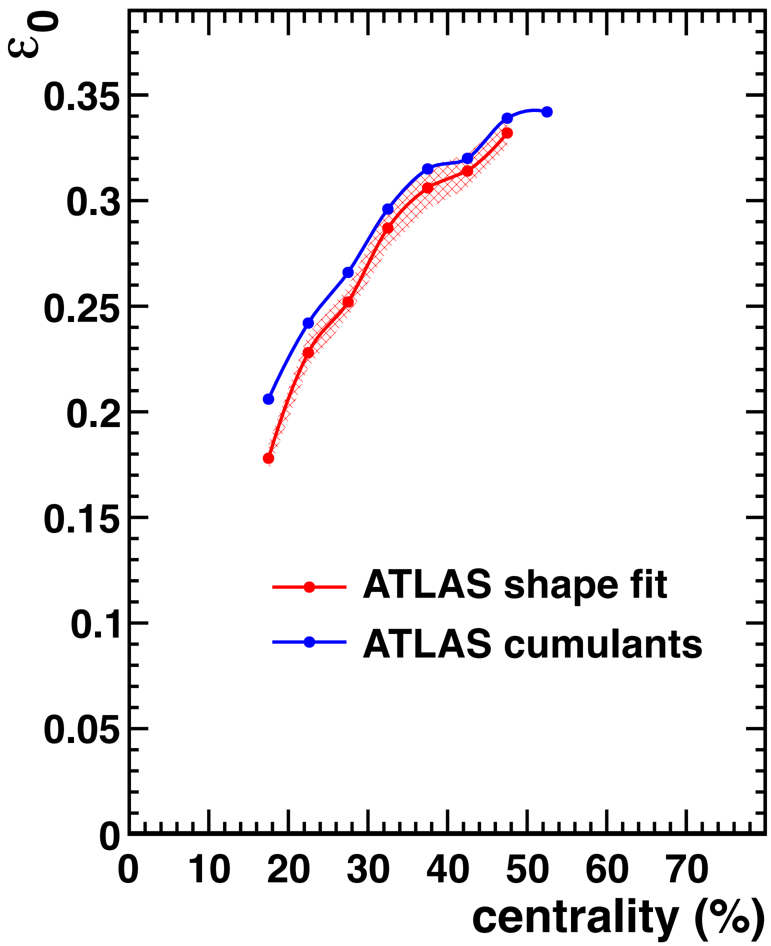
$$\kappa_n = v_n\{2\}\sqrt{2\left(\frac{v_n\{2\}}{v_n\{4\}}\right)^4 - 1}$$

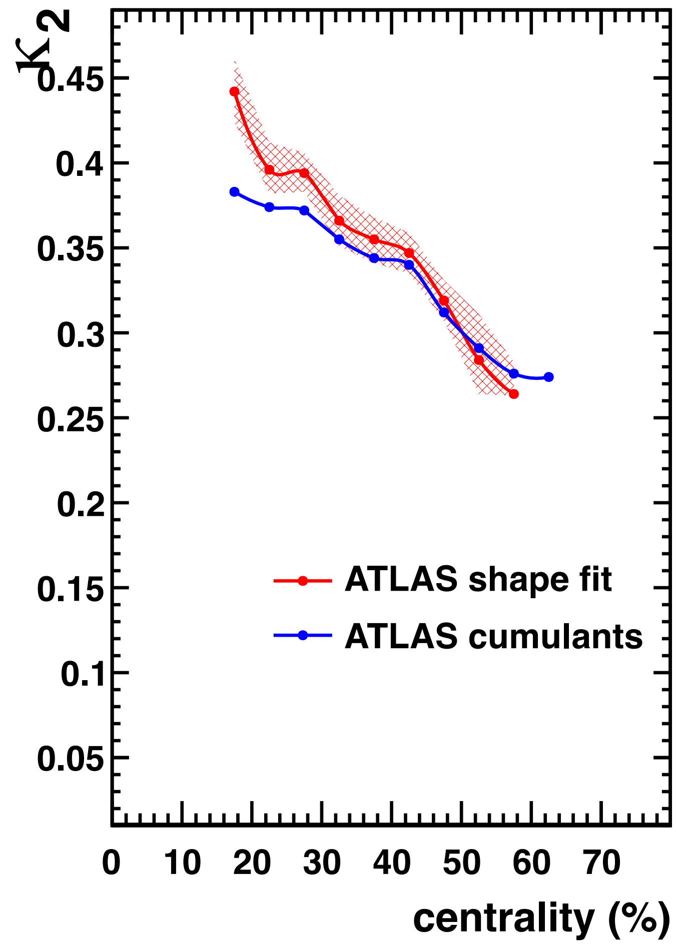
The cumulant ratio gives the non-Gaussian shape

Independent of the ε distribution!

Parameters from ATLAS Cumulants

2, 4, and 6 particle cumulants





Shape fits and cumulants give the same result

No errors for cumulants

Cumulants of Elliptic Power

$$f_k \equiv \langle (1-\varepsilon_n^2)^k \rangle = \frac{\alpha}{\alpha + k} \left(1 - {\varepsilon_0}^2\right)^k {}_2F_1\left(k + \frac{1}{2}, k; \alpha + k + 1; {\varepsilon_0}^2\right)$$

$$\varepsilon_n\{2\} = (1 - f_1)^{1/2}$$

$$\varepsilon_n\{4\} = (1 - 2f_1 + 2f_1^2 - f_2)^{1/4}$$

$$\varepsilon_n\{6\} = \left(1 + \frac{9}{2} f_1^2 - 3f_1^3 + 3f_1(\frac{3}{4} f_2 - 1) - \frac{3}{2} f_2 - \frac{1}{4} f_3\right)^{1/6}$$

need

k above is the order

Data does not need unfolding

but now 3 parameters

2 ratio equations in 2 unknowns

Then scale by kappa for hydro response

$v_n\{2\}$ may have non-flow (need large η-gap)